Optimum Design Method of Viscous Dampers in Building Frames Using Calibration Model

M. Yamakawa[1], Y. Nagano[2], Y. Lee[3], K. Uetani[4]

[1]*Assistant Professor, Dept. of Architecture and Architectural Engineering, Kyoto University, Kyoto, Japan*
[2]*Associate Professor, Dept. of Architecture and Civil Eng., Fukui University of Technology, Fukui, Japan*
[3]*Researcher, Dept. of Architecture and Architectural Engineering, Kyoto University, Kyoto, Japan*
[4] *Professor, Dept. of Architecture and Architectural Engineering, Kyoto University, Kyoto, Japan*
Email: yamakawa@archi.kyoto-u.ac.jp, nagano@fukui-ut.ac.jp, archi_yujin@yahoo.co.jp, uetani@archi.kyoto-u.ac.jp

ABSTRACT :

We present a new practical optimum design method of viscous dampers in building frames. In the design field of structural engineering, structural designers always try to build mathematical models to simulate the exact behavior of real world system which are often very complex for the verification analysis and are not suited for the optimum design owing to the fact that they come with a good deal of computational cost. Therefore, we present a method using calibrated response model, which is simpler than the verification model. The calibration model is a statistical prediction of the verification model and with its usage the optimum design method of dampers can be made efficient and numerically stable. The efficiency of the presented method has been demonstrated by a numerical example.

KEYWORDS : Optimum design, Viscous damper, Calibration, Bayesian inference

1. INTRODUCTION

Structural designers, in structural design, always try to develop mathematical models which can accurately simulate the behavior of the real world system. However, these models for the verification analysis are often complex and are not suited for the optimum design owing to the face that they come with a good deal of computational cost. Furthermore, it is difficult to understand the characteristics of the optimum solution. For this reason some studies of an optimum design method of viscous dampers [for example, Tsuji et al., 2000] in building frames have dealt with simpler model than the model widely used for the verification analysis. Here, we present a new practical optimum design method of viscous dampers in building frames using statistical prediction model within Bayesian framework. The presented method is based on the calibrated output of the simple model and is very efficient in view of computational cost and numerically stability.

1.1. Multi-level Models

Kennedy and O'Hagan (2001) modeled the outputs of multi-level computer codes using a spatial autocorrelation structure within Bayesian framework. In this paper, we use this multi-level model: verification model and simple model. We define a model using nonlinear time history response analysis as verification model and a model using extended CQC method [Igusa et al., 1984] as simple model. The calibration model is defined as the scaled simple model based on computer experiments. The response by verification model is predicted as the scaled simple mode. The method is very efficient and numerically stable. Using this method, we formulate the optimum design problem, which finds a set of viscous dampers, the cross sections of columns and beams that minimize the cost function subject to some constraints.

2. CALIBRATION MODEL

From a Bayesian perspective, uncertainty about the output of complex model can be expressed by a stochastic process. The prediction method is assumed to be a function of a set of inputs denoted by $\boldsymbol{x} = (x_1, \cdots, x_{n_X}) \subset \mathbb{R}^{n_X}$, with output of complex model represented by $y \in \mathbb{R}$. Let us consider a model:

$$y(\boldsymbol{x}) = \eta(\boldsymbol{x}) + \varepsilon\,, \tag{2.1}$$

where ε is a random noise and follows an independent normal distribution: $\varepsilon \sim \mathcal{N}(0, \alpha\sigma^2/(1+\alpha))$. The parameters σ^2 and α is unknown scale parameter and variance ratio. Such models typically include an unknown smooth response surface, which is knows as *"nugget effect"*.

2.1. Gaussian Process

We assume that $\eta(\boldsymbol{x})$ is represented by Gaussian process [for example, Rasmussen et al., 2006] with mean $m_0(\boldsymbol{x})$ and covariance function $V_0(\boldsymbol{x}, \boldsymbol{x}')$. Gaussian process using a hierarchical formulation is expressed as

$$\eta(\boldsymbol{x}) \mid \boldsymbol{\beta}, \sigma^2, \psi \sim \mathcal{GP}\left(m_0(\boldsymbol{x}), V_0\left(\boldsymbol{x}, \boldsymbol{x}'\right)\right), \tag{2.2}$$

where $\mathcal{GP}(\cdot,\cdot)$ denotes Gaussian process distribution, and

$$m_0(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \boldsymbol{\beta}\,, \tag{2.3}$$

$$V_0\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \sigma^2 R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right) / (1+\alpha)\,, \tag{2.4}$$

where $\boldsymbol{h}(\boldsymbol{x}) : \mathbb{R}^{n_X} \to \mathbb{R}^q$ is a function of the inputs $\boldsymbol{x}$, which is the simple model. The parameter $\boldsymbol{\beta} \in \mathbb{R}^q$ is an unknown vector of coefficients, and $R(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi})$ is a given correlation function. In this paper, we use Gaussian correlation function as follows

$$R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right) = \exp\left[-\sum_{k=1}^{q}\left(h_k(\boldsymbol{x}) - h_k\left(\boldsymbol{x}'\right)\right)^2 / \psi_k\right], \tag{2.5}$$

where the hyper parameter $\boldsymbol{\psi} = \{\psi_1, \cdots, \psi_q\}$ are called as correlation length parameters. We have a training dataset $\mathcal{D} = \{\boldsymbol{x}^i \in \mathbb{R}^{n_X}, y_i = y(\boldsymbol{x}^i); i = 1, \cdots, n\}$. According to (2.2), the distribution of $\boldsymbol{\eta} = (\eta(\boldsymbol{x}^1), \cdots, \eta(\boldsymbol{x}^n))$ is multivariate normal as follows

$$\boldsymbol{\eta} \mid \boldsymbol{\beta}, \sigma^2, \boldsymbol{\psi} \sim \mathcal{N}\left(\boldsymbol{H}\boldsymbol{\beta}, \sigma^2/(1+\alpha)\boldsymbol{R}_0\right), \tag{2.6}$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}\left(\boldsymbol{x}^1\right)^T \\ \vdots \\ \boldsymbol{h}\left(\boldsymbol{x}^n\right)^T \end{bmatrix}, \quad \boldsymbol{R}_0 = \begin{bmatrix} R\left(\boldsymbol{x}^1, \boldsymbol{x}^1; \boldsymbol{\psi}\right) & \cdots & R\left(\boldsymbol{x}^1, \boldsymbol{x}^n; \boldsymbol{\psi}\right) \\ \vdots & \cdots & \vdots \\ R\left(\boldsymbol{x}^n, \boldsymbol{x}^1; \boldsymbol{\psi}\right) & \cdots & R\left(\boldsymbol{x}^n, \boldsymbol{x}^n; \boldsymbol{\psi}\right) \end{bmatrix}. \tag{2.7),(2.8}$$

Using standard techniques for conditioning in multivariate normal distributions, we get Gaussian process prediction with mean $m_0^*(\boldsymbol{x})$ and covariance function $V_0^*(\boldsymbol{x}, \boldsymbol{x}')$ as

$$\eta(\boldsymbol{x}) \mid \alpha, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\psi}, \boldsymbol{y} \sim \mathcal{GP}\left(m_0^*(\boldsymbol{x}), V_0^*\left(\boldsymbol{x}, \boldsymbol{x}'\right)\right), \tag{2.9}$$

where

$$m_0^*(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \boldsymbol{\beta} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{\beta})\,, \tag{2.10}$$

$$V_0^*\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \sigma^2\left\{R\left(\boldsymbol{x}, \boldsymbol{x}'; \boldsymbol{\psi}\right)/(1+\alpha) - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\boldsymbol{r}\left(\boldsymbol{x}'\right)\right\}, \tag{2.11}$$

$$\boldsymbol{R} = \left(\boldsymbol{R}_0 + \alpha \boldsymbol{I}_n\right) / \left(1+\alpha\right), \tag{2.12}$$

$$\boldsymbol{r}(\boldsymbol{x}) = \begin{pmatrix} R\left(\boldsymbol{x}, \boldsymbol{x}_1; \boldsymbol{\psi}\right) / \left(1+\alpha\right) & \cdots & R\left(\boldsymbol{x}, \boldsymbol{x}_n; \boldsymbol{\psi}\right) / \left(1+\alpha\right) \end{pmatrix}^T. \tag{2.13}$$

where $\boldsymbol{I}_n$ denotes the identity matrix of size n. The model is also known as *"kriging"*. *Kriging* is mostly used in two or three dimensional input spaces for spatial prediction, although Gaussian process prediction could be used in a general regression context.

2.2. Gaussian Process within Bayesian Framework

Using a weak prior for $p(\boldsymbol{\beta}, \sigma^2) \propto \sigma^{-2}$, integrating out $\boldsymbol{\beta}$ and σ^2 of (2.9) by Bayes' theorem, the Student's *t* process predictor with $n-q$ degrees of freedom, mean $m(\boldsymbol{x})$ and covariance function $V(\boldsymbol{x}, \boldsymbol{x}')$ can be shown as

$$\eta(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \psi \sim \mathcal{TP}\left(n-q, m(\boldsymbol{x}), V\left(\boldsymbol{x}, \boldsymbol{x}'\right)\right) \tag{2.14}$$

where $\mathcal{TP}(\cdot,\cdot,\cdot)$ denotes Student's *t* process distribution, and

$$m(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T \hat{\boldsymbol{\beta}} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\left(\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{\beta}}\right), \tag{2.15}$$

$$\begin{aligned} V\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \hat{\sigma}^2 \Big[& C\left(\boldsymbol{x}, \boldsymbol{x}'; \psi\right) / \left(1+\alpha\right) - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \boldsymbol{r}\left(\boldsymbol{x}'\right) \\ & + \left(\boldsymbol{h}(\boldsymbol{x})^T - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\boldsymbol{H}\right)\left(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\left(\boldsymbol{h}\left(\boldsymbol{x}'\right) - \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{r}\left(\boldsymbol{x}'\right)\right)\Big], \end{aligned} \tag{2.16}$$

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{y}, \tag{2.17}$$

$$\hat{\sigma}^2 = \boldsymbol{y}^T\left(\boldsymbol{R}^{-1} - \boldsymbol{R}^{-1}\boldsymbol{H}\left(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\right)\boldsymbol{y} / \left(n-q-2\right). \tag{2.18}$$

According to (2.1) and (2.14), conditional expectation and variance are obtained as follows:

$$\mathbb{E}\left[y(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \boldsymbol{\psi}\right] = \boldsymbol{h}(\boldsymbol{x})^T \hat{\boldsymbol{\beta}} + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\left(\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{\beta}}\right), \tag{2.19}$$

$$\begin{aligned} \mathbb{V}\left[y(\boldsymbol{x}) \mid \boldsymbol{y}, \alpha, \boldsymbol{\psi}\right] = \hat{\sigma}^2 \Big[& 1 - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1}\boldsymbol{r}(\boldsymbol{x}) \\ & + \left(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{r}(\boldsymbol{x})\right)^T\left(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\left(\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{r}(\boldsymbol{x})\right)\Big]. \end{aligned} \tag{2.20}$$

3. OPTIMUM DESIGN METHOD

We present an optimum design method which enables to find minimum cost so as to satisfy member-end strain constraints, story drift constraints, and side constraints. Consider a planar steel building frame added Maxwell-type viscous dampers as shown in Figure 1. The nodal mass is taken into account in every node. Rigid diaphragms are assumed for floor slabs and the dampers connect these rigid diaphragms.

3.1. Design Variable

The cross sectional area of the i th member A_i is associated with the design variables $\boldsymbol{x} = \{x_l; l = 1, \cdots, n_X\}$ as $A_i = x_l \ (i \in \mathcal{I}_{Al})$ with the index set $\mathcal{I}_{Al}$. We simply write it as $A_i(\boldsymbol{x})$. The cross sectional area $A_i(\boldsymbol{x})$, second moment area $I_i(\boldsymbol{x})$ and section modulus $Z_i(\boldsymbol{x})$ are assumed to satisfy:

for beams: $I_i(\boldsymbol{x}) = 4.0\left(A_i(\boldsymbol{x})\right)^2$, $Z_i(\boldsymbol{x}) = 1.5\left(A_i(\boldsymbol{x})\right)^{1.5}$ $(i = 1, \cdots, n_M)$, (3.1),(3.2)

for columns: $I_i(\boldsymbol{x}) = 1.2\left(A_i(\boldsymbol{x})\right)^2$, $Z_i(\boldsymbol{x}) = 0.8\left(A_i(\boldsymbol{x})\right)^{1.5}$ $(i = 1, \cdots, n_M)$, (3.3),(3.4) where n_M denotes the number of members. Maxwell-type bilinear viscous elements which depend on velocity such as oil damper are added to the frame shown in Figure 1. The jth damper has bilinear damping coefficient: c_{Dj} and $0.1\times c_{Dj}$, relief load f_{Rj}, maximum load f_{Dj} as shown in Figure 2 and the stiffness $k_{Dj}(\boldsymbol{x})$. The f_{Dj} is also associated with $\boldsymbol{x}$ as $f_{Dj}=x_l\ (j\in\mathcal{I}_{Dl})$ with the index set $\mathcal{I}_{Dl}$, which we simply write as $f_{Dj}(\boldsymbol{x})$. The damping coefficient $c_{Dj}(\boldsymbol{x})$ and stiffness $k_{Dj}(\boldsymbol{x})$ are given as follows:

$$c_{Dj}(\boldsymbol{x})=(1/4.38)f_{Dj}(\boldsymbol{x}),\quad k_{Dj}(\boldsymbol{x})=2.0f_{Dj}(\boldsymbol{x})\qquad (j=1,\cdots,n_F) \tag{3.5),(3.6}$$

where n_F denotes the number of stories, and the units of $f_{Dj}(\boldsymbol{x}), c_{Dj}(\boldsymbol{x})$ and $k_{Dj}(\boldsymbol{x})$ are kN, $\mathrm{kN/(cm/sec)}$ and $\mathrm{kN/cm}$, respectively.

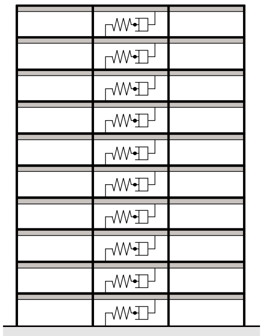

Figure 1 Planar steel frame added Maxwell-type viscous dampers

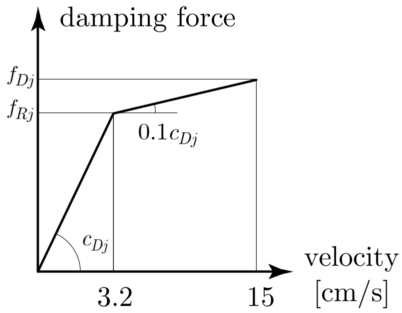

Figure 2 Damping characteristic of damper

3.2. *Verification Model*

We perform the nonlinear time history response analysis by Newmark-β method as the verification model. The design earthquake motions and the amplitude of the scaled design earthquakes are as shown in Table 1.

Table 1 Design earthquakes (by Building Center of Japan)

	Maximum amplitude	
	seismic velocity $(\mathrm{cm/s})$	seismic acceleration $(\mathrm{cm/s^2})$
EL CENTRO 1940 NS	255	25
TAFT 1952 EW	248	25
BCJ-L1 (artificial seismic wave)	207	29

Let $\delta^V_{j\max}(\boldsymbol{x})$ denote the maximum story drift of the jth story to the design earthquakes by time history response analysis. In this model, the dampers are bilinear viscous elements, however, the columns and the beams are elastic elements for simplicity. We suppose that the nonlinear elasto-plastic elements can be used.

3.3. *Simple Model*

We use the extended CQC method as simple model. Let ω and h denote an eigen frequency and a modal damping ratio. Design response spectrum [Newmark and Hall, 1982] is modified for the design earthquakes shown in Table 1. The following design displacement response spectrum is used here.

$$S_D(\omega,h)=\min\left[S_D^A(\omega,\mathrm{h}),S_D^V(\omega,\mathrm{h})\right] \tag{3.7}$$

$$S_D^A(\omega,\mathrm{h})=261.1\left\{4.42\text{-}1.00\ln(100\mathrm{h})\right\}/\omega^2\ (\mathrm{cm}) \tag{3.8}$$

$$S_D^V(\omega,\mathrm{h})=32.4\left\{2.62\text{-}0.51\ln(100\mathrm{h})\right\}/\omega\quad (\mathrm{cm}) \tag{3.9}$$

Let $\delta_{j\max}^{S}(\boldsymbol{x})$ denote the maximum story drift to the design response spectrum by extended CQC method. Note that, in this simple model, the j th damper is linear viscous element which has the initial damping coefficient of the verification model c_{Dj}. Additionally, we also use output of CQC method with no damper model, that is, $c_{Dj}=0$ as simple model. Let $\delta_{j\max}^{S0}(\boldsymbol{x})$ denote the maximum story drift to the design response spectrum by CQC method in the no damper model. $\delta_{j\max}^{V}(\boldsymbol{x})$ will be in between $\delta_{j\max}^{S}(\boldsymbol{x})$ and $\delta_{j\max}^{S0}(\boldsymbol{x})$.

3.4. *Original Optimum Design Problem*

The cost function of the frame model is given as

$$f(\boldsymbol{x})=\sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x})+\sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.10}$$

wehre l_i, ρ, w_F and w_D are the length of the i th member, the density of steel, the cost factor of frame and damper, respectively. We define the *Original Optimum Design Problem* (*OODP*) as follows:

OODP

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x})=\sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x})+\sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.11}$$

$$\text{subject to} \quad -\bar{\delta}_{j\max} \le \delta_{j\max}^{V}(\boldsymbol{x}) \le \bar{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.12}$$

$$-\bar{\varepsilon}_{\max} \le \varepsilon_{i\max}(\boldsymbol{x}) \le \bar{\varepsilon}_{\max} \quad (j=1,\cdots,n_F) \tag{3.13}$$

$$\bar{x}_l^L \le x_l \le \bar{x}_l^U \quad (l=1,\cdots,n_X) \tag{3.14}$$

where $\varepsilon_{i\max}(\boldsymbol{x})$ denotes the maximum member-end strain of the i th member. $\bar{\delta}_{j\max}$, $\bar{\varepsilon}_{\max}$, $\bar{x}_l^L$ and $\bar{x}_l^U$ denote the maximum story drift, the maximum member-end strain, lower and upper bound of the design variable, respectively. For simplicity, we use extended CQC method for $\varepsilon_{i\max}(\boldsymbol{x})$.

3.5. *Solution Algorithm with Bayesian Inference*

Time history response analysis needs much computational cost than extended CQC method. For this reason, the directly solving *OODP* is expensive in terms of the computer time. Besides, the time history response analysis is strongly nonlinear and non-convex, hence solving *OODP* is not numerically stable. Therefore, we present an optimization method with the statistical prediction model instead of solving *OODP* directly. The solution algorithm is shown as follows:

Step 1. Solving the *Simple Optimum Design Problem*

Firstly, we solve the following *Simple Optimum Design Problem* (*SODP*).

SODP

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x})=\sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x})+\sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.15}$$

$$\text{subject to} \quad -\bar{\delta}_{j\max} \le \delta_{j\max}^{S}(\boldsymbol{x}) \le \bar{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.16}$$

$$-\bar{\varepsilon}_{\max} \le \varepsilon_{i\max}(\boldsymbol{x}) \le \bar{\varepsilon}_{\max} \quad (i=1,\cdots,n_M) \tag{3.17}$$

$$\bar{x}_l^L \le x_l \le \bar{x}_l^U \quad (l=1,\cdots,n_X) \tag{3.18}$$

The solution of the *SODP* is denoted by $\hat{\boldsymbol{x}}^0=\{\hat{x}_l^0; l=1,\cdots,n_X\}$. Extended CQC method is modal analysis for linear model, as a result solving *SODP* needs low computational cost.

Step 2. Carrying out computer experiments

We carry out the computer analyses using both the simple model and the verification model at the set of the points $\{\boldsymbol{x}^i; i=1,\cdots,n\}$. The points are chosen to be satisfied

$$\overline{x}_l^L \le x_l^i \le \overline{x}_l^U \quad (i=1,\cdots,n,\ l=1,\cdots,n_X). \tag{3.19),(3.20}$$

$$\hat{x}_l^0 - \Delta\overline{r}_l \le x_l^i \le \hat{x}_l^0 - \Delta\overline{r} \quad (i=1,\cdots,n,\ l=1,\cdots,n_X). \tag{3.21),(3.22}$$

where x_l^i and $\Delta\overline{r}_l$ denote the l th element of $\boldsymbol{x}^i$ and change of the l th design variable, respectively. Here, we set the equations presented in section 2 as follows:

$$y^{(j)}(\boldsymbol{x}) = \delta_{j\max}^V(\boldsymbol{x}) \quad (j=1,\cdots,n_F), \tag{3.23}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \left(\delta_{1\max}^S(\boldsymbol{x}),\cdots,\delta_{n_F\max}^S(\boldsymbol{x}),\delta_{1\max}^{S0}(\boldsymbol{x}),\cdots,\delta_{n_F\max}^{S0}(\boldsymbol{x})\right)^T. \tag{3.24}$$

In equation (3.23), the method in section 2 is applied for each $j=1,\cdots,n_F$. Thus, we get the training dataset of j th story $\mathcal{D}_j = \{\boldsymbol{x}^i, y_i^{(j)}; i=1,\cdots,n\}$. Various methods of choosing points are presented [for example, Santner, et al., 2003]. Here, the points are generated randomly for simplicity.

Step 3. Finding the hyper parameters by cross validation

We find the optimum hyper parameters of each $y^{(j)}(\boldsymbol{x})$ by minimizing leave-one-out cross-validated sum of squared error as follows:

$$\hat{\alpha}^{(j)},\hat{\boldsymbol{\psi}}^{(j)} = \underset{\alpha^{(j)},\boldsymbol{\psi}^{(j)}}{\arg\min} \sum_{k=1}^{n} \sum_{l\in\{\mathcal{I}_k\}} \left\{y_l^{(j)} - \mathbb{E}\left[y(\boldsymbol{x}_l) \mid \boldsymbol{y}_{\mathcal{I}_k}^{(j)},\alpha^{(j)},\boldsymbol{\psi}^{(j)}\right]\right\}^2, \tag{3.25}$$

where $\mathcal{I}_k = \{1,\cdots,n\}\setminus\{k\}$ and $\boldsymbol{y}_{\mathcal{I}_k}^{(j)} = \left(y_k^{(j)}; k\in\mathcal{I}_k\right)$.

Step 4. Finding the optimum solution with Bayesian inference

We solve the following *Optimum Design Problem with Bayesian Inference* (*ODPBI*).

<u>*ODPBI*</u>

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}) = \sum_{i=1}^{n_M} w_F \rho l_i A_i(\boldsymbol{x}) + \sum_{j=1}^{n_F} w_D f_{Dj}(\boldsymbol{x}) \tag{3.26}$$

$$\text{subject to} \quad \left|\mathbb{E}\left[\delta_{j\max}^V(\boldsymbol{x}) \mid \boldsymbol{y}^{(j)},\hat{\alpha}^{(j)},\hat{\psi}^{(j)}\right]\right| + \sqrt{\mathbb{V}\left[\delta_{j\max}^V(\boldsymbol{x}) \mid \boldsymbol{y}^{(j)},\hat{\alpha}^{(j)},\hat{\boldsymbol{\psi}}^{(j)}\right]} \le \overline{\delta}_{j\max} \quad (j=1,\cdots,n_F) \tag{3.27}$$

$$-\overline{\varepsilon}_{\max} \le \varepsilon_i(\boldsymbol{x}) \le \overline{\varepsilon}_{\max} \quad (i=1,\cdots,n_M) \tag{3.28}$$

$$\overline{x}_l^L \le x_l \le \overline{x}_l^U \quad (l=1,\cdots,n_X) \tag{3.29}$$

$$\hat{x}_l^0 - \Delta r_l \le x_l \le \hat{x}_l^0 + \Delta r_l \quad (l=1,\cdots,n_X) \tag{3.30}$$

The solution of *ODPBI* is denoted by $\hat{\boldsymbol{x}} = \{\hat{x}_l; l=1,\cdots,n_X\}$. *ODPBI* is based on the calibrated simple model, besides, the output of the calibration model is smoothed out, as a result solving *ODPBI* needs low computational cost and is numerically stable.

Step 5. Verification of the solution

We analyze the solution $\hat{\boldsymbol{x}}$ by verification model. If the output $\{\hat{y}^{(j)}(\hat{\boldsymbol{x}}); j=1,\cdots,n_F\}$ doesn't satisfy the design criteria with sufficient accuracy, $\{\hat{\boldsymbol{x}},\hat{y}^{(j)}(\hat{\boldsymbol{x}})\}$ is added to the training data set $\mathcal{D}_j$ as a new data. Moreover, we repeat from step3 until the solution satisfies the design criteria.

4. NUMERICAL EXAMPLE

Consider 10-story and 3-span planar steel frames as shown in Figure 3. Cross sections of columns and beams are denoted by C1-C8 and G1-G10, respectively. The dampers of the 1-10th story are denoted by D1-D10.

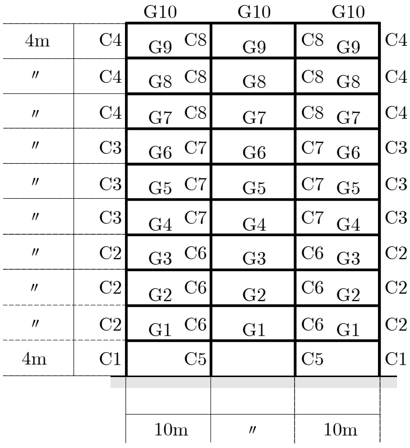

Figure 3 10-stories and 3-span model

The structural damping ratio is assumed as $h = 2\%$. A lamped mass of 326.5kg and 163.3kg are placed on every interior node and exterior node, respectively. The parameters of side constrains are as shown:

for beams: $\bar{x}_l^L = 100\text{cm}^2$, $\bar{x}_l^U = 400\text{cm}^2$, $\Delta r_l = 90\text{cm}^2$,
for columns: $\bar{x}_l^L = 200\text{cm}^2$, $\bar{x}_l^U = 800\text{cm}^2$, $\Delta r_l = 180\text{cm}^2$,
for dampers: $\bar{x}_l^L = 0\text{kN}$, $\bar{x}_l^U = 100\text{kN}$, $\Delta r_l = 30\text{kN}$.

The other parameters are as follows: $\bar{\varepsilon}_{\max} = 0.00157$, $\bar{\delta}_{j\max} = 2\text{cm}$, $w_F = 25/\text{ton}$, $w_D = 0.15/\text{kN}$. Here, practical constraint conditions are added to *SODP* and *ODPBI*. The condition is that a cross section of a column is smaller than cross sections of the lower columns ($C1 > C2 > C3 > C4$, $C5 > C6 > C7 > C8$).
Firstly, we obtain the initial solution $\hat{\boldsymbol{x}}^0$ by solving *SODP* with sequential quadratic programming (step1). Next, computer experiments, of which the number is given by $n = 100$, are carried out (step2). Lastly, we obtain the optimum solution $\hat{\boldsymbol{x}}$ by 10 iterations of solving and updating *ODPBI* with sequential quadratic programming (from step3 to step 5). The solutions are shown in Table 2. The costs and maximum story drift angles by time history response analysis of the optimum solutions $\hat{\boldsymbol{x}}^0$ and $\hat{\boldsymbol{x}}$ are shown in Table 3, where $H = 400\text{cm}$ denotes the story height. It can be observed that the costs of $\hat{\boldsymbol{x}}^0$ and $\hat{\boldsymbol{x}}$ are nearly the same. The maximum story drift angles of the solution $\hat{\boldsymbol{x}}^0$ violate the criteria $\bar{\delta}_{j\max}/H = 5/1000\,\text{rad}$ largely, by contrast, the maximum story drift angles of the solution $\hat{\boldsymbol{x}}$ approximately satisfy the criteria $5/1000\,\text{rad}$ because the predicted mean and variance have good accuracy.

5. CONCLUSION

The conclusion may be summarized as follows:

(1) We present a new optimum design method of a building frame with viscous dampers using the calibration model, which is based on the statistical multi-level analysis. The method has similar accuracy as the verification analysis and the computational cost is much smaller than the verification analysis.

(2) The efficiency of the presented method is demonstrated by numerical example. In the example, the predicted mean and variance have good accuracy. Consequently, the solution approximately satisfies the constraints.

Table 2 Optimum solutions

Cross sectional area of Colum (cm^2)			Cross sectional area of Beam (cm^2)			Maximum load of Damper (kN)		
	$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$		$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$		$\hat{\boldsymbol{x}}^0$	$\hat{\boldsymbol{x}}$
C1	268.9	254.7	G1	253.7	256.1	D1	100.0	70.0
C2	268.9	254.7	G2	323.4	321.6	D2	100.0	100.0
C3	217.9	209.6	G3	238.1	244.8	D3	100.0	100.0
C4	200.0	200.0	G4	281.6	310.4	D4	100.0	100.0
C5	404.2	370.7	G5	250.3	250.3	D5	100.0	100.0
C6	404.2	370.7	G6	235.8	247.1	D6	100.0	87.1
C7	380.9	352.7	G7	191.7	231.3	D7	100.0	83.9
C8	281.0	316.6	G8	248.6	201.8	D8	100.0	100.0
			G9	108.8	162.4	D9	100.0	70.0
			G10	100.0	118.2	D10	100.0	70.0

Table 3 Cost and maximum story drift angle by time history response analysis of the optimum solution

	Cost		Maximum story drift angles [1/1000 rad]									
			1F	2F	3F	4F	5F	6F	7F	8F	9F	10F
$\hat{\boldsymbol{x}}^0$	2381.2	$\delta^V_{\max}(\hat{\boldsymbol{x}}^0)\,/\,H$	3.00	4.16	4.32	4.42	4.74	4.39	4.64	5.20	5.09	6.59
$\hat{\boldsymbol{x}}$	2398.6	$\delta^V_{\max}(\hat{\boldsymbol{x}})\,/\,H$	**3.75**	**5.09**	**4.94**	**4.90**	**4.98**	**4.80**	**4.35**	**4.77**	**4.55**	**4.78**
		$\mathbb{E}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid \boldsymbol{y}\right]/\,H$	3.74	4.93	4.95	4.81	4.99	4.58	4.61	4.84	4.65	4.56
		$\sqrt{\mathbb{V}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid \boldsymbol{y}\right]}\,/\,H$	0.00	0.07	0.03	0.19	0.01	0.03	0.39	0.16	0.10	0.44
		$\left\lvert\delta^V_{\max}(\hat{\boldsymbol{x}})-\mathbb{E}\left[\delta^V_{\max}(\hat{\boldsymbol{x}})\mid \boldsymbol{y}\right]/\,H\right\rvert$	0.02	0.17	0.01	0.09	0.00	0.23	0.26	0.07	0.10	0.23

Acknowledgments This work was partially supported by the grant from the Kyoto University Foundation and Grant-in-Aid for Scientific Research (No. 17206058) from Japan Society for the Promotion of Science. The authors are grateful for advice from Shiresta K. C. (Research Student, Kyoto University).

REFERENCES

Igusa, T., Der Kiureghian, A. and Sackman, J. L. (1984). Modal decomposition method for stationary response of non-Classically damped systems. *Earthquake Engng. Struct. Dynamics*, 12, 121-136

Kennedy, M. C., and O'hagan, A. (2001). A Bayesian calibration of computer models (with discussion). *Journal of the Royal Statistical Society*, B, 63, 425-464

Newmark, N. M., and Hall, W. J. (1982). Earthquake spectra and design, Earthquake Engineering Research Institute, Berkeley, CA.

Rasmussen, C. E., and Williams, C. K. (2006). Gaussian Processes for Machine Learning, The MIT Press.

Santner, T. J., Williams, B. J., and Notz, W. I. (2003). The design and Analysis of Computer Experiments, Springer-Verlag.

Tsuji, M., Nagano, Y., Ohsaki, M. and Uetani, K. (2000) Optimum Design Method for High-Rise Building