PROBABILISTIC RESPONSE AND RELIABILITY EVALUATION OF NONLINEAR STRUCTURES UNDER EARTHQUAKE

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ABSTRACT:

The first part of the paper introduces an orthogonal expansion method for earthquake ground motion. In the method, seismic acceleration process is represented as a linear combination of deterministic functions modulated by 10 uncorrelated random variables. In the second part of the paper, the recently developed probability density evolution method (PDEM) is employed to study nonlinear random response of structures which are subjected to the external excitations. In the PDEM, a completely uncoupled one-dimensional governing partial differential equation, the generalized density evolution equation, is derived first with regard to evolutionary probability density function of the stochastic response for nonlinear structures. The solution of this equation can put out the instantaneous probability density function. So it is natural to combine the PDEM and the orthogonal expansion of seismic ground motion to study the nonlinear random earthquake response. Furthermore, the aseismatic reliability of structures is assessed using the idea of equivalent extreme-value, which can be used accurately to evaluate structural systems under compound failure criterion. Finally, an example, which deals with a nonlinear frame structure subjected to ground motions, is illustrated to validate the proposed method.

KEYWORDS: earthquake ground motion, nonlinear structures, random response, reliability, orthogonal expansion, probability density evolution method

1. INTRODUCTION

In the past decades, the dynamic response analysis methods of engineering structures and the random vibration theory, including dynamic reliability assessment approach, have been extensively developed. As far as the second-order statistics of the responses are concerned, the statistical approaches, e.g. the Monte Carlo simulation method (MCM), and the non-statistical approaches, such as the random perturbation method and the orthogonal polynomials expansion method, etc., have been extensively studied (Kleiber and Hien, 1992; Ghanem and Spanos, 1991; Li, 1996). However, the structures in service usually exhibit nonlinear, say, in strong earthquakes. Notwithstanding the paramount importance of analysis of nonlinear stochastic structures, quite insufficient studies, where the MCM, the equivalent linearization method and the random perturbation method have been investigated, were carried out. As a result, only preliminary results have been gained so far.

In recent years, a family of probability density evolution method (PDEM), which is capable of capturing the instantaneous probability density evolution (PDF) and its evolution of the response of structures involving random parameters, has been developed and used successfully in linear and nonlinear dynamical systems (Li and Chen, 2004; Chen and Li, 2005; Li and Chen, 2006). In the present paper, the PDEM is employed as a basis. Using the approach for evaluation of the extreme-value distribution of a set of random variables and /or a stochastic process and the idea of equivalent extreme-value event (Li et al, 2007), the structural system reliability could be evaluated requiring neither the joint PDF of the response and its velocity, nor the assumptions on properties of the level-crossing events. An example which nonlinear structures subjected to earthquake excitations is studied to exemplify and validate the proposed approach.

2. ORTHOGONAL EXPANSION MODEL OF EARTHQUAKE GROUND MOTION

A proper definition of the design ground motion time history is a very important concern in structural earthquake engineering. In order to account for local site properties and a dominant frequency in the ground motion, stationary nonwhite process models were suggested by Kanai (1957) and Tajimi (1960), namely, the well-known Kanai-Tajimi (K-T) acceleration power spectrum. At the same time, it is noted that the K-T acceleration power spectrum has a finite amplitude at the zero frequency. Thus, strong singularities are present at the zero frequency. For the power spectral density (PSD) function of the ground acceleration process, we consider a modified version of the K-T acceleration power spectrum model suggested by Clough and Penzien (1975), the Clough-Penzien (C-P) acceleration power spectrum is expressed as

$$S_{\ddot{x}_g}(\omega)=\frac{\omega_g^4+4\zeta_g^2\omega_g^2\omega^2}{(\omega^2-\omega_g^2)^2+4\zeta_g^2\omega_g^2\omega^2}\cdot\frac{\omega^4}{(\omega^2-\omega_f^2)^2+4\zeta_f^2\omega_f^2\omega^2}S_0 \tag{2.1}$$

where $S_{\ddot{x}_g}(\omega)$ is a single-side power spectral density function of earthquake acceleration; S_0 is the spectral intensity factor; ω_g and ζ_g are the filter parameters of the well-known K-T power spectrum model representing, respectively, the dominant frequency and critical damping of the soil layer; ω_f and ζ_f are parameters of a second filter which is introduced to assure a finite power for the ground displacement. For $\omega_f \ll \omega_g$, the second filter influences only the region of very low frequencies. The filter parameter values suggested by Deodatis (1996) for medium soil condition will be used in this study for demonstration purposes

$$\omega_g=5\pi\ \text{rad/s},\quad \zeta_g=0.60,\quad \omega_f=0.5\pi\ \text{rad/s},\quad \zeta_f=0.60 \tag{2.2}$$

According to random vibration theory, the spectral intensity factor S_0 may be expressed as

$$S_0=\frac{\bar{a}_{\max}^2}{v^2\omega_e} \tag{2.3}$$

where $\bar{a}_{\max}$ is the peak ground acceleration (PGA) value ; v is the peak factor. The value ω_e is given by

$$\omega_e=\int_0^{\infty}S_{\ddot{x}_g}(\omega)/S_0\mathrm{d}\omega \tag{2.4}$$

If the PGA value is specified, then S_0 is not a random variable. Thus, we can treat the spectral intensity factor S_0 as a determinate variable. The values of $\bar{a}_{\max}$ and v for medium soil condition are given in this study as

$$\bar{a}_{\max}=0.2g=196\ \text{cm/s}^2,\qquad v=3.1 \tag{2.5}$$

So, the spectral intensity factor S_0 may be solved from Eqns. (2.3)-(2.5), i.e. $S_0=81.15\ \text{cm}^2/\text{s}^3$.

Based on the C-P acceleration power spectrum, the orthogonal expansion model for earthquake ground motion was recently proposed as (Li and Liu, 2006; Liu and Li, 2008)

$$\ddot{x}_g(\mathbf{\Theta},t)=\sqrt{2S_0}\sum_{j=1}^{r}\sqrt{\lambda_j}\Theta_j f_j(t) \tag{2.6}$$

$$f_j(t)=-\sum_{m=1}^{N-1}\left(\frac{2\pi m}{T_s}\right)^2\eta_{m+1}\varphi_{j,m+1}\phi_m(t) \tag{2.7}$$

where r is the number of truncated terms (r = 10); N is the number of expanded terms (N =501); λ_j and $\varphi_{j,m+1}$ are the eigenvalues and the (m+1)th component of corresponding orthonormal eigenvectors $\mathbf{\Psi}_j$ of the seismic displacement correlation matrix, respectively; T_s is 90% energy duration or stationary duration of the ground motion, the value suggested by Seya et al (1993) for medium soil condition is 20 sec; η_{m+1} are defined as harmonic modulated coefficients; random variables Θ_j ($j=1,2,\cdots,r$) are mutually independent standard Gaussian variables; $\phi_m(t)$ is the Hartley orthogonal series, i.e. $\phi_m(t)=\frac{1}{\sqrt{T_s}}\mathrm{cas}(\frac{2\pi mt}{T_s})$, m = 0, 1,⋯, (N-1).

3. PDEM-BASED SEISMIC RESPONSE ANALYSIS OF NONLINEAR STRUCTURES

Without loss of generality, the governing equation of a nonlinear MDOF structural system under earthquake excitations is

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{F}(\mathbf{X})=-\mathbf{MI}\ddot{x}_g(\mathbf{\Theta},t) \tag{3.1}$$

with the deterministic initial condition

$$\mathbf{X}(t)\big|_{t=0}=\mathbf{x}_0,\quad \dot{\mathbf{X}}(t)\big|_{t=0}=\dot{\mathbf{x}}_0 \tag{3.2}$$

where $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$, $\mathbf{X}$ are the $n\times 1$ acceleration, velocity and displacement vector with the over dot denoting the derivation with regard to time; $\mathbf{M}$, $\mathbf{C}$ are $n\times n$ mass and damping matrix, respectively; $\mathbf{F}$ is the restoring force vector; $\mathbf{I}$ is the $n\times 1$ unit column vector; $\ddot{x}_g$ is the stochastic ground acceleration excitation, i.e. Eqns. (2.6) and (2.7); $\mathbf{\Theta}$ is the $r\times 1$ random vector with known probability density function (PDF) $p_{\mathbf{\Theta}}(\boldsymbol{\theta})$ where $\mathbf{\Theta}=(\Theta_1,\Theta_2,\cdots,\Theta_r)$.

Eqns. (3.1) and (3.2) can be rewritten as a stochastic state equation, i.e.

$$\dot{\mathbf{Y}}=\mathbf{A}(\mathbf{Y},\mathbf{\Theta},t) \tag{3.3}$$

$$\mathbf{Y}(t)\big|_{t=0}=\mathbf{y}_0 \tag{3.4}$$

where

$$\mathbf{Y}=\begin{bmatrix}\mathbf{X}\\ \dot{\mathbf{X}}\end{bmatrix},\quad \mathbf{A}=\begin{bmatrix}\dot{\mathbf{X}}\\ -\mathbf{M}^{-1}\mathbf{C}\dot{\mathbf{X}}-\mathbf{M}^{-1}\mathbf{F}-\mathbf{I}\ddot{x}_g\end{bmatrix} \tag{3.5}$$

For a well-posed dynamics problem, the solution to the system (3.3) exists, is unique and must be a function of $\mathbf{\Theta}$. It is convenient to assume the solution takes the form

$$\mathbf{Y}=\mathbf{H}(\mathbf{\Theta},t) \tag{3.6}$$

Likewise, the velocity of $\mathbf{Y}$ also exists and is a function of $\mathbf{\Theta}$, and could be assumed to take the form

$$\dot{\mathbf{Y}}=\mathbf{h}(\mathbf{\Theta},t) \tag{3.7}$$

Obviously, according to Eqns. (3.6) and (3.7), it follows that $\mathbf{h}(\mathbf{\Theta},t)=\partial\mathbf{H}(\mathbf{\Theta},t)/\partial t$.

The components of Eqns. (3.6), (3.7) read

$$Y_l=H_l(\mathbf{\Theta},t) \tag{3.8}$$

$$\dot{Y}_l=h_l(\mathbf{\Theta},t) \tag{3.9}$$

where Y_l, H_l, h_l (l=1,2, …,2n) are the lth component of **Y**, **H** and **h**, respectively. For simplicity of writing, the subscript l will be omitted hereafter without inducing confusions.

According to the derivation (Li and Chen, 2004; Chen and Li, 2005; Li and Chen, 2006), the joint PDF of the augmented state vector $(Y,\mathbf{\Theta})$ satisfies the governing partial differential equation

$$\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial t}+h(\boldsymbol{\theta},t)\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial y}=0 \tag{3.10}$$

with the initial condition

$$p_{Y\Theta}(y,\boldsymbol{\theta},t)\big|_{t=0}=\delta(y-y_0)\,p_{\Theta}(\boldsymbol{\theta}) \tag{3.11}$$

where y_0 is the component of $\mathbf{y}_0$ as a deterministic value.

The PDF of $Y(t)$ will then be given by

$$p_Y(y,t)=\int_{\Omega_\Theta} p_{Y\Theta}(y,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \tag{3.12}$$

where Ω_Θ is the distribution domain of $\boldsymbol{\Theta}$.

4. EVALUATION OF THE STRUCTURAL SYSTEM RELIABILITY

If the reliability of a structure is defined as the probability of a compound random event as combination of more than one random event, evaluation of the so-called system reliability is encountered. Using the idea of equivalent extreme-value event (Li et al, 2007), the system reliability could be evaluated through computing the probability of an equivalent extreme-value event, leading to one-dimensional integration of the PDF of the equivalent extreme-value random variable.

For the first-passage problem, the dynamic reliability against the response index $Y(t)$ reads

$$R=\Pr\{Y(\boldsymbol{\Theta},t)\in\Omega_{\mathrm{s}},t\in[0,T]\} \tag{4.1}$$

where Ω_{s} is the safe domain. In general cases, it is easy to rewrite Eqn. (4.1) into

$$R=\Pr\{G(\boldsymbol{\Theta},t)>0,t\in[0,T]\} \tag{4.2}$$

Here $G(\cdot)$ is a time dependent limit state function. For instance, if Eqn. (4.1) takes the form (as a double boundary condition):

$$R=\Pr\{|Y(\boldsymbol{\Theta},t)|<y_{\mathrm{b}},t\in[0,T]\} \tag{4.3}$$

where y_{b} is the boundary, then one can get

$$G(\boldsymbol{\Theta},t)=y_{\mathrm{b}}-|Y(\boldsymbol{\Theta},t)| \tag{4.4}$$

Eqn. (4.2) could also be written equivalently in a different form as

$$R=\Pr\{\bigcap_{t\in[0,T]}(G(\boldsymbol{\Theta},t)>0)\} \tag{4.5}$$

According to the idea of equivalent extreme-value event (Li et al, 2007), if one defines an extreme value as

$$W_{\min}=\min_{t\in[0,T]}(G(\boldsymbol{\Theta},t)) \tag{4.6}$$

where PDF can be captured through the PDEM, then the reliability in Eqn. (4.5) equals

$$R=\Pr\{W_{\min}>0\}=\int_0^{+\infty}p_{W_{\min}}(W)\mathrm{d}W \tag{4.7}$$

It is worth pointing out that if one wants to evaluate the reliability in Eqn. (4.5) directly with the probability integration, the infinite-dimensional joint PDF of the stochastic process $G(\boldsymbol{\Theta},t)$ is needed, i.e., the correlation information among any different time instants is required. However, using the equivalent extreme-value event, total information of the correlation is inherent and exact solution can be derived easily.

Likewise, if there is more than one limit state function combined together in the dynamic reliability evaluation, say,

$$R = \Pr\{\bigcap_{j=1}^{m} (G_j(\mathbf{\Theta}, t) > 0, t \in [0, T_j])\} \tag{4.8}$$

where T_j is the time duration corresponding to $G_j(\mathbf{\Theta}, t)$. According to the idea of equivalent extreme-value event (Li et al, 2007), one can define the equivalent extreme value as

$$W_{\text{ext}} = \min_{1 \leqslant j \leqslant m} \left(\min_{t \in [0, T_j]} (G_j(\mathbf{\Theta}, t)) \right) \tag{4.9}$$

So, the reliability in Eqn. (4.8) can be computed directly by

$$R = \Pr\{W_{\text{ext}} > 0\} \tag{4.10}$$

5. NUMERICAL IMPLEMENTATION

As discussed above, the random response and reliability evaluation of nonlinear structures could be easily implemented through the numerical solution. The steps include:

(i) Select representative point set in the domain $\Omega_{\mathbf{\Theta}}$ and denote it as $\mathbf{\theta}_q = (\theta_{q,1}, \theta_{q,2}, \cdots, \theta_{q,s})$ (q = 1,2, … , N_{sec}), where N_{sec} is the cardinal number of the point set. Simultaneously, determine the corresponding assigned probability P_q.

(ii) Let $\mathbf{\Theta} = \mathbf{\theta}_q$ and carry out dynamic response analysis on Eqn. (3.1) (or Eqn. (3.3)) to obtain the time-variant velocity $h(\mathbf{\theta}_q, t)$ and the value of the equivalent extreme value $W_{\min}(\mathbf{\theta}_q, T)$ (see Eqn. (4.6)) or $W_{\text{ext}}(\mathbf{\theta}_q, T)$ (see Eqn. (4.9)).

(iii) Introduce $h(\mathbf{\theta}_q, t)$ into Eqn. (3.10) and solve it under the initial condition (3.11).

(iv) Iterate Steps (ii) and (iii) running over all q's and obtain the PDF of $Y(t)$ by

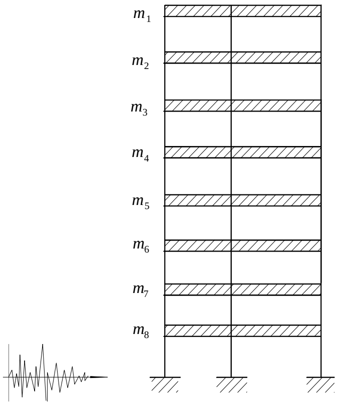
$$p_Y(y, t) = \sum_{q=1}^{N_{\text{sel}}} p_{Y\mathbf{\Theta}}(y, \mathbf{\theta}_q, t) \tag{5.1}$$

(v) Using PDEM to get the extreme-value distribution and compute the reliability with the one-dimensional numerical integration (see Eqn. (4.7) or (4.10)).

In step (i), the strategy of selecting representative points in $\Omega_{\mathbf{\Theta}}$ needs special techniques, the Number-Theoretical-Method-based algorithm has been developed by Li and Chen (2007). In step (iii), the TVD schemes are appropriate in dynamic response analysis while the one-sided difference scheme is preferred in evaluation of the extreme value distribution and the reliability.

6. NUMERICAL EXAMPLE

In order to verify and validate the proposed approach, an 8-storey frame structure shown in Figure 1 is investigated. The masses of each story, m_1- m_8, are, respectively, 0.6, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.2 ($\times 10^5$ kg). The heights h_1=4.0m, h=3.0m, section size of columns 500$\times$500 mm^2, the beams are assumed to be completely rigid. Rayleigh's damping, $\mathbf{C}$ = $a\mathbf{M}$ + $b\mathbf{K}_t$, where a = 0.01, b=0.005, $\mathbf{K}_t$ is the tangent stiffness matrix, is employed. The Young's modulus E of the column is 3.0$\times 10^{10}$ P$_a$. The restoring force relationship is the bilinear hysteretic force model shown in Figure 2 with Δ_x denoting the initial yielding displacement (Δ_x = 6mm), and $\alpha = K_1/K_0 = 0.1$.



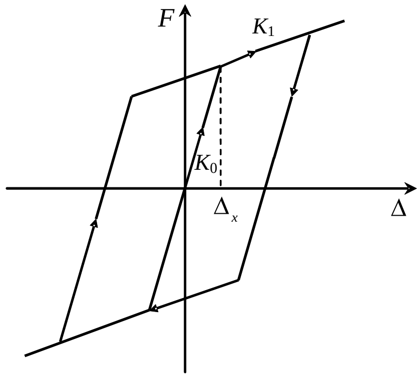


Figure 1 Structural model   Figure 2 The bilinear hysteretic restoring force model

In employing the PDEM, one will first select representative points in the random space and then use the orthogonal expansion model to generate the representative acceleration time history of ground motion. In the paper, 305 representative acceleration time history of ground motion from medium soil are picked. The probabilistic information, including the typical PDFs at certain time instants, the PDF evolution surface and the contour of the PDF surface, of the 4th inter-story drift of the structure is depicted in Figure 3.

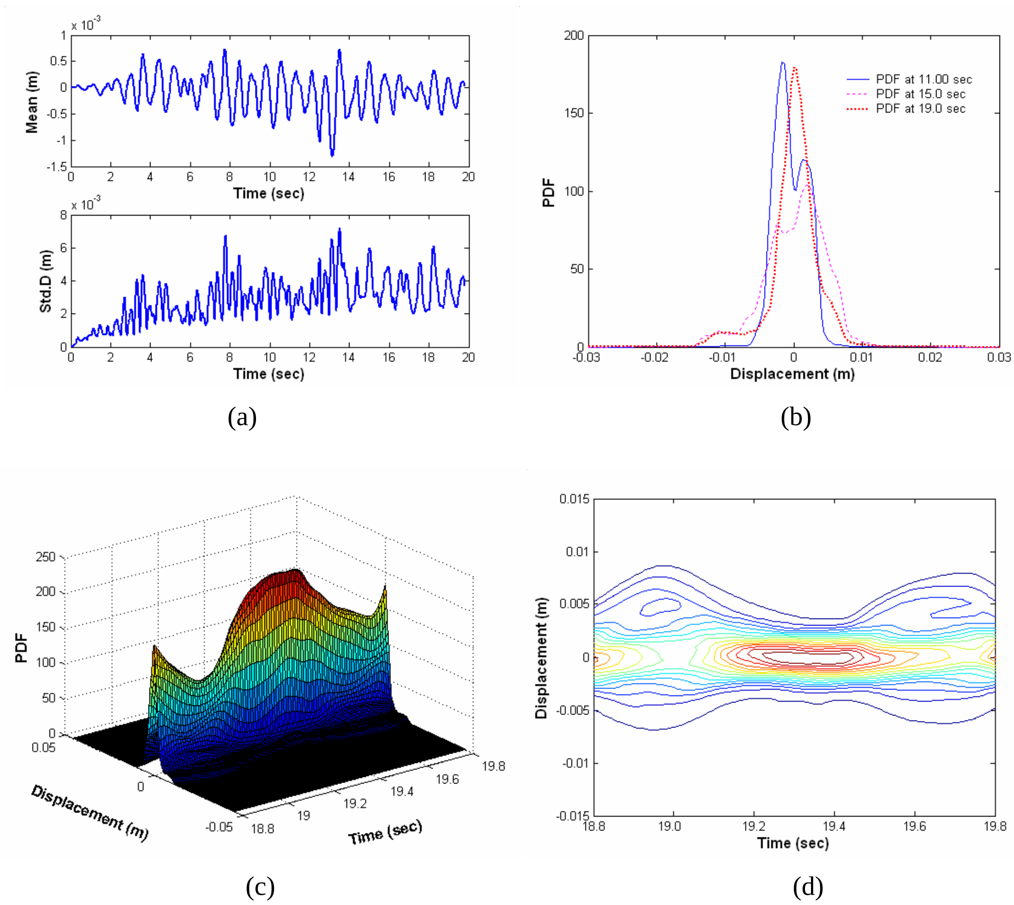


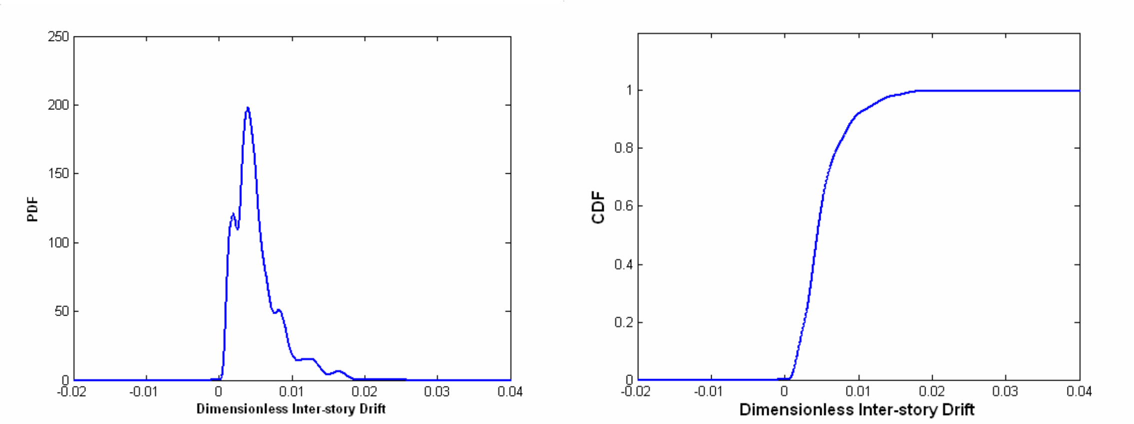
Figure 3 The probabilistic information of the response for the story drift of 4th floor: (a) the mean and the standard deviation, (b) typical instantaneous PDFs at certain instants of time, (c) evolution of PDF against time: the PDF surface and (d) the contour to the PDF surface.

The steps elaborated in the above section could be implemented to carry out the dynamic reliability evaluation. The probabilities of failure and reliability of the structure over time interval [0, 20] sec against inter-story drift are listed in Table 1. Pictured in figure 4 are the PDF and cumulated probability distribution function (CDF) of the equivalent extreme values of the structure system.

Table 1 the failure probability and reliability of the structure against inter-story drifts

Number of story	Probability of failure	Reliability
8th	0.0000	1.0000
7th	0.0000	1.0000
6th	0.0000	1.0000
5th	0.0000	1.0000
4th	0.0003	0.9997
3rd	0.0133	0.9867
2nd	0.0398	0.9602
1st	0.0556	0.9444
The structure system	0.0796	0.9204

(note: the threshold of dimensionless inter-story drift is 1/100)



(a) (b)

Figure 4 the equivalent extreme value of the dimensionless inter-story drift for the structure system: (a) the PDF and (b) the CDF.

7. CONCLUSIONS

In the present paper, the probability density evolution method and the idea of equivalent extreme-value event are applied to study dynamic response and reliability evaluation of nonlinear structures. This makes it possible to transform computation of the probability of the compound random event, usually leading to multi-dimensional integration of the joint PDF, to a one-dimensional integration of the PDF of the equivalent extreme value. The approach avoids the complicated computations in traditional structural system reliability assessment, and more importantly, avoids the difficulty in dealing with the correlation among the component random events. An example, where nonlinear 8-story frame structure under earthquake excitations, is studied to exemplify and validate the proposed approach.

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REFERENCES

Chen, J.B. and Li, J. (2005). Dynamic response and reliability analysis of non-linear stochastic structures. *Probabilistic Engineering Mechanics* **20:1**, 33-44.

Clough, R.W. and Penzien, J. (1975). Dynamics of structures, McGraw-Hill, New York.

Deodatis, G. (1996). Non-stationary stochastic vector processes: seismic ground motion applications. *Probabilistic Engineering Mechanics* **11:3**, 149-168.

Ghanem, R. and Spanos, P.D. (1991). Stochastic finite element: a spectral approach, Springer, Berlin.

Kanai, K. (1957). Semi-empirical formula for the seismic characteristics of the ground, Bulletin of the Earthquake Research Institute, University of Tokyo, Japan, **35**: 309-325.

Kleiber, M. and Hien, T.D. (1992). The stochastic finite element method: basic perturbation technique and computer implementation, Wiley, Chishcester.

Li, J. (1996). Stochastic structural systems: analysis and modeling, Science Press, Beijing, China.

Li, J. and Chen, J.B. (2004). Probability density evolution method for dynamic response analysis of structures with uncertain parameters. *Computational Mechanics* **34:5**, 400-409.

Li, J. and Chen, J.B. (2006). The probability density evolution method for dynamic response analysis of non-linear stochastic structures. *International Journal for Numerical Methods in Engineering* **65:6**, 882-903.

Li, J. and Chen, J.B. (2007). The number theoretical method in response analysis of nonlinear stochastic structures. *Computational Mechanics* **39:6**, 693-708.

Li, J., Chen, J.B. and Fan, W.L. (2007). The equivalent extreme-value event and evaluation of the structural system reliability. *Structural Safety* **29:2**, 112-131.

Li, J. and Liu, Z.J. (2006). Expansion method of stochastic processes based on normalized orthogonal bases. *Journal of Tongji University* (*Natural Science*) **34:10**, 1279-1283.

Liu, Z.J. and Li, J. (2008). Orthogonal expansion of stochastic processes for earthquake ground motion. *Journal of Tongji University* (*Natural Science*) **36:9**.

Seya, H., Talbott, M.E. and Hwang, H.H.M. (1993). Probabilistic seismic analysis of a steel frame structure. *Probabilistic Engineering Mechanics* **8:2**, 127-136.

Tajimi, H. (1960). A statistical method of determining the maximum response of a building structure during an earthquake. *Proceedings of the Second World Conference on Earthquake Engineering*, Tokyo and Kyoto, **2**, 781 -798.