ANALYSIS OF SEISMIC RESPONSE AND CATASTROPHIC BEHAVIOR ON SUSPENDED STRUCTURE SYSTEMS

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ABSTRACT:

To consider the influence of substructure's nonlinear driving force on primary structure, an improved dynamic analytic model containing dynamic stiffness is presented, and it is solved by means of Lindstedt-Poincaré method (L-P). Analysis on amplitude versus frequency based on arithmetic solution and catastrophe theory shows that small changes of system parameters may cause remarkable changes of amplitude, amplitude catastrophic value and unstable region of system response. It means that the response behaviors of system are very sensitive to the variation of system parameters. The longer suspender the smaller nonlinear parameter, and catastrophic behavior of system disappears and pseudo-linear behavior arises.

KEYWORDS: suspended structure, nonlinear restoring force, catastrophe, dynamic response, pseudo-linear behavior

1. INTRODUCTION

To reduce the seismic response of structures, the masses are suspended on one or more floors, or the whole stories of structure are suspended. As a new structural system with quality damping performance, suspended structures have been concerned increasingly by construction engineering. Theory analysis and practice show that the sub structure and hinge bar respond as gravity pendulum around the suspension centre, when they are excited by exciting force. The restoring force of pendulum depends on gravity stiffness of it. Some complex dynamic behaviors of suspended structures excited by earthquake, such as catastrophe and hysteresis, are caused by nonlinear action [1-4]. The existing method to study suspended structure is to simplify it as gravity pendulum and elastic staff model without considering nonlinear restoring force of sub structure, and cannot describe some nonlinear behavior above [5,6]. Therefore, the existing dynamic model is improved by including nonlinear restoring force in this paper. The stable solution of dynamic response of suspended structure is derived by means of Lindstedt-Poincaré method (L-P), and then the calculation results of amplitude-frequency response and its catastrophic complexity analysis are presented. Analysis shows that small changes of system parameters may cause remarkable changes of amplitude catastrophic value. It means that the behaviors of system response are very sensitive to the changes of system parameters. On the other hand, the catastrophic behavior disappears and the pseudo-linear behavior arises in the case of longer hinge bar. The results obtained in the paper are of significant value to the design of suspended structures.

2. DYNAMIC MODEL OF SUSPENDED STRUCTURE

Suspended structures consist of mega structure, sub structure and suspender, depicted in figure 1. The mass of mega structure is concentrated to the beam, and the beam is simplified as translational mass rigid body. Sub structure is simplified as gravity pendulum and the pendulum length equal to suspender, depicted in figure 2 (a). Based on d'alembert's principle, the dynamic equations of suspended structure system are

$$\begin{cases} m_1\ddot{x}_1 + k_1x_1 - m_2gx_2\sqrt{l^2 - x_2^2}\,/\,l^2 = -m_1\ddot{x}_g \\ m_2\ddot{x}_2 + m_2gx_2\sqrt{l^2 - x_2^2}\,/\,l^2 = -m_2\ddot{x}_g \end{cases} \quad (2.1)$$

Where x_g is the displacement of ground; x_1 the displacement of mega structure relative to ground; x_2 the displacement of sub structure relative to mega structures; k_1 lateral rigidity of mega structures; g the gravity acceleration; m_1 and m_2 the lumped mass mega and sub structures.

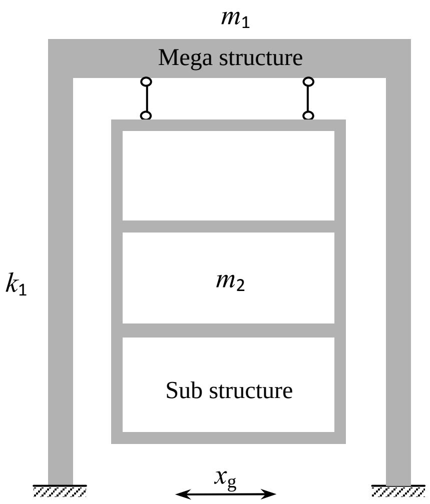


Figure 1 Schematic structure

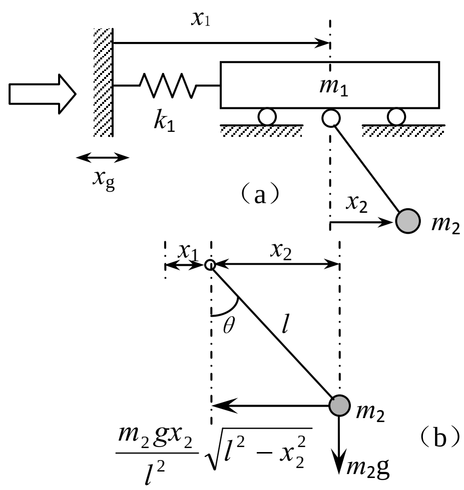


Figure 2 Mechanical model

To make pointed analysis on impact of substructure's nonlinear restoring force on dynamic behavior of structure, the second equation of Eqn.2.1 is overwritten as the following form, considering figure 2(b),

$$m_2(\ddot{x}_1 + \ddot{x}_2) + m_2gx_2\sqrt{l^2 - x_2^2}\,/\,l^2 = 0 \quad (2.2)$$

Make substitution [7,8]

$$\ddot{x}_2 + \omega_{20}^2x_2 + \varepsilon kx_2^3 = -\ddot{x}_1 \quad (2.3)$$

Where $\omega_{20}^2x_2 + \varepsilon kx_2^3 \cong gx_2\sqrt{l^2 - x_2^2}\,/\,l^2$; $\omega_{20}^2 = g/l^2$; $k = 1/l^2$; ε is a smaller parameter. Non-dimensional Eqn.2.3 is solved by means of L-P in the next step.

3. SOLUTION AND ANALYZE ON DYNAMIC EQUATION OF SUSPENDED SUBSTRUCTURE

3.1. Main Harmonic Response

Assume external excitation $-\ddot{x}_1 = \varepsilon p \cos \Omega t$, and $\tau = \Omega t$, then Eqn.2.3 can be given as the following form

$$\Omega^2 x_2'' + \omega_{20}^2 x_2 + \varepsilon k x_2^3 = \varepsilon p \cos \tau \tag{3.1}$$

Where x_2'' is the second derivative of τ .

Assume the solution of Eqn.3.1as

$$x_2 = x_{20}(\tau) + \varepsilon x_{21}(\tau) + \varepsilon^2 x_{22}(\tau) + \cdots \tag{3.2}$$

$$\Omega = \omega_{20} + \varepsilon \omega_{21} + \varepsilon^2 \omega_{22} + \cdots \tag{3.3}$$

Substitute Eqn.3.2 and Eqn.3.3 into Eqn.3.1, and make Taylor series expansion with respect to ε for $\varepsilon k x_2^3$.

In the case of sum of coefficient, the same power of ε , equal to zero

$$x_{20}'' + x_{20} = 0 \tag{3.4}$$

$$\omega_{20}^2 (x_{21}'' + x_{21}) = -k x_{20}^3 - 2\omega_{20}\omega_{21} x_{20}'' + p \cos \tau \tag{3.5}$$

The solution form of Eqn.3.4

$$x_{20} = a \cos(\tau + \theta) \tag{3.6}$$

Submitting Eqn.3.6 into Eqn.3.5

$$\omega_{20}^2 (x_{21}'' + x_{21}) = p \sin\theta \sin(\tau + \theta) + (2\omega_{20}\omega_{21} a - \frac{3}{4} k a^3 + p \cos\theta) \cos(\tau + \theta) - \frac{1}{4} k a^3 \cos 3(\tau + \theta) \tag{3.7}$$

To eliminate secular term assuming that

$$p \sin\theta = 0 \tag{3.8}$$

$$2\omega_{20}\omega_{21} a - \frac{3}{4} k a^3 + p \cos\theta = 0 \tag{3.9}$$

Eliminate unknown quality θ according to Eqn.3.8 and Eqn.3.9

$$\omega_{21} = \frac{3ka^3 \pm 4p}{8a\omega_{20}} \tag{3.10}$$

Eqn.3.10 is called as frequency-amplitude response equation. Response curve of frequency-amplitude can be derived with varied length of suspender and amplitude of exciting, depicted in Figure 3 and Figure 4.

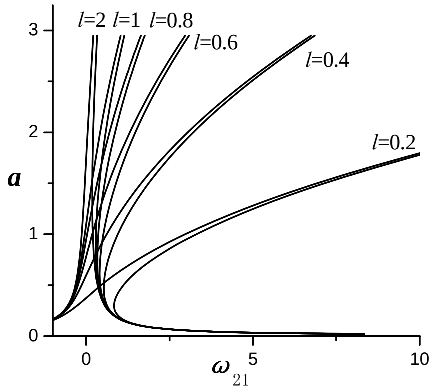


Figure 3 Frequency-amplitude curves

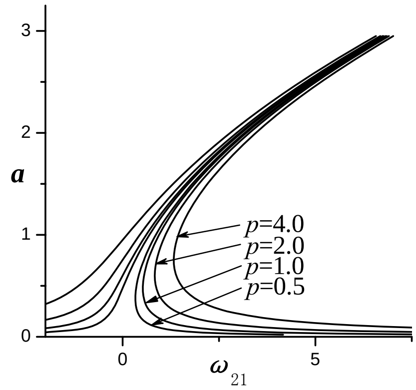


Figure 4 Frequency-amplitude curves

3.2. Ultra Harmonics Response

For the convenience of solution, make substitute $\varepsilon p = F$ into Eqn.3.1

$$\Omega^2 x_2'' + \omega_{20}^2 x_2 + \varepsilon k x_2^3 = F\cos\tau \tag{3.11}$$

Assume that

$$\Omega = \frac{1}{3}\omega_{20} + \varepsilon\omega_{21} + \varepsilon^2\omega_{22} + \cdots \tag{3.12}$$

Submitting Eqn.3.2 and Eqn.3.12 into Eqn.3.11

$$x_{20}'' + 9x_{20} = \frac{9}{\omega_{20}^2} F\cos\tau \tag{3.13}$$

$$x_{21}'' + 9x_{21} = \frac{9}{\omega_{20}^2}\left(-kx_{20}^3 - \frac{2}{3}\omega_{20}\omega_{21}x_{20}''\right) \tag{3.14}$$

The solution form of Eqn.3.13 is

$$x_{20} = a\cos[3(\tau+\theta)] + A\cos\tau \tag{3.15}$$

Where $A = \dfrac{9F}{8\omega_{20}^2}$

Submitting Eqn.3.15 into Eqn.3.14

$$
\begin{aligned}
x''_{21} + 9x_{21} = \frac{9}{\omega_{20}^2}\Big\{ & \frac{2}{3}\omega_{20}\omega_{21}\left[9a\cos 3(\tau+\theta) + A\cos\tau\right] \\
& - \frac{1}{4}ka^3\left[3\cos 3(\tau+\theta) + \cos 9(\tau+\theta)\right] \\
& - \frac{3}{4}ka^2 A\left[\cos(7\tau+6\theta) + \cos 5\tau + 6\theta + 2\cos\tau\right] \\
& - \frac{3}{4}kaA^2\left[\cos(5\tau+3\theta) + \cos(\tau+3\theta) + 2\cos 3(\tau+\theta)\right] \\
& - \frac{1}{4}kA^3\left[3\cos\tau + \cos 3\tau\right]\Big\}
\end{aligned}
\quad (3.16)
$$

Eliminate secular term and equation of frequency-amplitude can derived

$$
\left(\omega_{21} - \frac{1}{4\omega_{20}}kA^2 - \frac{1}{8\omega_{20}}ka^2\right)^2 a^2 = \frac{k^2A^6}{576\omega_{20}^2} \quad (3.17)
$$

According to Eqn.3.17, response curve of frequency-amplitude can be derived with varied length of suspender and amplitude of exciting, depicted in Figure 7 and Figure 8.

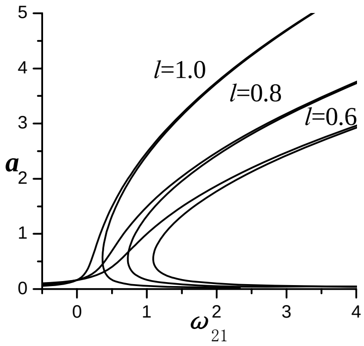


Figure 5 Frequency-amplitude curves

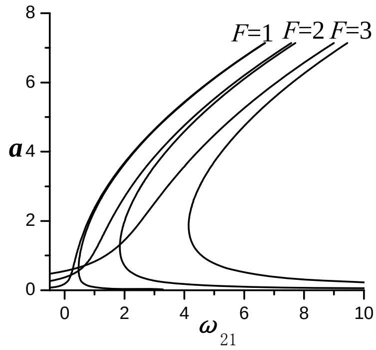


Figure 6 Frequency-amplitude curves

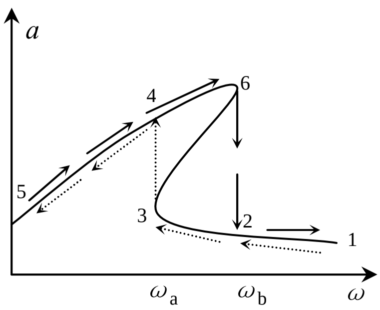


Figure 7 Sketch of frequency-amplitude

3.3. Analysis on Solution

(1) Distortion of resonance region: According to Figure 3-Figure 6, under the exciting of main harmonics and ultra harmonics, the exciting frequency with the maximum value of amplitude of substructure response is not close to natural vibration frequency of substructure, and it fall in a certain frequency rang more than natural vibration frequency. The shorter length of suspender, the farther distance between natural vibration frequency of substructure and maximum value point is. Compared with the solution of linear system, resonance region is distorted.

(2) Catastrophe of response amplitude: According to Figure 7, in the case of amplitude of exciting p is constant and ω_{21} is increasing continuously, response amplitude a varies from point 5 to 6 via 4 along the curve, and it suddenly drops down from point 6 to 2 and decreases with increasing ω_{21}. Inversely, when ω_{21} decreases from a higher frequency, response amplitude a varies from point 1 to 3 via 2. If ω_{21} decreases continuously then response amplitude a suddenly jump up from point 3 to 4 and going to decrease. As has been analyzed, response amplitude has catastrophic behavior.

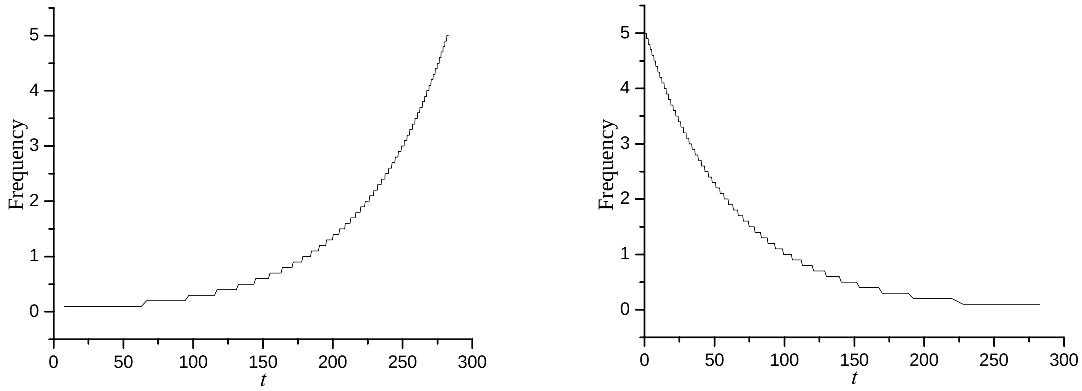
(3) Pseudo-linear resonance: if the length of suspender is longer than a certain value, there is no catastrophe in response amplitude and the extreme point of it very close natural vibration frequency. In this case, the behavior of frequency-amplitude is similarly linear, so call it as pseudo-linear resonance.

(4) "Path" and lag effect: As the frequency of exciting vary from a lower to a higher or invert, catastrophic behavior would occur on response amplitude. But the frequency points of jumping or dropping are not identical, ω_a and $\omega_{b.,}$ nor the values of catastrophe. That is to say there is "path" effect on the direction of frequency variation influencing response amplitude. On the other hand, the catastrophe of response amplitude as exciting frequency returns always falls behind the forwards, so there is a lag effect.

(5) Response amplitude's sensitivity to parameters: according to Figure 3-Figure 6, the little difference in taking parameters (such as 0.2) can lead to significant changes in response amplitude. It shows that response amplitude is very sensitive to the little changes of parameters.

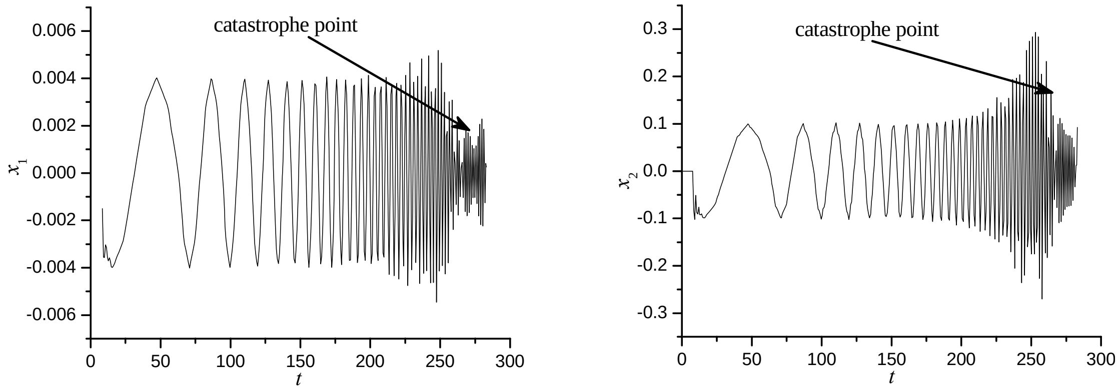
4. ANALYSIS ON DYNAMIC RESPONSE OF SUSPENDED STRUCTURE SYSTEM

To analyze the impact of substructure's catastrophic behavior on mega structure, numerical calculation on the structure system presented in Figure 1 is carried out. Parameter values: l=1, F=1, frequency from 0.1 to 5.0 and invert. Loads are sine-conversion acceleration loaded on foundation of the structure system. The different directions of frequency variation are used to derive "path" and lag effect of response amplitude. The increment of exciting frequency is 0.1, and one period in every level of frequency. Exciting frequency-time history is plot in Figure 8. The solutions of different loads are recorded in Figure 9 and Figure 10.



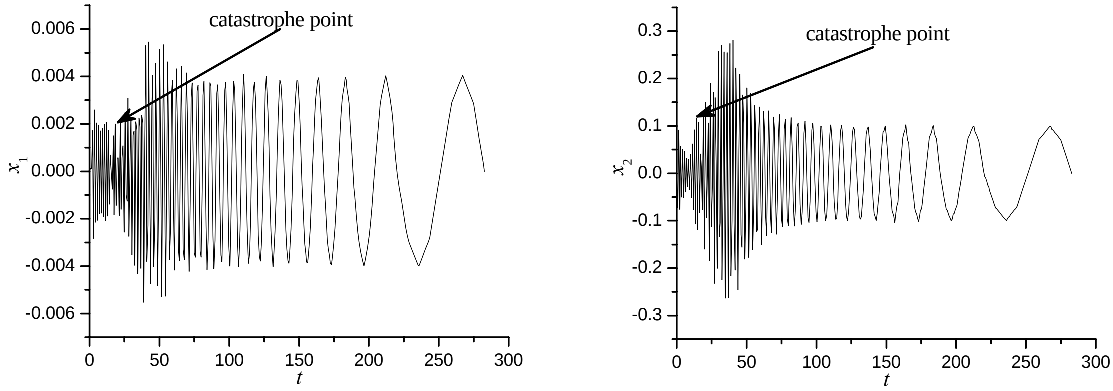
(a) Frequency increasing　　(b) Frequency decreasing

Figure 8 Frequency-time Loads



(a) Mega structure (b) Substructure

Figure 9 Response amplitudes excited by increasing frequency



(a) Mega structure (b) Substructure

Figure 10 Response amplitudes excited by decreasing frequency

Base on Figure 9 and Figure 10, the response amplitude of suspended substructure and mega structure are steady in the frequency range from 0.1 to 0.3, and response frequency equal to exciting. As exciting frequency increasing to about 0.4, the response amplitude of suspended substructure surges, and a magnificent catastrophe occurs, as a result, the response amplitude of mega structure is catastrophic in a short time. The same goes as exciting frequency decreasing to about 0.4, but the frequency values are little different, it shows up “path” and lag effects mentioned above.

5. CONCLUSIONS

Due to the consideration theoretical analysis and calculation, several conclusions can be derived:

(1) The dynamic responses of suspended structure system are very complex, and there is no opportunity to derive some extraordinary nonlinear behavior of dynamic responses, such as resonance region distortion, catastrophic amplitude, “path” and lag effect, if including linear restoring force only. The analysis on suspended structures above leads to more reasonable and optimal design parameters could be selected.

(2) The substructures can play a part in reducing the dynamic response amplitude of suspended structure system, but the catastrophic amplitude of substructures can lead to catastrophic amplitude of mega structures. If it is not controlled effectively, the dynamic responses of system will be unstable, even structural damage.

(3) The dynamic responses of suspended structures are hypersensitive to the length of suspenders; a little different could result in catastrophe of the response peak and resonance region of structure system.

REFERENCE

[1] Kenneth T.W. Yiu, Sai On Cheung.(2006).Instability leading to coal bumps and nonlinear evolutionary mechanisms for a coal-pillar-and-roof system. *International Journal of Solids and Structures* **43**,7407-7423

[2] M Amabilis, F. Pellicano, M. P. Paig Doussis. (2000). Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid. part Iv: large-amplitude vibrations with flow. *Journal of Sound and* vibration **237:4,**641-666

[3] Marco Carricato, Joseph Duffy, Vincenzo Parenti-Castelli. (2002). Catastrophe analysis of a planar system with flexural pivots. *Mechanism and Machine Theory* **37,**693-716

[4] AnthonyN. Kounadis. (2002). Dynamic buckling of simple two-bar frames using catastrophe theory. *International Journal of Non-Linear Mechanics* **37,**1249-1259

[5] X. Lignos, G. Ioannidis, A.N. Kounadis. (2003). Non-linear bucklingof simple models with tilted cusp catastrophe. *International Journal of Non-Linear Mechanics* **38,**1163-1172

[6] András Lengyel, Zhong You. (2004). Bifurcations of SDOF mechanisms using catastrophe theory. *International Journal of Solids and Structures* **41,**559-568

[7] Fang yingguang.(2004). analysis of seismic response properties and catastrophe of nonlinear sliding isolation structures. *china cicil engineering journal* **37:9,**22-26

[8] Qiang Xu, Grunqiu Huan.(1997). Catastrophic analysis of nonlinear response of structure under earthquake. *Chinese journal of geotechnical engineering*.**19:4,**25-29