EFFECTS OF DEGRADING MODELS FOR DUCTILITY AND DISSIPATED HYSTERETIC ENERGY IN UNIFORM ANNUAL FAILURE RATE SPECTRA

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ABSTRACT:

One of the most important tools for seismic design purposes is the response spectrum because it represents the maximum demands on structures. Nevertheless, two aspects should be included on seismic response spectra provided by codes. The first aspect is the inclusion of specific reliability levels on the structures designed. The second is the inclusion of cumulative demands through plastic deformation. While uniform annual failure rate (UAFR) spectra can be used to account for the structural reliability, cumulative demand can be accounted through dissipated hysteretic energy spectra. In this paper the effects of degrading models for ductility and dissipated hysteretic energy UAFR spectra are studied. Narrow-band motions from the soft soil of the valley of Mexico were selected due to the large energy amount that demands to the structures. The implications to use simplify models as the well-known elasto-plastic to obtain the requirements of lateral strength are analyzed. Especially, the larges differences on the seismic coefficient obtained through simplify hysteretic models and degrading models are discussed. It is observed in structures that exhibit low to moderate degradation in strength, and low degradation in stiffness that the use of simplify hysteretic models can be adequate. However, with the aim to obtain satisfactory designs, for structures with important levels of degradation in stiffness, constitutive laws to represent with good accuracy the hysteretic behavior of structures are necessaries in the design spectra, or specific factors to account for the degradation in the mechanical characteristics.

KEYWORDS: Uniform annual failure rate spectra, dissipated hysteretic energy, degradation function

1. INTRODUCTION

A fundamental piece in the seismic design codes is the use of response spectra obtained through single degree of freedom (SDOF) systems. The spectra provided by the most of codes were derived using elastoplastic or bilinear hysteretic behavior; however, the structures built with materials as masonry, concrete and steel, exhibit strength and stiffness degradation. This implies in some circumstances an unrealistic modeling of the structures subjected to earthquakes when simplification assumptions are considered. The most important aspect is when due to the use of non degrading simple nonlinear models, the earthquake effects on structures are underestimated, for example: for buildings with low cycle capacity or subjected to long ground motion duration (Terán-Gilmore and Jirsa, 2005; Bojórquez et al, 2008), where the influence of earthquake's duration on structural response has been observed in many studies (Fajfar, 1992; Manfredi, 2001; Chai, 2005; Iervolino et al, 2006; Bojórquez et al, 2006, Hancock and Boomer 2006); furthermore, especial attention is necessary for the case of narrow-band motions, because the energy demands can be three or four times greater than firm-soil (Terán-Gilmore and Jirsa, 2005). For these reasons, the motivation of this paper is to observe the effects of strength and stiffness degradation in UAFR spectra related not only for ductility, also for a performance parameter to account for ground motion duration, as the case of dissipated hysteretic energy.

2. DEGRADATION FUNCTIONS AND SDOF MODELS

Several hysteretic models have been proposed to represent the behavior of structures, and many of these models that account for strength and stiffness degradation (e.g. Otani, 1974; Saiidi and Sozen 1979; other). This work is not focusing in the study of such models, if not illustrate how general conditions affect in the constitutive laws for example: reduction in the levels of strength and stiffness due to plastic behavior. All this, based in the degradation levels observed in experimental tests of structures commonly used in the engineering practice. For this purpose, a degradation function consistent with specific degrading levels of the system is used. A degradation function f_d describe the lost of the mechanical characteristics of a structure in terms of some performance parameter (e.g. maximum displacement), and it represents the strength or stiffness of the system in some instant with respect to the initial values. For example: figure 1 illustrates a function $f_{dk}(d_m)$ for the stiffness k of a system in terms of the maximum displacement d_m; it means, increasing the maximum displacement increase the stiffness degradation of the system. It can be observed in figure 1 when d_m equals zero, $f_{dk}(d_m)$ is equal to 1, this implies that the system no exhibit stiffness degradation. The stiffness in some stage will be given by $k = f_{dk}(d_m)\, k_o$, where k_o represents the initial stiffness of the system.

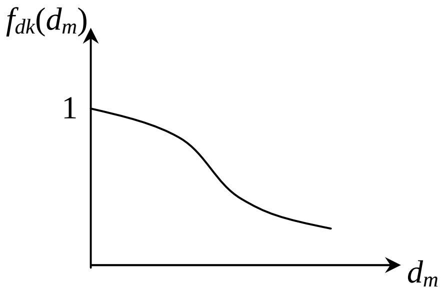


Figure 1 Stiffness degradation function for a system based on maximum displacement demand

In some cases, the degradation functions could not represents adequately the mechanical characteristics. In particular, representing the behavior of structures using a degradation function based on maximum displacements results in some circumstances in an underestimation of the lost in strength and stiffness, as can be observed in figures 2a and 2b from experimental tests for steel and concrete elements subjected to constant ductility. It is observed how the strength and stiffness in the elements is reduced for a constant maximum displacement demand. For this reason, it is important to use parameters that characterize with good accuracy the levels of degrading in strength and stiffness in the structures subjected to seismic load. In this case, SDOF with bilinear behavior and 5% of critical damping are analyzed, which are modified through a degradation function used in previous studies for normalized dissipated hysteretic energy with respect to the initial strength F_{yo} and yielding displacement d_{yo} (Bojórquez and Rivera, 2005; Bojórquez et al 2006). Considering the degradation in strength and stiffness for SDOF with an initial bilinear hysteretic behavior, it can be obtained a diversity of hysteretic models with different degradation levels as can be shown in figure 3.

The general degradation function in terms of a damage index to account for cumulative demands used is given by the next equation (Bojórquez and Rivera, 2005):

$$f_d(I_D) = \left(\frac{a}{a + I_D} \right)^{\gamma} \tag{1}$$

where I_D represents the damage parameter; a and γ are constant that indicate the shape of the function, and represents the rate of degradation of the system. For a specific value of a, the rate of degradation of the system will depend on the value assigned to γ. To represents in general the hysteretic behavior of structures, in this study parameters a and γ consistent with two cases are used: 1) a maximum strength degradation equals to 80% of the initial strength of the system; 2) three degradation levels in stiffness from low to large degradations. Levels of degradation low to moderate are typical for steel members with stable cycles of behavior (see figure

2a), and moderate to large degradation levels are typical for concrete structures as can be appreciated in figure 2b.

Due to the clear relation of the normalized dissipated hysteretic energy with the structural damage (e.g. the damage index based in this parameter; Teran and Jirsa, 2005), equation 1 can be expressed in terms of normalized dissipated hysteretic energy as:

$$f_d = \left(\frac{E_{HNC}}{E_{HNC} + E_{HND}} \right)^{\gamma} \tag{2}$$

Equation 2 is used in this study to characterize different levels of strength and stiffness degradation. In this equation E_{HNC} is the normalized hysteretic energy capacity, and E_{HND} is the normalized hysteretic energy demand.

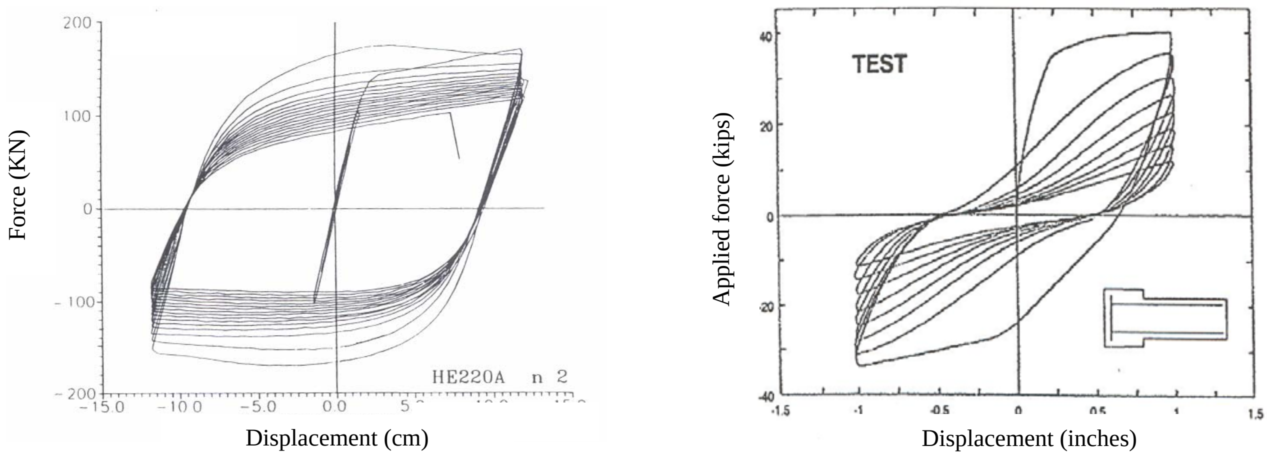


Figure 2 Plot force-displacement for: a) steel element subjected to constant displacement plastic cycles (Ballio and Castiglioni, 1994); b) reinforced concrete element subjected to constant displacement (Hwang, 1982)

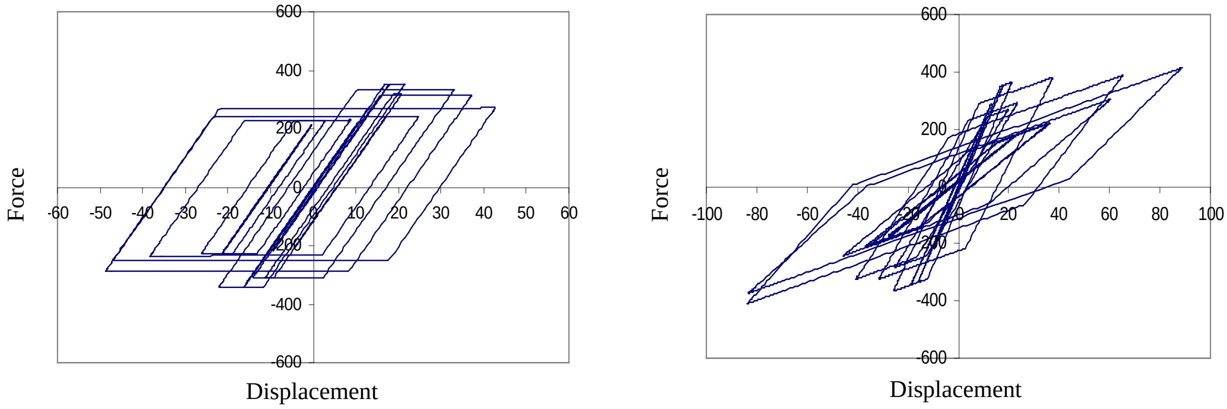


Figure 3 Hysteretic model with degradation in: a) strength and b) strength and stiffness

For all the cases, the degradation function has the form described in figure 4. In this figure, it can be observed that low degradation levels are considered for structures with a 10% maximum reduction of their mechanical characteristics typical for steel elements, moderate for 30% of maximum reduction an finally, larges degradation levels imply a 50% of maximum reduction in the mechanical characteristics (concrete members). The summary of the hysteretic behavior models considered for all the analyses is illustrated in Table 2.1.

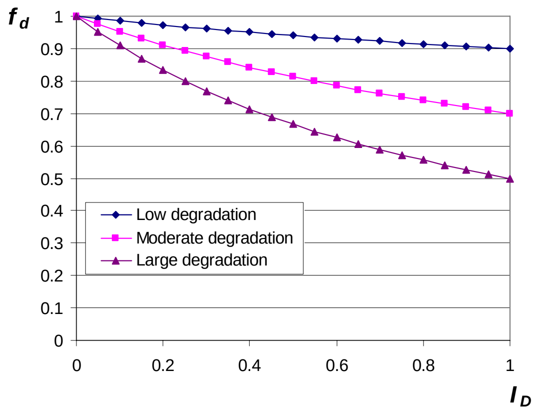


Figure 4 Degradation levels used in this study

Table 2.1 hysteretic behavior models considered

Hysteretic Model	Post-yielding Stiffness (%)	Maximum Strength Degradation	Maximum Stiffness Degradation	Description
EPP	0	0	0	Non degrading EPP model
B15	15%	0	0	Non degrading B15 model
B30	30%	0	0	Non degrading B30 model
B5D10	5%	20%	10%	Strength degradation and low stiffness degradation typical for steel elements
B5D30	5%	20%	30%	Strength and moderate stiffness degradation typical for concrete elements
B5D50	5%	20%	50%	Strength degradation and large stiffness degradation typical for concrete elements with high loss in the mechanical characteristics

3. GROUND MOTION RECORDS

For the aim to obtain UAFR spectra, in this study a set of 31 narrow-band motions corresponds to Mexico City are used. The magnitude range is from 6.9 to 8.1 (including the 1985 Mexican Earthquake). The records previously used by Bojórquez et al (2008) were selected for a soil period close to 2 seconds. Scale the seismic records to different intensity is necessary to obtain UAFR spectra, where the procedure to obtain UAFR is described in Rivera and Ruiz (2007). Here, the records are scaled for different intensity levels in terms of spectral acceleration ($Sa(T_1)$) for the vibration period of the SDOF analyzed (Shome, 1999). Some studies demonstrated the efficacy of this scaling criterion (Iervolino and Cornell, 2005). However, special attention is required for narrow-band motions (Montiel and Ruiz, 2007), especially it must be selected an adequate number of records to establish clear relations between the intensity ($Sa(T_1)$), and the parameters used as indicator of the seismic demands. In the following, the procedure to obtain UAFR spectra is described.

4. ALGORITHM TO OBTAIN UAFR SPECTRA

The annual structural failure rate is evaluated based on previous works developed by (Esteva, 1967; Cornell, 1968), and in the total probability theorem as following:

$$\nu_F = \int P\left(Q \geq 1 \middle| y\right) \left| \frac{d\nu_Y(y)}{dy} \right| dy \tag{3}$$

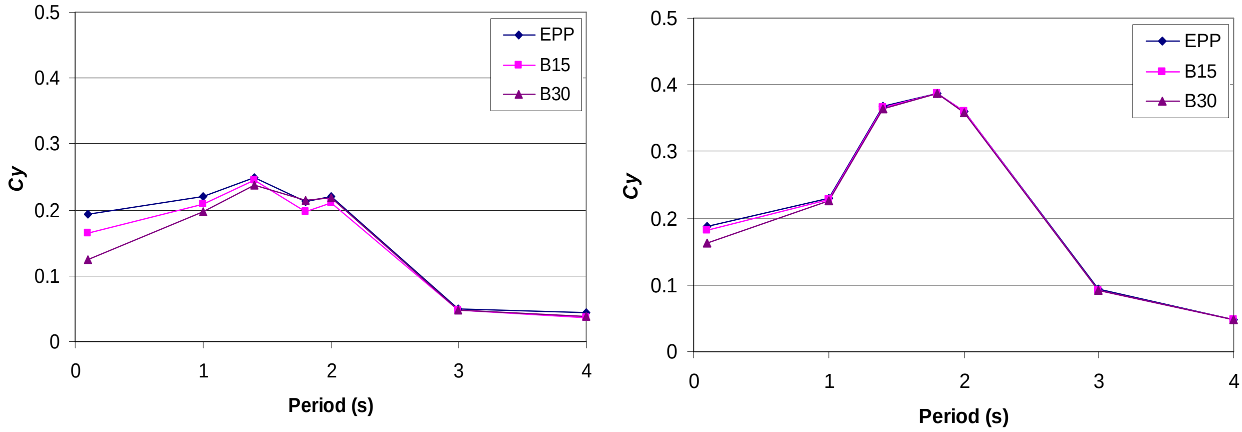
where $\left|\frac{d\nu_Y(y)}{dy}\right|$ is the absolute value of the derivative of the site seismic hazard curve, $P(Q \geq 1|y)$ is the conditional probability of failure, given a seismic intensity y, the structural failure occurs when capacity is smaller than demand, in other words: $\frac{demanda}{capacidad} = Q \geq 1$.

The following steps need to be accomplished in order to obtain the UAFR spectra:

1) Seismic hazard curve of the site studied. In this paper, the seismic hazard curve for the Ministry of Communications and Transportation (SCT) station was used (Alamilla, 2001).
2) Obtain the response of the system by Incremental Dynamic Analysis IDAs, for a selected period T_1 and yield force coefficient *Cy* subjected to a scaled ground motion records. Scaling is so that the spectral acceleration associated with the period (T_1) of the system under study corresponds to specific value of intensity ($Sa(T_1)$) as it was discussed before. The responses obtained, in this paper are: the ductility and normalized dissipated hysteretic energy.
3) Propose specific values of ductility capacity and normalized dissipated hysteretic energy.
4) Failure probability was obtained dividing the number of ground motion records in which ductility demanded is greater than ductility capacity (step 3) between the total numbers of records used for that specific intensity.
5) The asses of the vulnerability curves for the capacity proposed in the step 3.
6) Evaluate numerically the annual failure rate by using equation 3. Repeat step 2 to 5 for other systems (other T_1 and *Cy*) to obtain the structural annual failure rate curves.
7) The UAFR spectra for a specific value are calculated for each of the parameter here studied.

5. POST-YIELDING STIFFNESS INFLUENCE IN UAFR SPECTRA

The influence of post-yielding stiffness in the UAFR spectra is obtained by comparison of the elasto-plastic (EPP), bilinear model with 15% of post-yielding stiffness (B15), and finally a bilinear model with 30% of post-yielding stiffness (B30). Figures 5a and 5b compare the spectra for μ and E_{HN}. All the spectra are associated with a UAFR ν_F=0.008, that it corresponds to a 125 years return period. In order to plot the spectra of ductility (μ), we consider a ductility capacity μ=4 that is representative value in structures with level moderate to high of ductility capacity. In the case of hysteretic energy, it was use a capacity E_{HNC}=9, recommended by Terán-Gilmore and Jirsa (2005) for structures with good detailing (ductile structures with stable hysteretic cycles given by the parameter b=1.5 in the Terán and Jirsa damage index), a more detailed explanation is given in the described paper.



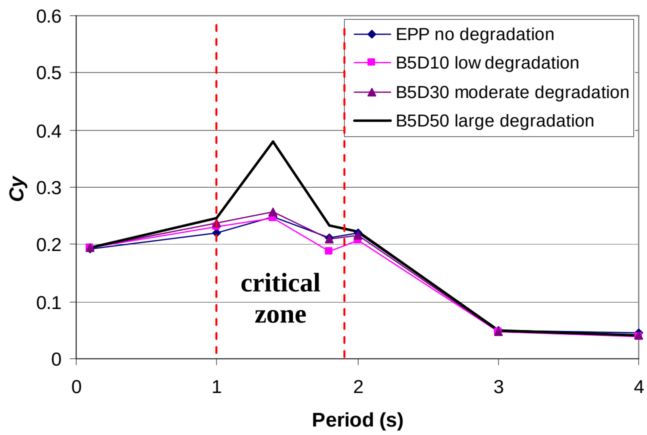
a) Spectrum for μ=4 b) Spectrum for E_{HNC}=9

Figure 5 UAFR spectra with ν_F=0.008 (125 years return period) and different levels of post-yield stiffness

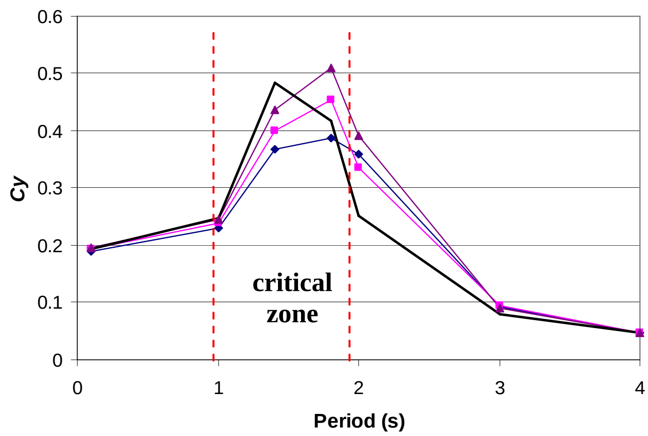
It can be seen in figure 5 for both parameters analyzed and most of the periods no significant influence of the post-yield stiffness, except in the case of rigid structures and for the ductility UAFR spectra in a range of periods from 0.1 to 1 sec, in these cases important reductions are observed in the seismic coefficient *Cy* when it is increased the post-yield stiffness. However, the use of EPP model can estimated reasonably the lateral strength required in structures with bilinear behavior and with different post-yield stiffness. In the case of normalized hysteretic energy there is not evidence to show effect of the post-yielding stiffness. For ductility and hysteretic energy, the large values of seismic coefficient required are observed for structures with periods near to the period of the soil (T_s=2s), especially those with the softening effects.

6. EFFECTS OF STRENGTH AND STIFFNESS DEGRADATION FOR UAFR SPECTRA

In this part, the effect of structures modeled with different levels in strength and stiffness in UAFR spectra is studied. For this purpose, the constitutive laws EPP, B5D10, B5D30 and B5D50 are selected to account for the mechanical characteristics degradation. As in the case illustrated before, the UAFR spectra are obtained for μ=4, but the E_{HNC} was selected in function of the level of degradation, because structures with high degradation levels would have a less plastic energy capacity, which also should be reflected in the degradation function. For this study, the E_{HNC} used are: 9, 7.5, 5.6 and 4.5 for models EPP, B5D10, B5D30 and B5D50, these values were obtained for b=1.5, 1.4, 1.2 and 1.0, where b is a parameter to account for the characteristics of the hysteretic cycle in the Terán and Jirsa damage index). A UAFR ν_F=0.008 corresponding to a 125 years return period is used. Figure 6 shows the results obtained. For UAFR ductility spectra, in structures with large degrading in stiffness, it can be observed larger values of the seismic coefficient, when the vibration periods are in the region of the softening effect (periods smaller than the soil period). It is interesting to note that in a previous study developed by Meli and Avila (1989), they conclude that the most of damage recorded in the 1985 Mexican Earthquake were for structures in a range of periods from 0.5 to 1.5 (structures very potential influenced by the effect of softening). For this reason, especial careful is necessary in this kind of structures. It is recommended for structures sensitive to the effect of softening the use of constitutive laws to account for strength and stiffness degradation, or alternatively the use of a corrective factor to account the mechanical characteristics degradation. In structures with low degradation in strength and stiffness (e.g. steel members or steel frames designed with capacity requirements), the use of simplify hysteretic models as the elastoplastic give reasonable results. For the UAFR normalized hysteretic energy spectra to account for ground motion duration, the evidence suggests that similar results are valid that in the case of ductility UAFR spectra. That is, the structures in the softening zone required special attention (critical zone in figure 6). In particular, the lateral resistance required in structures with moderate to high degradation is larger than those derived from simplified systems. It is important to observe that the requirements of the seismic coefficient by using the dissipated hysteretic energy are larger than those for ductility for structures with periods close to the soil period, which represent the importance to account for parameters related with the effect of ground motion duration.



a) Spectrum for ductility



b) Spectrum for hysteretic energy

Figure 6 UAFR spectra with ν_F=0.008 (125 years return period) and different levels of degradation

7. CONCLUSIONS

The influence of hysteretic models with degradation in strength and stiffness in the evaluation of UAFR spectra was studied. This paper contemplates not only parameters to account for maximum demands (displacement ductility), also parameters related with the ground motion duration effects (normalized dissipated hysteretic energy). From the results here presented it can be concluded the following:

1. There is not evidence supporting important influence of the post-yielding stiffness in the assessment of UAFR spectra for ductility and normalized dissipated hysteretic energy. Except, for structures with short periods; however, the elasto-plastic model give reasonable results compared with bilinear models with different post-yielding stiffness.

2. Structures in the region of softening are more sensitive to the effects of strength and stiffness degradation, as it was observed for the UAFR ductility and normalized dissipated hysteretic energy spectra evaluated. It is recommended further studies focusing to know the real normalized hysteretic energy capacity of structures, because the effect of the reduction in the plastic cycles can provokes important reductions in the hysteretic energy capacity of the system.

3. For structures with low degradation in strength and stiffness, as in the case of steel members and/or steel frames designed with the capacity requirements, the use of simplify hysteretic models as EPP result appropriated in the evaluation of UAFR spectra to account for maximum and cumulative demands.

4. In the case of structures with important degrading levels (e.g. concrete structures) and with periods near to the soil period that suffer the effect of softening, it is necessary the use of more sophisticated constitutive laws. Nevertheless, due to all the uncertainties associated in the response and in the assessment of sophisticated rules where an important influence of the load history must be accounted, it could be more appropriated considerer the most critical situation in the degradation of the mechanical characteristics, and with this to use specific factors to consider different levels of strength and stiffness degradation to provide the requirements of lateral strength for earthquake resistant structures, accounted for cumulative demand and with the incorporation of the structural reliability, which can be done though UAFR spectra. In those structures that do not fall in this case, the use of simplify models can be an important tool for the knowledge of the lateral requirements for seismic design of structures.

5. Finally, it was observed the importance to consider some structural parameters related with the effect of ground motion duration for structures with period near to the soil period, where the lateral resistance requirements are larger than in the case of the ductility displacement control.

ACKNOWLEDGEMENTS

The study presented in this paper was developed within the activities of Rete dei Laboratori Universitari di Ingegneria Sismica – ReLUIS for the research program founded by the Dipartimento di Protezione Civile.

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