DYNAMIC STRESS CONCENTRATION FOR THE SHALLOW BURIED TUNNEL WITHOUT LINER IMPACTED BY SH-WAVE

Chen Zhigang[1], Yan Qiuhong[2] and Yang Zailin[3]

[1] *Department of Mechanics and Civil Engineering, Jinan University / Key Lab of Disaster Forecast and Control in Engineering, Ministry of Education,Guang Zhou, China*
[2] *College of Liberal Arts, Jinan University,Guang Zhou, China*
[3] *School of Civil Engineering, Harbin Engineering University,Harbin, China*
Email: czgjnu@163.com

ABSTRACT :

The dynamic response for quadratic and U-shaped cavities in half space, which are similar to the cross-section of the tunnels, is studied in this paper impacted by SH-wave. The analytical solution for the cavity in elastic half space is gained by the complex function method. In the complex plane, the scattering wave which satisfies the stress-free condition at the horizontal surface can be constructed, the problem can be inverted into the analysis for the single hole in whole space impacted by SH-wave and a set of infinite algebraic equations to solve this problem can be obtained. For the earthquake-resistance researches, the numerical examples of the dynamic stress concentration around the quadratic and U-shaped cavities impacted by SH-wave are given. The influences of the dynamic stress concentration by the incident wave number and angle, the depth and shape of the cavity are discussed. It is showed that the interaction among the wave, the surface and the shallow buried tunnels should be cared in half space. In this situation, the dynamic stress concentration around the tunnel is greater obvious than the whole space. The maximum value of the dynamic stress concentration around the tunnel is 10.0 impacted by SH-wave horizontally.

KEYWORDS: Scattering of SH-waves, Dynamic stress concentration, Shallow buried tunnel

1. INTRODUCTION

Scattering of elastic wave caused by cavity in half space is one of interesting problems, it has great theory and application value in the earthquake resistance question of underground structure. At present, analytical solution to scattering of elastic wave by a cavity in half space mostly choose circular structure as object. V.W.Lee and M.D.Trifunac(1979)obtained analytical solution to incident SH-wave by a circular tunnel in half space by using the wave function expansion method and symmetry of SH-wave; Liu Diankui *et al*(Liu Diankui and Lin Hong 2003; Wang Guoqing and Liu Diankui 2003; Wang Guoqing and Liu Diankui 2002;Wang Yan and Liu Diankui 2002) studied dynamic stress response of shallow buried single circular cavity, multiple circular cavity and circular lining structure to incident SH-wave in half space by using the complex function method and multi-polar coordinates; V.W.Lee and J.Kar(2004) acquired approximate analytic solution of scattering of P-waves, SV-waves by circular cavity in half space by using mortal circular arc to simulate the surface of half space; Liang Jianwen *et al* (2004)studied the scattering of P-waves by multiple circular cavity in half space by using mortal circular arc to simulate the surface of half space. But analytic investigations to noncircular cavity in half space are less comparatively, M.E.Manoonan and V.W.Lee(1996) solved displacement of elastic half space to circular, elliptic and quadratic inclusion to incident SH-wave by using the weighted residual method; Chen Zhigang(2007) solved anti-plane ground motion caused by shallow buried elliptic and quadratic cavity in half space by using the complex function method.

Scattering of SH-wave by quadratic and U-shaped cavities in elastic half space are investigated by using conformal mapping method. For the earthquake-resistance researches, the numerical results of the dynamic stress concentration factor around the cavities impacted by SH-wave with arbitrary incidence angle in half space are given and discussed.

2. THE MODEL OF ANALYSIS

In elastic half space, there is a square or horseshoe hole, the distance between center of the hole and horizontal surface is *h*, the model is shown in Figure.1,steady state SH wave incident with angle α_0 to axis o_1x_1. In elastic half space without cavity, the reflection wave $w^{(r)}$ will be produced because of the horizontal interface, in complex Z ,it can be expressed as

$$w^{(i)} = w_0 e^{\frac{ik}{2}\left[(z-ih)e^{-i\alpha_0}+(\overline{z}+ih)e^{i\alpha_0}\right]} \tag{2.1}$$

$$w^{(r)} = w_0 e^{\frac{ik}{2}\left[(z-ih)e^{i\alpha_0}+(\overline{z}+ih)e^{-i\alpha_0}\right]} \tag{2.2}$$

where: w_0 is the amplitude of the incident wave, $i=\sqrt{-1}$, $e^{-i\omega t}$ is omitted bellow.

The SH-wave propagation in homogeneous and isotropic continuous media should satisfy Helmholtz equation, in complex Z ,it can be expressed as:

$$\frac{\partial^2 w}{\partial z \partial \overline{z}} + \frac{k^2}{4} w = 0 \tag{2.3}$$

where: $w(x,y)$ is the displacement function, the displacement function is $w(x,y)e^{-i\omega t}$ in steady case. $k=\omega/c_s$, ω is circular frequency of the displacement function, $c_s=\sqrt{\mu/\rho}$ is shear wave velocity, ρ , μ are mass density of the medium and shear elastic modulus respectively.

The constitutive equations are:

$$\tau_{xz} = \mu\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial \overline{z}}\right) \quad , \quad \tau_{yz} = \mu\left(\frac{\partial w}{\partial z} - \frac{\partial w}{\partial \overline{z}}\right) \tag{2.4}$$

Let us introduce mapping functions

$$z = \omega(\zeta) = 1.11a\left(\zeta - \frac{1}{9\zeta^3}\right) \tag{2.5}$$

or
$$z = \omega(\zeta) = 1.2987a\left(\zeta - \frac{0.13134}{\zeta} - \frac{0.06094i}{\zeta^2} - \frac{0.10324}{\zeta^3} + \frac{0.0425i}{\zeta^4}\right) \tag{2.6}$$

We can transform the domain on plane Z into unit circle on mapping plane ζ by conformal mapping, then Eqs(2.3) and (2.4) can be written in the following form:

$$\frac{1}{\omega'(\zeta)\overline{\omega'(\zeta)}}\frac{\partial^2 w}{\partial \zeta \partial \overline{\zeta}} + \frac{k^2}{4}w = 0 \tag{2.7}$$

$$\tau_{rz} = \frac{\mu}{|\omega'(\zeta)|}\left(\zeta\frac{\partial w}{\partial \zeta} + \overline{\zeta}\frac{\partial w}{\partial \overline{\zeta}}\right)$$
$$\tau_{\theta z} = \frac{i\mu}{|\omega'(\zeta)|}\left(\zeta\frac{\partial w}{\partial \zeta} - \overline{\zeta}\frac{\partial w}{\partial \overline{\zeta}}\right) \tag{2.8}$$

In which: (r,θ) is curvilinear coordinates correspond to pole coordinates in complex plane.
In complex plane ζ, the incident wave $w^{(i)}$ and the reflected wave $w^{(r)}$ can be written as:

$$w^{(i)} = w_0 e^{\frac{ik}{2}\left\{[\omega(\zeta)-ih]e^{-i\alpha_0} + \left[\overline{\omega(\zeta)}+ih\right]e^{i\alpha_0}\right\}} \tag{2.9}$$

$$w^{(r)} = w_0 e^{\frac{ik}{2}\left\{[\omega(\zeta)-ih]e^{i\alpha_0} + \left[\overline{\omega(\zeta)}+ih\right]e^{-i\alpha_0}\right\}} \tag{2.10}$$

3. SCATTERING WAVE

In complex plane ζ, using the symmetry of the scattering wave and multi-polar coordinates, the scattering wave $w^{(s)}$ caused by cavity in elastic half space (see Figure.1) can be constructed, which should satisfy the equation (2.7), Sommerfeld radiation condition in infinite distance and the stress free condition on the horizontal interface. $w^{(s)}$ can be expressed as

$$w^{(s)} = \sum_{n=-\infty}^{\infty} A_n \left\{ H_n^{(1)}\left[k|\omega(\zeta)|\right]\left[\frac{\omega(\zeta)}{|\omega(\zeta)|}\right]^n + H_n^{(1)}\left[k|\omega(\zeta)-2hi|\right]\left[\frac{\omega(\zeta)-2hi}{|\omega(\zeta)-2hi|}\right]^{-n} \right\} \tag{3.1}$$

where A_n are unknown coefficients; $H_n^{(1)}(\cdot)$ is the first kind Hankel function of n-th order.

4. SOLUTION

The corresponding stresses of the incident wave $w^{(i)}$, the reflected wave $w^{(r)}$ and the scattered wave $w^{(s)}$ expressed by Eqs (2.9), (2.10) and (3.1) respectively satisfy the stress free boundary condition on the horizontal interface. The stress along the circumference of cavity in elastic half space is free, in curvilinear coordinates, the stress free condition on the surface of cavity *S* can be expressed as:

$$\tau_{rz} = \tau_{rz}^{(i)} + \tau_{rz}^{(r)} + \tau_{rz}^{(s)} = 0 \tag{4.1}$$

Substituting Eq.(2.9), Eq.(2.10) and Eq.(3.1)into Eq.(2.8), the corresponding stresses is obtained, and substituting these stresses into boundary condition (12), we have

$$\sum_{n=-\infty}^{\infty} A_n \varepsilon_n - \varepsilon = 0 \tag{4.2}$$

Where:

$$\varepsilon = -iw_0 e^{\frac{ik}{2}\left[\omega(\zeta)e^{-i\alpha_0} + \overline{\omega(\zeta)}e^{i\alpha_0}\right]} \cdot \left[\frac{\zeta\omega'(\zeta)}{|\omega'(\zeta)|}e^{-i\alpha_0} + \frac{\overline{\zeta}\overline{\omega'(\zeta)}}{|\omega'(\zeta)|}e^{i\alpha_0}\right]$$
$$- iw_0 e^{\frac{ik}{2}\left[\omega(\zeta)e^{i\alpha_0} + \overline{\omega(\zeta)}e^{-i\alpha_0}\right]} \cdot \left[\frac{\zeta\omega'(\zeta)}{|\omega'(\zeta)|}e^{i\alpha_0} + \frac{\overline{\zeta}\overline{\omega'(\zeta)}}{|\omega'(\zeta)|}e^{-i\alpha_0}\right]$$

$$\varepsilon_n = \left[H_{n-1}^{(1)}\left(k|\omega(\zeta)|\right)\left(\frac{\omega(\zeta)}{|\omega(\zeta)|}\right)^{n-1} - H_{n+1}^{(1)}\left(k|\omega(\zeta)-2hi|\right)\left(\frac{\omega(\zeta)-2hi}{|\omega(\zeta)-2hi|}\right)^{-(n+1)}\right] \cdot \frac{\zeta\omega'(\zeta)}{|\omega'(\zeta)|}$$
$$- \left[H_{n+1}^{(1)}\left(k|\omega(\zeta)|\right)\left(\frac{\omega(\zeta)}{|\omega(\zeta)|}\right)^{n+1} - H_{n-1}^{(1)}\left(k|\omega(\zeta)-2hi|\right)\left(\frac{\omega(\zeta)-2hi}{|\omega(\zeta)-2hi|}\right)^{-(n-1)}\right] \cdot \frac{\overline{\zeta}\overline{\omega'(\zeta)}}{|\omega'(\zeta)|}$$

Unknown coefficients A_n can be obtained by least square method:

$$\int_{-\pi}^{\pi} \overline{\varepsilon}_m \left(\sum_{n=-\infty}^{\infty} \varepsilon_n A_n - \varepsilon\right) d\theta = 0 \qquad m = 0, \pm 1, \pm 2 \cdots n \tag{4.3}$$

in which: $\overline{\varepsilon}_m$ is complex variable, ε_m is conjugate variable.
Through controlling precision and intercepting finite items, Eq.(14) can be translated into algebraic equations to gain unknown coefficients A_n.

5. DYNAMIC STRESS CONCENTRATION FACTOR

By incident SH-wave, the stress along the circumference of cavity S is

$$\tau_{rz} = 0$$
$$\tau_{\theta z} = \tau_{\theta z}^{(i)} + \tau_{\theta z}^{(r)} + \tau_{\theta z}^{(s)} \tag{5.1}$$

The dynamic stress concentration factor along the circumference of cavity *S* is defined as $\tau_{\theta z}^{*}$ and it can be written as:

$$\tau_{\theta z}^{*} = \left|\tau_{\theta z} / \tau_0\right| \tag{5.2}$$

In which: $\tau_0 = \mu k w_0$ stands for the maximum amplitude of the incident stresses.

6. NUMERICAL RESULTS AND CONCLUSIONS

(1)Figure.2-4 show the variations of DSCF around the square tunnel without liner when the distance between the tip of cavity and the horizontal interface is $h = 0.5a$ (a is half of the length of the square cavity border), and SH-wave incident with different angles and frequency. Here, it can be seen that the maximum value of DSCF always appears on the corner of the square tunnel, and the value of dynamic stress is bigger in the corner which is against the incident direction and close to the horizontal interface by horizontally incident SH-waves, when $ka = 1.0$,the value of DSCF is about 6.0.
(2) Figure.5-7 show the variations of DSCF around the U-shaped tunnel without liner when the distance between the tip of cavity and the horizontal interface is $h = 0.5a$ (a is radius of the semicircular arch of the U-shaped cavity), and SH-wave incident with different angles and frequency. It can be seen that the maximum value of DSCF always appears at the tip of the U-shaped tunnel closed to the horizontal interface by horizontally incident SH-waves. As the ratio of the height to width of the U-shaped tunnel is 1.33, comparing with the square tunnel, the height of the U-shaped tunnel is bigger, when SH-wave incident horizontally, severe dynamic stress concentration phenomenon will occur, so the ratio of the height to width of the tunnel cannot be too high. The maximum value of DSCF appears at the tip of the U-shaped tunnel closed to the horizontal interface by horizontally incident SH-waves, and it is about 10.
(3)Figure.8 shows the variation of DSCF, which is at the tip of the U-shaped tunnel with different wave number by horizontally incident SH-waves, change with h . It can be seen that the value of DSCF decrease obviously with h increasing, with different incident wave number. When $h > 6a$, the value of DSCF tends to be stable. In Fig.8, it also can be seen that the value of DSCF decrease with increasing wave number ka .
From these computational examples and analysis above, the following comments may be made: the interaction of shallow buried tunnel and horizontal interface is severe under dynamic loading, and severe dynamic stress concentration phenomenon will occur near the tunnel, in the corner of the tunnel closed to the horizontal interface, dynamic stress concentration phenomenon is more severe. So, we should pay more attention to the dynamic stress design of shallow buied tunnels.

7. ILLUSTRATIONS, DIAGRAMS AND PHOTOGRAPHS

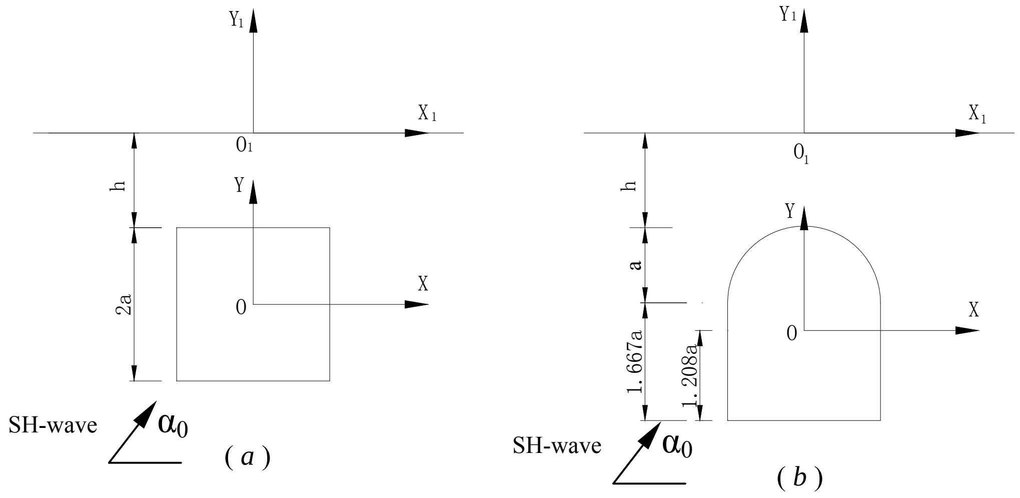

Figure.1 The shallow buried tunnel without liner impacted by SH-wave

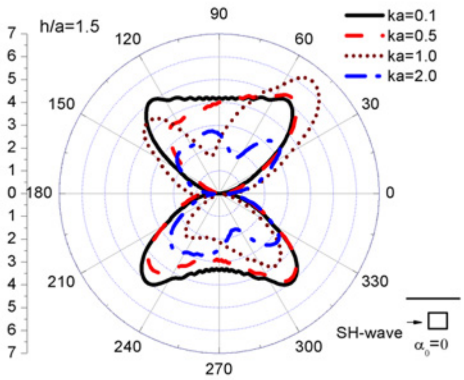

Figure.2 Distribution of DSCF around the edge of square tunnel by incident SH-wave horizontally

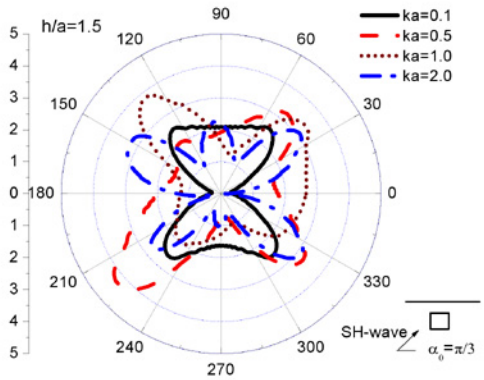

Figure.3 Distribution of DSCF around the edge of square tunnel by incident SH-wave obliquely

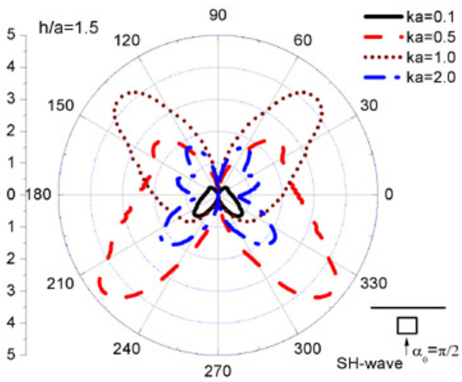

Figure.4 Distribution of DSCF around the edge of square tunnel by incident SH-wave vertically

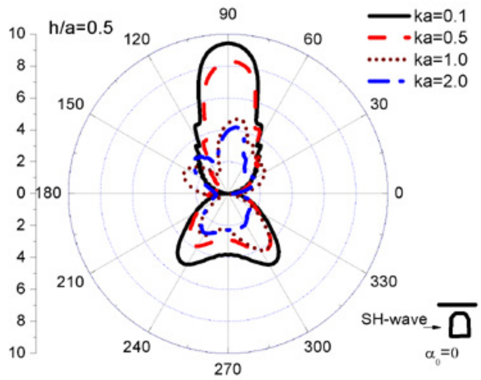

Figure 5 Distribution of DSCF around the edge of U-shaped tunnel by incident SH-wave horizontally

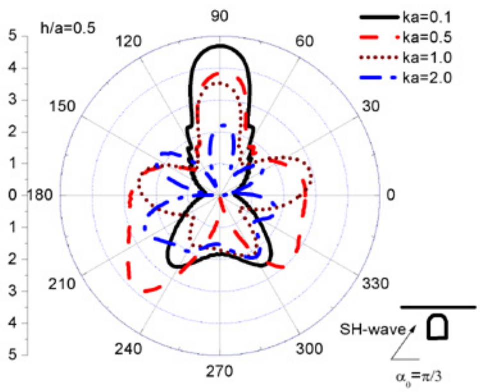

Figure.6 Distribution of DSCF around the edge of U-shaped tunnel by incident SH-wave obliquely

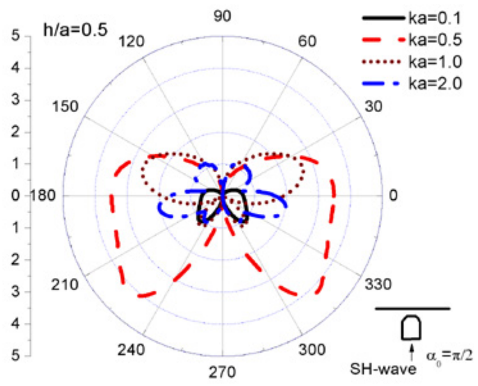

Figure.7 Distribution of DSCF around the edge of U-shaped tunnel by incident SH-wave vertically

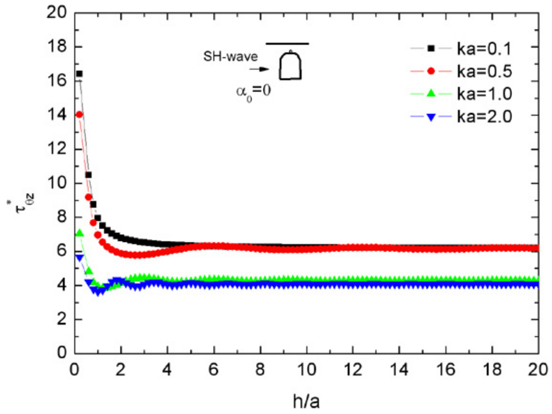

Figure.8 Variation of DSCF at the tip of U-shaped tunnel by incident SH-wave horizontally with h/a

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