FACTORS INFLUENCING ANALYTICAL CONTINUUM SIMULATION OF THREE-POINT BEND TEST OF A CONCRETE NOTCHED BEAM

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ABSTRACT :

Accurate determination of the response of concrete in tension is required for analysis of massive concrete structures designed based on philosophy of un-cracked concrete sections. The three-point bend test of a notched plain concrete beam is a standard test for determining concrete fracture energy and concrete tensile response. Data from these tests are used routinely to evaluate the accuracy with which finite element software and concrete constitutive models simulate response. Performing these analyses requires the analyst to make a number of modeling decisions with respect to the element type, size, and integration rules as well as the material model parameters to be used. In this study, the impact of these decisions on predicted response is investigated and the uncertainty associated with modeling decisions is demonstrated.

KEYWORDS: Continuum finite elements, concrete analysis, concrete tensile response, crack models for concrete.

1. INTRODUCTION

Typically in structural analysis many researchers and practitioners idealize the constitutive behavior of concrete as a material that does not possess any tensile stress. It was shown by Bazant (1996), the above assumption of no-tension design being safe may hold true for typical reinforced concrete structures, however does not hold true particularly for massive structures, such as dams, which are designed based on the philosophy of un-cracked concrete section. Fracture mechanics is required to ensure safety of those structures and nonlinear analysis of these structures required explicit modeling of behavior of concrete in tension. Infact, there exist a lot of concrete structures which are massive and are designed based on the philosophy of un-cracked concrete sections. The examples are massive concrete gravity dams, nuclear containers and silos, offshore concrete platforms, concrete foundation of offshore rigs and offshore wind turbines. The following research concentrates on considerations of tensile response for nonlinear analysis of concrete structures specifically for structures designed based on the philosophy of un-cracked concrete sections.
A RILEM benchmark test of a three-point bend test of a notched concrete beam was considered as a standard test for determining concrete fracture energy and concrete tensile response. Data from these tests are used routinely to evaluate the accuracy with which finite element software and concrete constitutive models simulate response. Performing these analyses within a continuum finite element software framework requires the analyst to make a number of modeling decisions with respect to the element type, size, and integration rules as well as the material model parameters to be used. The current study was performed using commercial finite element software DIANA 9.1. The analyst’s modeling decisions with regards to variation of smeared models for concrete, different element types used for simulation, gauss-quadrature rules, shear retention parameter, threshold angle for subsequent crack formation in multi-directional fixed crack formulation are being investigated in this study.

2. EXPERIMENTAL AND ANALYTICAL PROTOTYPE MODEL

2.1. Experimental test setup

Experimental investigations of a three point notched beam test were carried out in University of Washington. Details of the investigation can be found in Martin et al.(2007). A typical fracture energy test specimen is shown in figure 1. The specimen FR33R1, from experimental investigations by Martin et al. (2007), chosen for the purpose of simulation and parametric evaluation had geometry of length in between pinned supports equal to 18 in, height 6 in, width 3 in, notch depth 3 in and the in-plane width of the notch as 0.25 in. The material parameters as obtained from the experimental investigations were fracture energy G_f equal to 0.80 lb/in, tensile strength f_t as 526.5 psi and elastic modulus E_c as 5315 ksi.

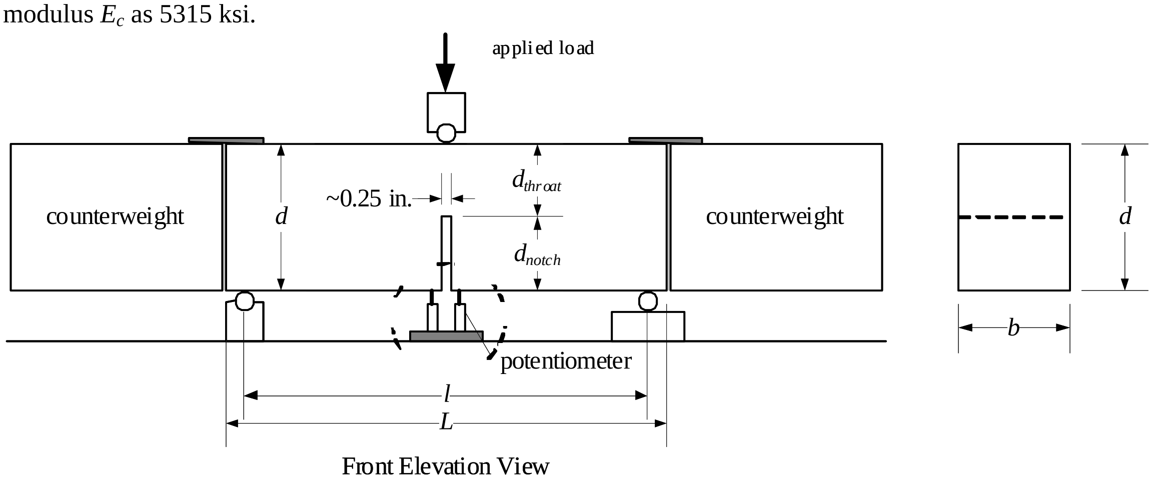


Figure 1. Fracture energy test at University of Washington (Martin et al. 2007)

2.2. Analytical prototype model

A prototype two-dimensional analytical model has been developed using commercial finite-element software DIANA 9.1. A mesh comprising 0.125x0.125 in. eight-node two-dimensional plane stress quadrilateral element using a 2x2 reduced integration scheme has been utilized for the prototype model. Since the compressive strength of concrete is typically 105 to 110 times the tensile strength, the energy absorption due to plastic compressive response is not expected to occur in these tests and all the energy dissipation is assumed to occur through concrete fracture (Hillerborg 1985); thereby the concrete response in compression has been assumed to be elastic with a measured modulus of elasticity for concrete, E_c. The tensile response of concrete in the prototype model has been idealized by a multi-directional smeared fixed crack formulation (deBorst & Nauta 1986) and Hordijk model (Hordijk 1991) for tension softening. A crack in a concrete element is assumed to be induced if the normal stress in a plane at a gauss integration point exceeds the value of the tensile strength of concrete. Successive cracks at a gauss point are assumed to originate if both the tensile strength and the threshold angle for the formation of crack were exceeded. A threshold angle of 60 deg has been chosen, since a large number of cracks would defeat the purpose of robust solution and would generate unnecessary convergence problems. The crack band width is taken as the element dimension perpendicular to the direction of loading; under the assumption that only one major crack in the specimen contributes to failure by cracking. It is also assumed that once cracking originates, the shear stiffness parallel to the crack's surface will remain constant and specified by a reduced stiffness ratio of 0.001.
According to RILEM specifications, controlled loading is to be applied to the top surface of the specimen to produce a specific rate of crack width opening. Since such load-control is not possible for typical finite element solution algorithms, thereby loading in the analytical model has been introduced by monotonically increasing vertical deflection (in the downward direction) at the point of load application.

2.3. Prototype model validation

To validate the modeling strategy employed in the prototype model, simulated results were compared to the experimentally observed results. Figure 2 shows beam mid-span load-deflection response as simulated using the prototype model with varying levels of mesh refinement. Meshes comprising element sizes of 0.125 x 0.25 in. (coarse), 0.125 x 0.1875 in. (fine) and 0.125 x 0.125 in. (superfine) were used in the simulation. Good correlation was observed between the simulated and the observed response. The identical response of different mesh sizes in figure confirms that the solution is not exhibiting mesh sensitivity and verifies that the level of mesh refinement provided in the prototype specimen results in a converged solution. Figure 2 also shows the data for the prototype model with an elastic concrete constitutive model in tension. Comparison of the data for the elastic and inelastic analyses indicates that both constitutive models provide identical response within the elastic range.

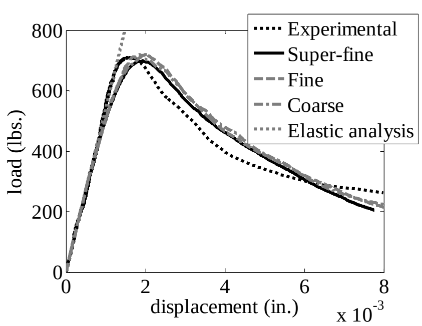


Figure 2. Simulation of fracture energy test

3. INVESTIGATION OF THE IMPACT OF ANALYTICAL MODELING DECISIONS

A number of uncertainties, termed as epistemic uncertainties, are associated with analytical modeling of concrete structures. These primarily arise from lack of accurate measuring observations; analytical modeling simplifications and also high level modeling decisions by the user. The impact of analytical modeling decisions by the user on the global response has been focused in this research paper. The following sections present parametric investigations on the effect of various modeling decisions by the user such as variation of smeared crack models for concrete, post-peak softening curves for concrete, different element types used for simulation, gauss quadrature rules, shear retention parameter, threshold angle for subsequent crack formation in multi-directional fixed crack formulation.

3.1. Impact of concrete tension post-peak softening curve envelope

The behavior of concrete in tension is not purely brittle but is characterized with rapid strength loss (de Borst & Nauta 1986, Feenstra & de Borst 1995). In order to characterize the strength loss associated with concrete, researchers primarily are interested in the fracture energy for the concrete sample; however, the shape of post-peak softening curve also plays a significant role in determination of the global response of concrete. A number of different post peak softening curves have been proposed by different researchers such as exponential decay curve by Corneliessen et al 1986, cubic exponential decay curve by Hordijk et al. 1991 and a linear curve. Figure 3 shows load-deflection response for the prototype model with each of the above post-peak response curves and with the experimentally observed load-deflection response. It is observed that the post-peak response model has a significant impact on predicted response, with models that exhibit rapid initial strength loss resulting in weaker maximum strength prediction and lower displacement at maximum strength when compared to those that exhibit less rapid initial strength loss. Of the three type of post-peak softening curves investigated, the tension softening model by Hordijk provides the best correlation with the experimental data.

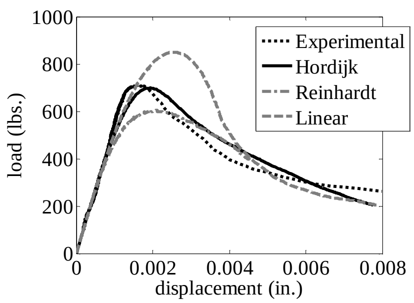


Figure 3. Variation with concrete tensile post-peak softening curve

3.2. Impact of different smeared crack modeling methodologies for concrete

Smeared crack models are based on the philosophy of smearing the crack over a certain region and changing the constitutive formulation in that region in an effort to characterize the physical geometrical discontinuity associated with a crack. Three different smeared crack models have been utilized to study the effect of the choice of user decision in modeling the global load-deformation response of the specimen. The different smeared crack models used are described as follows:

Decomposed strain multiple-fixed crack model (de Borst and Nauta 1986): As the name suggests, the concrete strain in this model is decomposed into a strain within the concrete continuum and strain due to crack width opening. Crack is initiated if the normal stress at a plane exceeds the tensile strength of concrete. This model also allows for multiple cracks to form at a gauss integration point if the threshold angle is exceeded and the normal stress at the plane exceeds the tensile strength of concrete. For this model, it is assumed that the threshold angle is 60 deg. so that robust solution is obtained. The shear retention parameter, β, for this model is taken as 0.001. This model is referred to as DSFMC in figure 4.

Total strain smeared single fixed crack model (Rots and Blaauwendraad 1989): In this model the total concrete strain is considered and crack is initiated at a gauss quadrature point if the normal stress corresponding to the strain exceeds the value of the tensile strength. Once a crack is formed, the crack remains at that orientation and no other crack forms at that gauss quadrature point. The shear retention parameter, β, for this model is taken as 0.001. This model is referred to as TSSFC in figure 4.

Total strain smeared co-axial rotating crack model (Crisfield and Wills 1989): In this model also, the total strain has been considered to determine stress at a gauss point. However, the difference with the fixed crack models is that the cracks are formed at any orientation as the normal stress in that orientation exceeds the tensile strength and also the previously formed cracks are assumed to be closed when cracks at other orientations form. The shear retention, β, associated with a coaxial rotating crack is 0. This model is referred to as TSCRC in figure 4.

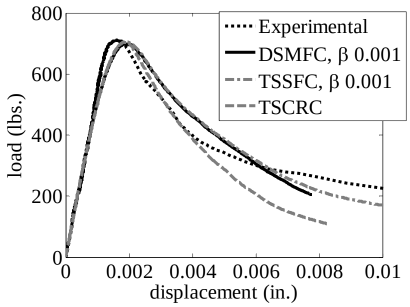


Figure 4. Variation with different smeared crack models

It can be observed from figure 4 that coaxial rotating crack performs better in comparison to both the total strain fixed crack model and also the decomposed strain multiple fixed crack model in the post peak regime since there is a release of energy associated with rotation of the primary crack (Rots and Blaauwendraad 1986). However, it has been previously reported in the literature (Crisfield and Wills 1989, Jirasek 2000) that coaxial rotating cracks exhibits drawbacks associated with numerical convergence in cases of multi-axial loading. Thereby multi-directional fixed crack has been considered in the prototype model since the response obtained is slightly better than the total strain single fixed crack model due to partial release of energy associated with cracks forming in different orientations and also this process exhibits less convergence problems in comparison to the rotating crack model. Moreover theoretically the decomposed strain multiple fixed crack model can be combined easily with other concrete characteristics. De-Borst (2002) also demonstrated that this decomposed strain multiple-fixed crack model is theoretically similar to mathematically sound formulations of damage plasticity models (Lubliner et al. 1989) and microplane model (Bazant and Prat 1988).

3.3. Impact of shear retention factor

The shear retention factor, β, refers to the amount of shear transfer across a crack. A shear retention factor of *0* refers to no aggregate interlock, whereas *1* represents full aggregate interlock. If the value of shear retention factor is specified near to *1*, there is a possibility of stress locking. Cope et al (1980) observed that the shear retention factor allows for the principal stress at a cracked gauss integration point to rotate on further loading. Shear retention in combination with tensile softening may also result in principle tensile stress to exceed the tensile strength in a direction other than normal to the crack. This explains the stiffening in the post peak response when β is increased from 0.001 to 0.05 in figure 5. Thereby, it is being concluded that the shear retention parameter is an important parameter to be calibrated for the post-peak response. It depends upon the type of loading and structure based on which an analyst should make a judgment in selecting this parameter.

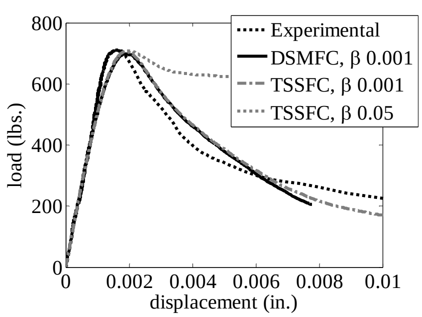


Figure 5. Variation with shear retention factor

3.4. Impact of cracking threshold angle

In a multiple fixed crack model the angle between the normal to the crack and the direction of the principle tensile stress must exceed the threshold angle for another crack surface to form. As the threshold angle, θ, becomes close to *0* deg, a multi-directional fixed crack model exhibits response similar to a rotating crack model. Rots and Blaauwendraad (1989), Jirasek (2000) explains that with more number of cracks, there will be more energy dissipation and thereby will result in less stiffened load-deformation response. On the other hand, more cracks will result in more numerical convergence problems (Crisfield and Wills 1989). Figure 6 demonstrates that simulation with θ equal to 60 deg gives a close resemblance with experimental results. Thereby, in order to strike a balance, a threshold angle of 60 deg has been chosen for the prototype model and is being recommended for further analysis simulations using this material model for concrete.

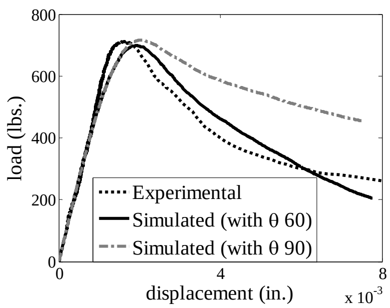
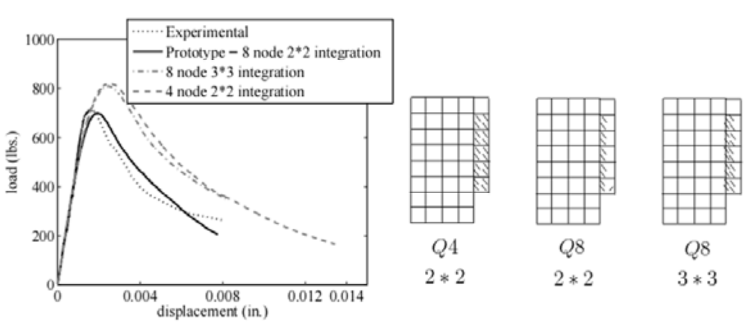


Figure 6. Variation with different threshold angle

3.5. Impact of element types and integration rules

It was also observed that the use of different elements and integration rules varied the load-deformation response of the specimen. Simulation with a 8-node quadrilateral with a 2x2 gauss quadrature integration scheme produced the best correlation with the experimental data in comparison to simulations by using a 4-node quadrilateral with a 2x2 integration rule or a 8-node quadrilateral with a 3x3 integration rule (Figure 7a). Ideally only one set of crack should develop in the notched region, which is observed in Q8 (2*2) in Figure 7b and thereby the use of characteristic length or crack band width equal to the width of the element perpendicular to the crack is justified. But in other element types and integration rules two sets of cracks are observed and thereby the characteristic length used in these cases should be changed from that of the width of the element perpendicular to the crack (which has been used in the simulation) to half it's value in order to obtain good correlation with the observed response. Thereby, it is being recommended that analyst should analyze the cracks in the elements rigorously to decide about the choice of element types, integration rules and crack band width. Based on experience of the authors, a higher order element with reduced integration (8-node quadrilateral with a 2x2 gauss quadrature integration) is being recommended for further simulations of concrete structures.

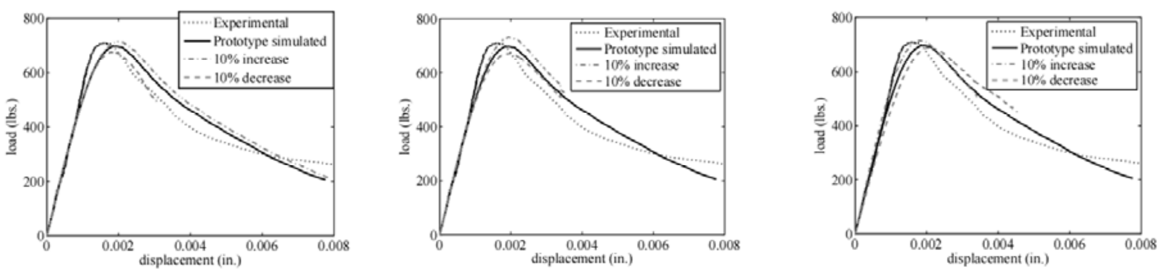


a) Load-displacement response b) Crack formation in notched region

Figure 7. Variation with different element types

4. INVESTIGATION OF THE IMPACT OF UNCERTAINTY IN LABORATORY DATA

Epistemic uncertainty can exist in lack of accuracy of measuring observations from experimental investigations. A parametric study has been conducted to determine the effect of lack of accuracy in measuring material model parameters (e.g. modulus of elasticity, fracture energy and tensile strength) on the numerical global response. Thereby all the parameters are varied by 10% higher and lower of the actual value provided by the experimental investigation. Simulations have been carried out for specimens in which the fracture energy parameter, G_f, (figure 8a); tensile strength of concrete, f_t, (figure 8b); modulus of elasticity of concrete, E_c, (figure 8c) are varied by approximately 10% of the measured value. These parameter variations are considered to represent the potential experimental error in measured values. It has been observed from figure 8 that an approximate 10% variation in the material parameter obtained from experiments results in less than 10% variations in the simulated response. Thereby, it is being concluded that even though the uncertainty in laboratory data is important but it is far less important in comparison to the analyst's high level modeling choices.



a) Fracture energy, G_f b) Tensile strength, f_t c) Modulus of elasticity, E_c

Figure 8. Variation with uncertainty in measurement of material parameters

5. CONCLUSIONS

Determination of tensile response of concrete structures is important in many applications. The research shows variations of response observed with analyst's high level modeling decisions such as concrete tension post-peak softening curve envelope, different smeared crack modeling methodologies for concrete, shear retention factor, cracking threshold angle and element types and integration rules has significant impacts on predicted response for a three-point bend test of a notched plain concrete beam. Infact these high level modeling decisions have significant impact in comparison to uncertainties associated with measurement of material parameters. There exists a lot of literature which deals with the concept of uncertainty due to lack of accuracy in measuring observations; however very few papers (if any) could be found for highlighting and/or categorizing uncertainty associated with analyst's high level modeling decisions. This paper represents a first step in that direction.

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