Response-Spectrum-Based Analysis for Generally Damped Linear Structures

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ABSTRACT :

When civil structures are equipped with earthquake protective systems such as damping devices and base isolators, they are likely to be non-classically damped. In addition, when the overall damping is increased to a certain level, some modes will become over-damped. In such cases, the conventional treatment by using the classical damping assumption (ignoring the non-classical damping effect and the over-damped modes) may result in unacceptable design errors on the unsafe side. Based on the general modal response history analysis formulation with over-damped modes developed by the authors (Song et al. 2008), this paper further extends it to a response-spectrum analysis approach and proposes a general modal combination rule GCQC. Examples to examine the accuracy and effectiveness of this approach are also given.

KEYWORDS: Response spectrum, Generally damped linear structures, Seismic analysis, Over-damped modes

1. INTRODUCTION

In earthquake response analysis of structures, the response spectrum method is commonly used as an alternative approach to the response history analysis for determining the maximum values of the seismic responses of classically damped structures. In this method, the modal peak responses are obtained using the prescribed response spectrum. These modal maxima are then appropriately combined to estimate the peak values of the responses of interest. The conventional response spectrum method is ideal to structures satisfying classical damping condition. For structures that are strongly non-classically damped, the accuracy of the square-root-of-sum-of-squares (SRSS) and the complete quadratic combination (CQC) rule becomes questionable (Clough and Mojtahedi 1976, Warburton and Soni 1977 and Veletsos and Ventura 1986). For this reason, several modal combination rules accounting for the effect of the non-classical damping are developed (Singh 1980, Igusa et al. 1984, Ventura 1985, Gupta and Jaw 1986, Maldonado and Singh 1991 and Zhou et al. 2004). However, all combination rules developed in these literatures did not incorporate the over-damped modes in the formulation and the response quantities considered in these rules are limited to deformation-related response quantities. In this paper, on the basis of the general modal response history analysis developed by the authors (Song et al. 2008) and the white noise input assumption as well as the theory of random vibration, a general modal combination rule for response spectrum method are formulated to deal with the non-classical damping and over-damped modes. This general modal combination rule is referred to as 'General-Complete-Quadratic-Combination' (GCQC) rule in this study. An over-damped modal response spectrum is introduced to account for the corresponding peak modal responses. The accuracy of the new rule is evaluated through an example by comparing it to the mean response history results.

2. ANALYTICAL FORMULATION

According to the general modal response analysis method formulated by Song et al. (2008), the responses of a generally damped linear structure under seismic excitations can be expressed as the response history combination of N_c complex modes and N_P over-damped modes ($2N_C + N_P = 2N$, N is the structural DOFs) :

$$\mathbf{x}_0(t) = \sum_{i=1}^{N_C}[\mathbf{A}_{0i}\dot{q}_i(t) + \mathbf{B}_{0i}q_i(t)] + \sum_{i=1}^{N_P}[\mathbf{A}_{0i}^{P}q_i^{P}(t)] \quad \in \mathbb{R}^N \tag{1}$$

where $\mathbf{x}_0(t)=[x_{01}(t),x_{02}(t),...,x_{0N}(t)]^{\mathrm{T}}$ $(\in\mathbb{R}^N$ -- belongs to N dimension vector space in real field) represents a response vector for most response quantity of interest for structural seismic evaluation and design, such as relative displacement and velocity, absolute acceleration, inter-story drift and damping force etc. And $\dot{q}_i(t)\in\mathbb{R}$, $q_i(t)\in\mathbb{R}$ and $q_i^{\mathrm{P}}(t)\in\mathbb{R}$ are the under-damped mode (complex mode or simply termed as mode) displacement, velocity and over-damped mode responses to seismic acceleration excitation $\ddot{x}_{\mathrm{g}}(t)\in\mathbb{R}$, respectively, that is, $\dot{q}_i(t)$, $q_i(t)$ and $q_i^{\mathrm{P}}(t)$ are the solutions of the following differential equations, respectively:

$$\ddot{q}_i(t)+2\xi_i\omega_i\dot{q}_i(t)+\omega_i^2 q_i(t)=-\ddot{x}_{\mathrm{g}}(t)\quad (i=1,2,...,N_{\mathrm{C}}) \tag{2}$$

$$\dot{q}_i^{\mathrm{P}}(t)+\omega_i^{\mathrm{P}} q_i^{\mathrm{P}}(t)=-\ddot{x}_{\mathrm{g}}(t)\qquad (i=1,2,...,N_{\mathrm{P}}) \tag{3}$$

in which, $\omega_i\in\mathbb{R}$ and $\xi_i\in\mathbb{R}$ are the circular natural frequency and damping ratio of the complex mode respectively and $\omega_i^{\mathrm{P}}\in\mathbb{R}$ is the over-damped modal circular frequency. In Eq. (1), $\mathbf{A}_{0i}\in\mathbb{R}^N$, $\mathbf{B}_{0i}\in\mathbb{R}^N$ and $\mathbf{A}_{0i}^{\mathrm{P}}\in\mathbb{R}^N$ are the coefficient vectors associated with $\dot{q}_i(t)$, $q_i(t)$ and $q_i^{\mathrm{P}}(t)$, respectively. These coefficient vectors only depend on the structural modal parameters and are time invariant. The expressions of these coefficient vectors for most response quantities can be found in Song et al. (2008).

2.1 Definition of vector operation symbols

For simplicity in subsequent formulation, we define symbol "•" as a vector element-wise operation. For example, $\mathbf{c}=\mathbf{a}\bullet\mathbf{b}$ means that each element in vector $\mathbf{c}$ is the product of the corresponding elements in $\mathbf{a}$ and $\mathbf{b}$, assuming that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ have the same dimension. $\mathbf{a}^{\bullet 2}$ means taking the square for each element in the vector $\mathbf{a}$.

2.2 Covariance of modal responses to stationary excitation

Consider the input ground acceleration $\ddot{x}_{\mathrm{g}}(t)$ as a wide-band stationary process. Based on the theory of random vibration, the responses of a linear system subjected to a stationary process are also stationary and the covariance or mean squares of the response $\mathbf{x}_0(t)$ from Eq. (1) is in the form of

$$\begin{aligned}
\mathrm{E}\left[\mathbf{x}_0^{\bullet 2}(t)\right] &= \mathrm{E}\left[\left\{\sum_{i=1}^{N_{\mathrm{C}}}[\mathbf{A}_{0i}\dot{q}_i(t)+\mathbf{B}_{0i}q_i(t)]+\sum_{i=1}^{N_{\mathrm{P}}}[\mathbf{A}_{0i}^{\mathrm{P}}q_i^{\mathrm{P}}(t)]\right\}^{\bullet 2}\right] \\
&=\sum_{i=1}^{N_{\mathrm{C}}}\sum_{j=1}^{N_{\mathrm{C}}}\left\{\mathbf{A}_{0i}\bullet\mathbf{A}_{0j}\,\mathrm{E}\left[\dot{q}_i(t)\dot{q}_j(t)\right]+\mathbf{B}_{0i}\bullet\mathbf{B}_{0j}\,\mathrm{E}\left[q_i(t)q_j(t)\right]+2\mathbf{A}_{0i}\bullet\mathbf{B}_{0j}\,\mathrm{E}\left[\dot{q}_i(t)q_j(t)\right]\right\} \\
&+2\sum_{i=1}^{N_{\mathrm{C}}}\sum_{j=1}^{N_{\mathrm{P}}}\left\{\mathbf{A}_{0i}\bullet\mathbf{A}_{0j}^{\mathrm{P}}\,\mathrm{E}\left[\dot{q}_i(t)q_j^{\mathrm{P}}(t)\right]+\mathbf{B}_{0i}\bullet\mathbf{A}_{0j}^{\mathrm{P}}\,\mathrm{E}\left[q_i(t)q_j^{\mathrm{P}}(t)\right]\right\} \\
&+\sum_{i=1}^{N_{\mathrm{P}}}\sum_{j=1}^{N_{\mathrm{P}}}\left\{\mathbf{A}_{0i}^{\mathrm{P}}\bullet\mathbf{A}_{0j}^{\mathrm{P}}\,\mathrm{E}\left[q_i^{\mathrm{P}}(t)q_j^{\mathrm{P}}(t)\right]\right\}\ \in\mathbb{R}^N
\end{aligned} \tag{4}$$

Eq. (4) shows that it is necessary to compute the covariance of the response produced by two modes (e.g. $\mathrm{E}\left[q_i(t)q_j(t)\right]$) in order to obtain the variance of $\mathbf{x}_0(t)$. Before proceeding to calculate the covariance produced by two modes, a number of expressions are listed as they would be required in the subsequent formulations. They are:

$$q_i(t)=\int_0^t h_i(t-\tau)\ddot{x}_{\mathrm{g}}(\tau)\mathrm{d}\tau \in \mathbb{R} \quad \text{and} \quad q_i^{\mathrm{P}}(t)=\int_0^t h_i^{\mathrm{P}}(t-\tau)\ddot{x}_{\mathrm{g}}(\tau)\mathrm{d}\tau \in \mathbb{R} \tag{5a,b}$$

$$H_i(\mathrm{j}\omega)=-\frac{1}{-\omega^2+\mathrm{j}2\xi_i\,\omega_i\,\omega+\omega_i^2} \in \mathbb{C} \quad \text{and} \quad H_{\mathrm{v}i}(\mathrm{j}\omega)=-\frac{\mathrm{j}\omega}{-\omega^2+\mathrm{j}2\xi_i\,\omega_i\,\omega+\omega_i^2} \in \mathbb{C} \tag{6a,b}$$

$$H_i^{\mathrm{P}}(\mathrm{j}\omega)=-\frac{1}{\mathrm{j}\omega+\omega_i^{\mathrm{P}}} \in \mathbb{C} \tag{7}$$

where $h_i(t)$ and $h_i^{\mathrm{P}}(t)$ are the unit impulse response function of a complex mode and an over-damped mode, respectively; $\mathrm{j}=\sqrt{-1}$ is the imaginary unit; $H_i(\mathrm{j}\omega)$ and $H_{\mathrm{v}i}(\mathrm{j}\omega)$ are the displacement and velocity frequency response function of a complex mode with respect to excitation $\ddot{x}_{\mathrm{g}}(t)$, respectively; and $H_i^{\mathrm{P}}(\mathrm{j}\omega)$ is the frequency response function of an over-damped mode with respect to $\ddot{x}_{\mathrm{g}}(t)$. The displacement response covariance term $\mathrm{E}\left[q_i(t)q_j(t)\right]$ in Eq. (4) is first examined. According to Eq. (5a), this term may be written as

$$\mathrm{E}\left[q_i(t)q_j(t)\right]=\int_0^t\int_0^t h_i(\tau_1)h_j(\tau_2)\mathrm{E}\left[\ddot{x}_{\mathrm{g}}(t-\tau_1)\ddot{x}_{\mathrm{g}}(t-\tau_2)\right]\mathrm{d}\tau_1\mathrm{d}\tau_2 \in \mathbb{R} \tag{8}$$

Knowing that the input ground excitation $\ddot{x}_{\mathrm{g}}(t)$ starts from zero at the time instant $t=0$ (i.e. $\ddot{x}_{\mathrm{g}}(t)=0$ when $t\le 0$), it is reasonable to extend the lower limit of the integration in Eq. (8) to negative infinity as

$$\mathrm{E}\left[q_i(t)q_j(t)\right]=\int_{-\infty}^t\int_{-\infty}^t h_i(\tau_1)h_j(\tau_2)\mathrm{E}\left[\ddot{x}_{\mathrm{g}}(t-\tau_1)\ddot{x}_{\mathrm{g}}(t-\tau_2)\right]\mathrm{d}\tau_1\mathrm{d}\tau_2 \tag{9}$$

Now, suppose that the ground excitation $\ddot{x}_{\mathrm{g}}(t)$ is further considered as a white noise process with zero mean, described by a constant power spectral density S_0. It follows that the term $\mathrm{E}\left[\ddot{x}_{\mathrm{g}}(t-\tau_1)\ddot{x}_{\mathrm{g}}(t-\tau_2)\right]$ in Eq. (9) becomes

$$\mathrm{E}\left[\ddot{x}_{\mathrm{g}}(t-\tau_1)\ddot{x}_{\mathrm{g}}(t-\tau_2)\right]=2\pi S_0\delta(\tau_1-\tau_2) \tag{10}$$

where $\delta(\tau)$ is the Dirac function and it is defined as follows.

$$\delta(\tau)=\begin{cases}+\infty & \tau=0\\ 0 & \tau\neq 0\end{cases} \qquad \text{and} \qquad \int_{-\infty}^{+\infty}\delta(\tau)\mathrm{d}\tau=1 \tag{11a,b}$$

In light of the inverse of Fourier transform, the Dirac function also can be expressed as

$$\delta(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{-\mathrm{j}\omega\tau}\mathrm{d}\omega \quad \text{or} \quad \delta(\tau_1-\tau_2)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{-\mathrm{j}\omega(\tau_1-\tau_2)}\mathrm{d}\omega \tag{12a,b}$$

Substituting Eq. (12b) together with Eq. (10) into Eq. (9) and setting the upper integral limit to infinity to retain the steady state response, Eq. (9) becomes

$$\mathrm{E}\left[q_i(+\infty)q_j(+\infty)\right]=S_0\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{+\infty}h_i(\tau_1)\mathrm{e}^{-\mathrm{j}\omega\tau_1}\,\mathrm{d}\tau_1\int_{-\infty}^{+\infty}h_j(\tau_2)\mathrm{e}^{\mathrm{j}\omega\tau_2}\,\mathrm{d}\tau_2\right)\mathrm{d}\omega \quad \in\mathbb{R} \tag{13}$$

Denoting $R_{ij}^{\mathrm{DD}}=\mathrm{E}\left[q_i(+\infty)q_j(+\infty)\right]$ and making use of Eqs. (5) and (6) along with contour integration in complex plane, the displacement covariance R_{ij}^{DD}, shown by Eq. (13), may be written as

$$R_{ij}^{\mathrm{DD}}=S_0\int_{-\infty}^{+\infty}H_i(-\mathrm{j}\omega)H_j(\mathrm{j}\omega)\mathrm{d}\omega=\frac{\pi S_0}{2\omega_i\omega_j\sqrt{\omega_i\omega_j\xi_i\xi_j}}\rho_{ij}^{\mathrm{DD}} \quad \in\mathbb{R} \tag{14}$$

where ρ_{ij}^{DD} is the well-known displacement correlation coefficient originally derived for the CQC rule (Der Kiureghian 1981). Further, let $i=j$ in Eq. (14), it can be entirely expressed in modal displacement variance terms. That is,

$$R_{ij}^{\mathrm{DD}}=\sqrt{R_{ii}^{\mathrm{DD}}R_{jj}^{\mathrm{DD}}}\rho_{ij}^{\mathrm{DD}}, \qquad (i,j=1,2,\ldots,N_{\mathrm{C}}) \tag{15}$$

Following the similar procedures for the derivation of the modal displacement response covariance R_{ij}^{DD}, the modal velocity response covariance R_{ij}^{VV} and the covariance of the *i*th modal velocity and the *j*th modal displacement R_{ij}^{VD} can also be derived as

$$R_{ij}^{\mathrm{VV}}=\omega_i\omega_j\sqrt{R_{ii}^{\mathrm{DD}}R_{jj}^{\mathrm{DD}}}\rho_{ij}^{\mathrm{VV}} \in\mathbb{R} \quad \text{and} \quad R_{ij}^{\mathrm{VD}}=\omega_i\sqrt{R_{ii}^{\mathrm{DD}}R_{jj}^{\mathrm{DD}}}\rho_{ij}^{\mathrm{VD}} \in\mathbb{R}, \qquad (i,j=1,2,\ldots,N_C) \tag{16a,b}$$

where ρ_{ij}^{VV} and ρ_{ij}^{VD} are the modal velocity correlation coefficient and modal velocity-displacement correlation coefficient. Their expressions and variations versus modal frequency ratio and modal damping ratio can be found in Zhou et al. (2004) and Song et al. (2008). Noted that when $i=j$, the variance of velocity response R_{ii}^{VV} and the covariance of velocity and displacement response R_{ii}^{VD} becomes

$$R_{ii}^{\mathrm{VV}}=\frac{\pi S_0}{2\omega_i\xi_i}=\omega_i^2R_{ii}^{\mathrm{DD}} \qquad \text{and} \qquad R_{ii}^{\mathrm{VD}}=0 \tag{17a,b}$$

It is clear from Eq. (17) that the velocity variance and the displacement variance of a SDOF system is related by the squares of its natural circular frequency and the modal displacement and velocity responses of a SDOF system are orthogonal with each other under the white noise excitation assumption. The presence of R_{ij}^{VV} and R_{ij}^{VD} reflects the non-classical damping effect.

Another important objective of this study is to consider the contributions of over-damped modes (if exist) to evaluate the complete structural responses in modal response combination method. According to linear control system theory, an over-damped mode corresponds to an independent first-order subsystem (integral unit), while an under-damped mode (complex mode) associates with a second-order subsystem (oscillation unit). This is clear if we compare Eq. (2) with Eq. (3) and Eq. (6) with Eq. (7). The detailed natural properties regarding to the over-damped modes are described in Song et al. (2008). Now, considering the over-damped modal response covariance term $\mathrm{E}\left[q_i^{\mathrm{P}}(t)q_j^{\mathrm{P}}(t)\right]$ in Eq. (4) and following the similar procedures as the above, we have

$$R_{ii}^{\mathrm{PP}} = \mathrm{E}\left[q_i^{\mathrm{P}}(t)q_j^{\mathrm{P}}(t)\right] = S_0\int_{-\infty}^{+\infty} H_i^{\mathrm{P}}(\mathrm{j}\omega)H_j^{\mathrm{P}}(-\mathrm{j}\omega)\mathrm{d}\omega \in \mathbb{R}, \qquad (i, j = 1,2,...N_{\mathrm{P}}) \tag{18}$$

Substitution of Eq. (7) into Eq. (18) and manipulation with contour integration in complex plane leads to

$$R_{ij}^{\mathrm{PP}} = \frac{2\pi S_0}{\omega_i^{\mathrm{P}} + \omega_j^{\mathrm{P}}} = \sqrt{R_{ii}^{\mathrm{PP}} R_{jj}^{\mathrm{PP}}}\,\rho_{ij}^{\mathrm{PP}}, \qquad (i, j = 1,2,...N_{\mathrm{P}}) \tag{19}$$

in which

$$\rho_{ij}^{\mathrm{PP}} = \frac{2\sqrt{\omega_i^{\mathrm{P}}\omega_j^{\mathrm{P}}}}{\omega_i^{\mathrm{P}} + \omega_j^{\mathrm{P}}} \in \mathbb{R}, \qquad (i, j = 1,2,..., N_{\mathrm{P}}) \tag{20}$$

is a newly derived correlation coefficient that accounts for the relationship between the over-damped mode responses. Similarly, the modal displacement and the over-damped modal response covariance term $\mathrm{E}\left[q_i(t)q_j^{\mathrm{P}}(t)\right]$ and the modal velocity and the over-damped modal response covariance term $\mathrm{E}\left[\dot{q}_i(t)q_j^{\mathrm{P}}(t)\right]$ in Eq. (6) can be derived as

$$R_{ij}^{\mathrm{DP}} = \sqrt{R_{ii}^{\mathrm{DD}} R_{jj}^{\mathrm{PP}}}\,\rho_{ij}^{\mathrm{DP}} \quad \text{and} \quad R_{ij}^{\mathrm{VP}} = \sqrt{R_{ii}^{\mathrm{DD}} R_{jj}^{\mathrm{PP}}}\,\omega_j^{\mathrm{P}}\rho_{ij}^{\mathrm{DP}} \in \mathbb{R}, \qquad (i = 1,2,...N_{\mathrm{C}}, j = 1,2,...N_{\mathrm{P}}) \tag{21}$$

where

$$\rho_{ij}^{\mathrm{DP}} = \frac{2\omega_i\sqrt{2\xi_i\omega_i\omega_j^{\mathrm{P}}}}{\omega_i^2 + 2\xi_i\omega_i\omega_j^{\mathrm{P}} + (\omega_j^{\mathrm{P}})^2} \in \mathbb{R}, \qquad (i = 1,2,...N_{\mathrm{C}}, j = 1,2,...N_{\mathrm{P}}) \tag{22}$$

is another newly developed correlation coefficient which accounts for the correlation between the complex modal displacements and the over-damped mode responses. Fig. 1 shows the variations of the correlation coefficient ρ_{ij}^{PP} versus $\omega_i^{\mathrm{P}}/\omega_j^{\mathrm{P}}$, from which it is observed that the value of ρ_{ij}^{PP} only depends on the over-damped modal frequencies and it remains to be a significant component across the range of $\omega_i^{\mathrm{P}}/\omega_j^{\mathrm{P}}$. Fig. 2 shows the variation of ρ_{ij}^{DP} with respect to $\omega_i/\omega_j^{\mathrm{P}}$ and damping ratio ξ_i. It is seen that the values of ρ_{ij}^{DP} are significant, particularly at large damping level. Also, ρ_{ij}^{DP} grows as the ratio $\omega_i/\omega_j^{\mathrm{P}}$ approaches two and decreases slowly beyond that value. The observations made from Figs. 1 and 2 suggest that the over-damped mode may contribute significantly to the overall structural response and should be considered appropriately. Finally, upon substitution of the above derived covariance into Eq. (4), one obtains

$$\begin{aligned}\mathrm{E}\left[\mathbf{x}_0^{\bullet 2}(t)\right] &= \sum_{i=1}^{N_{\mathrm{C}}}\sum_{j=1}^{N_{\mathrm{C}}}\left[\rho_{ij}^{\mathrm{VV}}\omega_i\omega_j\mathbf{A}_{0i}\bullet\mathbf{A}_{0j} + \rho_{ij}^{\mathrm{DD}}\mathbf{B}_{0i}\bullet\mathbf{B}_{0j} + 2\rho_{ij}^{\mathrm{VD}}\omega_i\mathbf{A}_{0i}\bullet\mathbf{B}_{0j}\right]\sqrt{R_{ii}^{\mathrm{DD}}R_{jj}^{\mathrm{DD}}} \\ &+ 2\sum_{i=1}^{N_{\mathrm{C}}}\sum_{j=1}^{N_{\mathrm{P}}}\rho_{ij}^{\mathrm{DP}}\left[\omega_j^{\mathrm{P}}\mathbf{A}_{0i}\bullet\mathbf{A}_{0j}^{\mathrm{P}} + \mathbf{B}_{0i}\bullet\mathbf{A}_{0j}^{\mathrm{P}}\right]\sqrt{R_{ii}^{\mathrm{DD}}R_{jj}^{\mathrm{PP}}} + \sum_{i=1}^{N_{\mathrm{P}}}\sum_{j=1}^{N_{\mathrm{P}}}\left[\rho_{ij}^{\mathrm{PP}}\mathbf{A}_{0i}^{\mathrm{P}}\bullet\mathbf{A}_{0j}^{\mathrm{P}}\right]\sqrt{R_{ii}^{\mathrm{PP}}R_{jj}^{\mathrm{PP}}} \in \mathbb{R}^N\end{aligned} \tag{23}$$

2.3 Development of response spectrum method

It has been shown that the mean maximum modal response of a linear system over a specified duration to stationary excitations is proportional to their respective root mean squares(Vanmarcke 1972), i.e.,

$$\left|q_i(t)\right|_{\max} = S_i = p_i\sqrt{R_{ii}^{\mathrm{DD}}} \in \mathbb{R} \quad \text{and} \quad \left|q_i^{\mathrm{P}}(t)\right|_{\max} = S_i^{\mathrm{P}} = p_i\sqrt{R_{ii}^{\mathrm{PP}}} \in \mathbb{R} \tag{24}$$

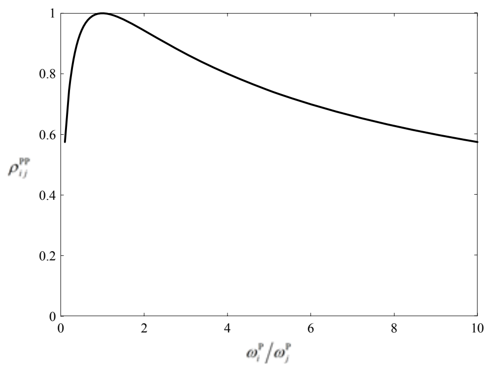

Figure 1 Variation of correlation coefficient ρ_{ij}^{PP}

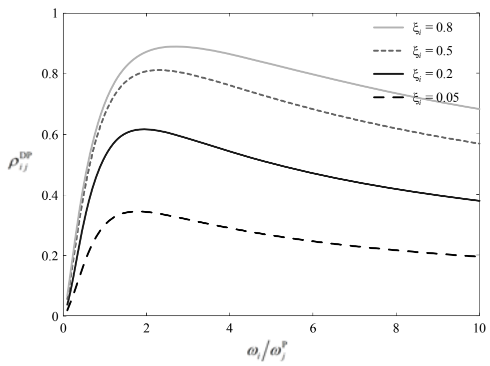

Figure 2 Variation of correlation coefficient ρ_{ij}^{DP}

where the S_i is the ordinate of the mean displacement response spectrum and S_i^{P} is the ordinate of the mean over-damped mode response spectrum. The idea of the over-damped mode response spectrum will be introduced in a later section. The numerical value of p_i, in general, does not differ greatly in magnitude from mode to mode. Thus, for practice applications, it is reasonable to assign the same value to p_i for each mode as well as for the combined responses. As a result, the following General-Complete-Quadratic-Combination rule (GCQC rule) applicable to structural systems with non-classical damping and over-damped modes is derived.

$$\left|\mathbf{x}_0(t)\right|_{\max} = \left\{ \begin{array}{l} \sum_{i=1}^{N_{\text{C}}}\sum_{j=1}^{N_{\text{C}}} \rho_{ij}^{\text{DD}} \left[\mu_{ij}\omega_i\omega_j \mathbf{A}_{0i}\bullet\mathbf{A}_{0j} + \mathbf{B}_{0i}\bullet\mathbf{B}_{0j} + 2\upsilon_{ij}\omega_i\mathbf{A}_{0i}\bullet\mathbf{B}_{0j} \right] S_i S_j \\ +2\sum_{i=1}^{N_{\text{C}}}\sum_{j=1}^{N_{\text{P}}} \rho_{ij}^{\text{DP}} \left[\omega_j^{\text{P}} \mathbf{A}_{0i}\bullet\mathbf{A}_{0j}^{\text{P}} + \mathbf{B}_{0i}\bullet\mathbf{A}_{0j}^{\text{P}} \right] S_i S_j^{\text{P}} + \sum_{i=1}^{N_{\text{P}}}\sum_{j=1}^{N_{\text{P}}} \left[\rho_{ij}^{\text{PP}} \mathbf{A}_{0i}^{\text{P}}\bullet\mathbf{A}_{0j}^{\text{P}} \right] S_i^{\text{P}} S_j^{\text{P}} \end{array} \right\}^{\bullet 1/2} \in \mathbb{R}^N \qquad (25)$$

where $\mu_{ij} = \dfrac{\rho_{ij}^{\text{VV}}}{\rho_{ij}^{\text{DD}}} = \dfrac{\xi_i + \xi_j\gamma_{ij}}{\xi_j + \xi_i\gamma_{ij}}$ and $\upsilon_{ij} = \dfrac{\rho_{ij}^{\text{VD}}}{\rho_{ij}^{\text{DD}}} = \dfrac{1-\gamma_{ij}^2}{2\gamma_{ij}(\xi_j + \xi_i\gamma_{ij})}$, $(i, j = 1,2,\ldots,N_{\text{C}})$.

As a special case of the GCQC, if the correlations between each mode are ignored; that is, when $i \neq j$ $\rho_{ij}^{\text{DD}} = 0$, $\rho_{ij}^{\text{PP}} = 0$ as well as $\upsilon_{ij} = 0$ and $\rho_{ij}^{\text{DP}} = 0$ for all *i* and *j*, Eq. (25) is reduced to

$$\left|\mathbf{x}_0(t)\right|_{\max} = \sqrt{\sum_{i=1}^{N_{\text{C}}} \left(\omega_i^2 \mathbf{A}_{0i}^{\bullet 2} + \mathbf{B}_{0i}^{\bullet 2} \right) S_i^2 + \sum_{i=1}^{N_{\text{P}}} \left(\mathbf{A}_{0i}^{\text{P}} \right)^{\bullet 2} \left(S_i^{\text{P}} \right)^2} \in \mathbb{R}^N \qquad (26)$$

Eq. (26) is termed as General-Square-Root-of-Sum-of-Square combination rule (GSRSS rule). If the damping matrix of a structure satisfies Caughey Criterion and all over-damped modes are ignored, Eqs. (25) and (26) can be reduced to the conventional CQC and SRSS rules for classically-damped structures. However, the formulation resulting from this study can be used to evaluate most peak response quantities of interest, such as relative displacement and velocity, absolute acceleration, inter-story drift, story shear and damping force etc. The most reduced form of Eq. (26), for example, can be used to evaluate the peak absolute acceleration of a linear SDOF system, that is

$$\left|\ddot{x}_{\text{A}}(t)\right|_{\max} = \sqrt{1+4\xi^2}\left[\omega_{\text{n}}^2 S(T_{\text{n}},\xi)\right] = \sqrt{1+4\xi^2}\, S_{\text{PA}}(T_{\text{n}},\xi) \in \mathbb{R} \qquad (27)$$

where ω_{n}, T_{n} and ξ are the SDOF system's natural frequency, period and damping ratio; and $S_{\text{PA}}(T_{\text{n}},\xi)$ is the ordinate of pseudo-acceleration spectrum. If ξ value is small, say, less than 15%, $\left|\ddot{x}_{\text{A}}(t)\right|_{\max} \approx S_{\text{PA}}(T_{\text{n}},\xi)$.

3. OVER-DAMPED MODE RESPONSE SPECTRUM

The over-damped mode response spectrum follows a similar definition as the conventional response spectrum used in earthquake engineering. The objective of the over-damped mode response spectrum is to account for the peak over-damped mode response contributions. Rewriting Eq. (3) as a general form of a first-order differential equation with excitation input $\ddot{x}_g(t)$, we have

$$\dot{q}^P(t) + \omega^P q^P(t) = -\ddot{x}_g(t) \tag{28}$$

Similar to the concept of conventional response spectrum, the over-damped mode response spectrum is defined as a plot of peak over-damped mode responses $q^P(t)$, as a function of over-damped modal frequency ω^P or over-damped modal period $T^P = 2\pi/\omega^P$ (actually, T^P is termed as time constant of a first-order system in control theory) under a given ground acceleration via Eq. (28). Unlike the conventional response spectrum, the over-damped mode response spectrum has only one parameter, ω^P, influencing the response and the over-damped response, q^P (t), which has velocity dimension. The procedure to directly construct the over-damped mode response spectrum consists of the following three steps: (1) Select the ground motion to be considered; (2) Determine the peak over-damped mode responses represented by Eq. (28) using the selected ground motion for different over-damped modal frequencies; and (3) The peak over-damped modal response obtained offers a point on the over-damped mode response. As a result, it is found that the construction of over-damped mode response spectrum relies on the availability of the ground acceleration histories. However, when using the response spectrum approach, the site response spectrum specified in design codes is used, which may vary from site to site, rather than ground acceleration histories. Therefore, the over-damped mode response spectrum cannot be directly generated due to the unavailability of ground acceleration records. A conversion approach to construct an over-damped mode response spectrum based on the 5% damping displacement spectrum (or pseudo-acceleration spectrum) is also established to address this issue. The central idea is derived from the fact that the ground motion power spectral density (PSD) that serves for input to either second-order subsystem or first-order subsystem is the same. Thus, after establishing the relationship between PSD and the peak response for both subsystems, we can further construct the connection between two peak responses and then use 5% damping displacement spectrum to predict the compatible over-damped mode response spectrum. The detailed procedure can be found in Song et al. (2008).

4. EVALUATION OF THE GCQC RULE

The accuracy and applicability of the proposed GCQC rule is evaluated by conducting response spectrum analyses of a steel frame example building shown in Fig. 3. The detailed information of this building and the ground motion acceleration ensemble used can be referred to Song et al. (2008). This example building is aimed to represent a highly non-classically damped structure with over-damped modes. It is noted that in order to evaluate the errors arising from the combination rule itself, the actual mean peak values of modal displacement responses to the acceleration ensemble (considered as the displacement response spectra) are used in the modal response combinations. The example building frame is analyzed by using linear response history analysis to each ground motion record listed in the ensemble. The mean response analysis results (considered as the exact values) are then used to examine the accuracy of the GCQC rule, including a comparison of the effect of: (1) using the forced classical damping assumption, and (2) ignoring the over-damped modes when they are present. Three sets of results are obtained and compared with the exact values. These three sets are obtained under the following conditions: (a) results of the first set are obtained based on the proposed GCQC rule. The state space approach is used to derive the mode shapes, modal frequencies and modal damping ratios. These modal parameters are then used to generate the correlation coefficients and peak modal responses required in the GCQC rule. The contributions from the over-damped modes are considered; (b) results of the second set are based on the modal parameters obtained under the forced classical damping assumption. Similar to the GCQC rule, these properties are used to generate the data required in the modal combination rule. The over-damped modes are ignored when they are present. This process is often used for the design and analysis of structures with added damping devices. This rule is referred to as the CDA (forced classical damping assumption); and (c) results of the third set are identical to the GCQC rule except that the over-damped modes are not taken into account in the modal combination process. This consideration is aimed to examine the effects of the over-damped modes in terms of response quantities. This rule is referred to as the EOM (exclude over-damped modes).

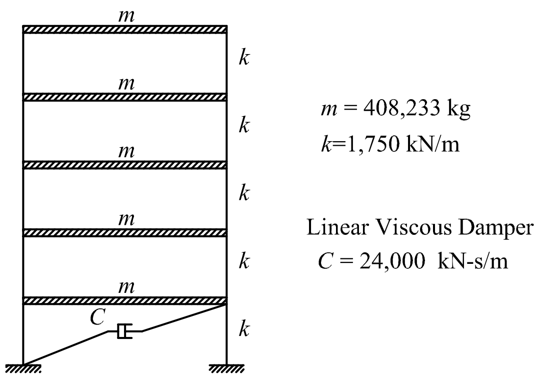

Figure 3 Configuration of the example building

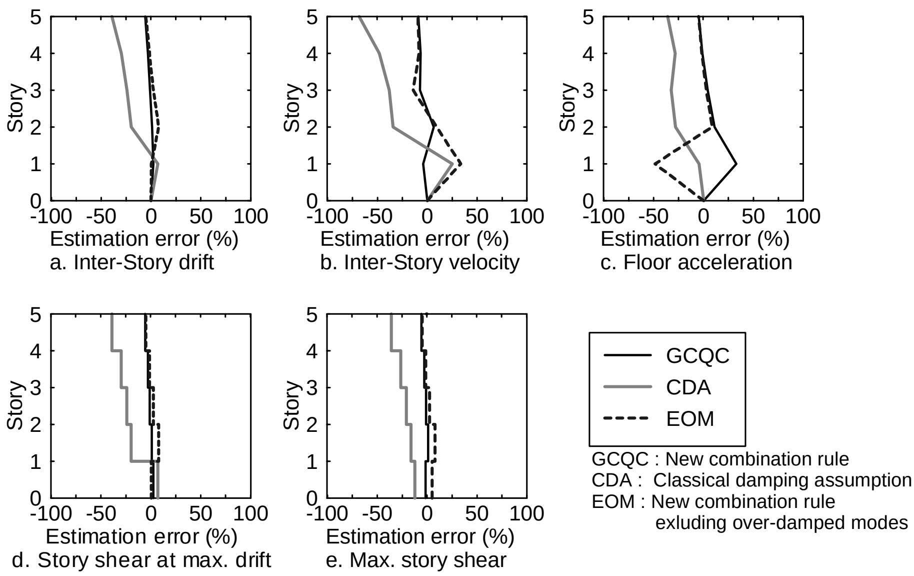

Figure 4 Estimated errors due to GCQC,CDA and EOM

Fig. 4 shows the estimation errors of each combination rule. It is shown that the GCQC provided excellent results except it overestimated the first floor acceleration by about 35% where the damper is added. The CDA, however, considerably underestimated the peak responses with one exception (overestimated the peak inter-story velocity of the first floor by 25%). The error increases as the level of story increases. This overestimation is more profound for inter-story velocity and floor acceleration. On the other hand, EOM overestimated the inter-story velocity while it underestimated the floor acceleration at the first floor. For the rest of the response quantities, the EOM provided conservative estimates. In general, the results show that using GCQC, in which the over-damped modes, if exist, are considered, can estimate the peak responses more accurately. It is found that the inter-story velocity and floor acceleration are significantly influenced by the over-damped modes. This is particularly true for the floors at which dampers are installed. The responses estimated by using the forced classical damping assumption deviate substantially from the exact values. Most of the responses are underestimated, which is understandable, because the complex modal effects and over-damped modal contributions are ignored by using this method. This implies that the utility of the forced classical damping assumption should be further examined in the design and analysis of structures supplemented with dampers.

5. SUMMARY AND CONCLUSION

There are many design and analysis approaches for damping device applications in new construction and rehabilitation of civil engineering structures. The response spectrum method is one of the most common approaches. When damping devices are added to complex and irregular structures, the structures are, in general, heavily non-classically damped and some over-damped modes may develop. Under such circumstances, the conventional CQC or SRSS rules for the response spectrum analysis method, assuming the structures are classically-damped, may not provide accurate results. A general modal combination rule for the response spectrum method, denoted as GCQC, is developed to accommodate the presence of non-classical damping and over-damped modes. This GCQC rule retains the conceptual simplicity of the conventional CQC rule and offers an efficient and accurate estimation of the peak responses of structures with added damping devices. In addition, a transformation principle to construct the over-damped mode response spectrum from the given design spectrum is also introduced briefly. This ensures the applicability of the GCQC rule in engineering practice. Example study shows that structures with added dampers should be modeled as non-classically damped and the over-damped modes should be included in the analysis in order to achieve more reliable estimates. In this paper, the formulation is focused on a planar structure subjected to single directional excitation. This formulation has been extended to the 3D generally damped structures under multi-component excitation by the authors (Chu et al. 2008).

6. ACKNOWLEDGEMENT

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