EVALUATION OF CRITICAL RESPONSES AND CRITICAL INCIDENCE ANGLES OBTAINED WITH RSA AND RHA

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ABSTRACT :

Critical responses and critical angles for a structure can be obtained by response history analysis (RHA) or by response spectrum analysis (RSA). Results obtained with RHA can be considered as a good approximation to actual responses, but they require a great amount of numerical effort. Results obtained with RSA are easily obtained, but they have the limitations associated with spectrum analysis. The goal of this investigation is to compare the critical incidence angles and the maximum critical responses obtained with response history analysis (RHA) and response spectrum analysis (RSA). The responses are obtained for several one story reinforced concrete structures with linear behavior, viscous damping, and natural periods ranging between 0.10 and 3.00 sec. An ensemble of far field ground motions recorded on rock was selected to perform the analyses. The analyses are performed considering the action of one horizontal component, the major component, and two horizontal orthogonal components, the major and the minor components, for the selected ground motions. The RHA results are obtained covering all possible incidence angles for each ground motion. The RSA results are obtained with the critical incidence angle formula given by the CQC3 combination rule, using the mean spectra for the ensemble of the recorded ground motions. The results show that RSA is adequate to estimate the critical responses and corresponding critical incidence angles for design purposes. More refined values of the critical responses could be obtained using RHA in a reduced incidence angle range that is given by the RSA results thus reducing the numerical computations required.

KEYWORDS: critical incidence angle, critical response, response spectrum analysis, response history analysis

1. INTRODUCTION

An earthquake can be described in the horizontal plane as two orthogonal acceleration components of different intensities, which can excite a structure with any horizontal incidence angle. This situation is accounted for in several codes considering two orthogonal components of equal intensities, usually defined by response spectra, and oriented along the principal axis of the structure. As an example, Venezuelan seismic code (Covenin 1756, 2001) allows to obtain the seismic forces with the "square root of square sum rule" (SRSS) or the "30% rule"; while "complete quadratic combination rule with three seismic components" (CQC3) can be optionally used. When the difference in the intensity of the two horizontal components is considered, the first and second criteria do not guarantee that structures are designed for the critical condition due to variability in earthquake incidence angle.

Critical responses are defined as maximum and minimum structural responses considering any earthquake incidence angle. Critical angles are earthquake incidence angles producing critical responses. Procedures to obtain critical angles and critical responses based on response spectrum analysis methods (RSA) are known. Such procedures were proposed by Smeby and Der Kiureghian (1985) and López and Torres (1997) based on different approaches. These procedures are usually identified in technical literature as complete quadratic combination rule with three seismic components or CQC3. A more accurate structural response can be obtained with response history analysis (RHA); however, for practical applications it requires the use of several ground motions and hence enormous numerical efforts. Several examples of this procedure are presented in technical literature. See for instance MacRae and Mattheis (2000), Lobos and Fernández-Dávila (2000), and Fernández-Dávila *et al.* (2000).

Comparison between results obtained with RSA and RHA is of paramount importance due to time-saving characteristics of spectrum based procedures and its convenience to design purposes. However, there is little information in technical literature referred to comparison between RSA and RHA. Biggs *et al.* (1977) compared the results obtained with RHA, using several ground motions, and those obtained with RSA, using the mean spectrum for the ground motions, for 2D reinforced concrete frames. The authors concluded that RSA is adequate to design purposes. More recently, Chopra and Chintanapakdee (2001) compared the results obtained with RHA and RSA for a shear beam considering a near-field ground motion and a far-field ground motion. The authors concluded that RSA is adequate to practical design purposes.

The goal of this investigation is to compare the critical incidence angles and the maximum critical responses obtained with response history analysis (RHA) and response spectrum analysis (RSA).

2. SELECTED STRUCTURES

Selected structures were one-story reinforced concrete structures of 2.40m height. A plan view for the structures is shown in Figure 1. Columns had 0.30m x 0.30m cross-sections and beams had 0.30m x 0.50m cross-sections. Slabs were considered as rigid diaphragms and they concentrated all the mass of the structures. The structures were considered to behave linearly with a 5% viscous damping. Natural periods of the structures ranged between 0.10 and 3.00 sec as can be seen in Table 1.

Table 1 - Natural periods *Tx* and *Ty* for selected structures

Structure #	***Tx* (sec)**	***Ty* (s)**	**Structure**	***Tx* (sec)**	***Ty* (sec)**	**Structure**	***Tx* (sec)**	***Ty* (sec)**
1	0,10	0,50	**11**	0,10	1,00	**21**	0,10	3,00
2	0,20		**12**	0,20		**22**	0,20	
3	0,30		**13**	0,30		**23**	0,30	
4	0,40		**14**	0,40		**24**	0,40	

5	0,50		**15**	0,50		**25**	0,50	
6	1,00		**16**	1,00		**26**	1,00	
7	2,00		**17**	2,00		**27**	2,00	
8	3,00		**18**	3,00		**28**	3,00	
9	4,00		**19**	4,00		**29**	4,00	
10	5,00		**20**	5,00		**30**	5,00	

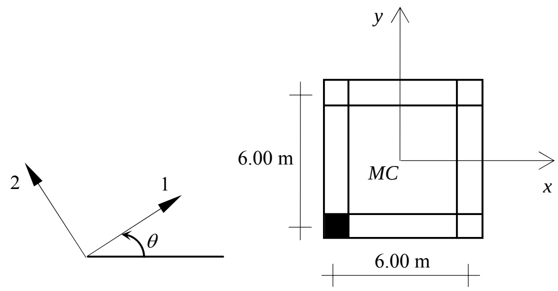


Figure 1 - Plan view for selected structures.

3. SELECTED GROUND MOTIONS

An ensemble of ten horizontal ground motion records was selected to perform the analyses. Nine of them are far field ground motions recorded on rock. Imperial Valley ground motion recorded at soil was also included. Selected ground motions are shown in Table 2. Each pair of horizontal acceleration records was rotated to its principal uncorrelated major and minor directions. Each major acceleration component was scaled to a peak ground acceleration of 0.30g. Then each minor acceleration component was multiplied by the same factor used to scale the corresponding major component. Pseudo-acceleration spectra for each major and minor component are presented in Figures 2 and 3, respectively. Figures 2 and 3 also show the mean pseudo-acceleration spectra for the ensemble.

Table 2 - Selected ground motions

Earthquake	Station	Date (d/m/y)
Kern County	Taft	21/07/1952
San Fernando	Lake Hughes St. No. 4	09/02/1971
San Fernando	Lake Hughes St. No. 12	09/02/1971
Miyagi-Oki	Ofunato Bochi	12/06/1978
Michoacán	Caletas de Campos	19/09/1985
Loma Prieta	Santa Cruz	18/10/1989
Northridge	Mt. Wilson	17/01/1994
Northridge	Lake Hughes St. No. 9	17/01/1994
Chi-Chi	TCU046	21/09/1999
Imperial Valley	El Centro	18/05/1940

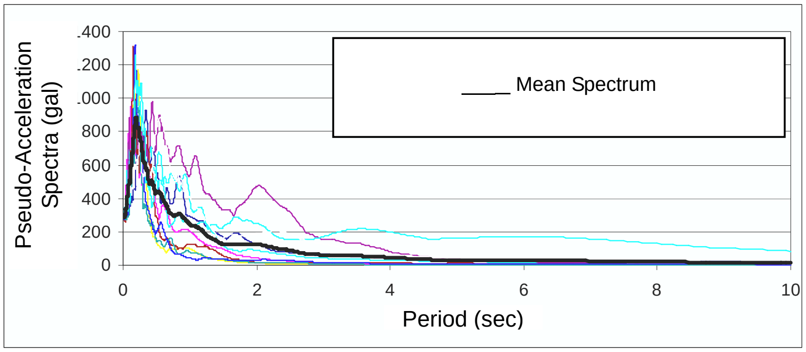


Figure 2 - Pseudo-acceleration spectra for major components and mean spectrum (5% damping)

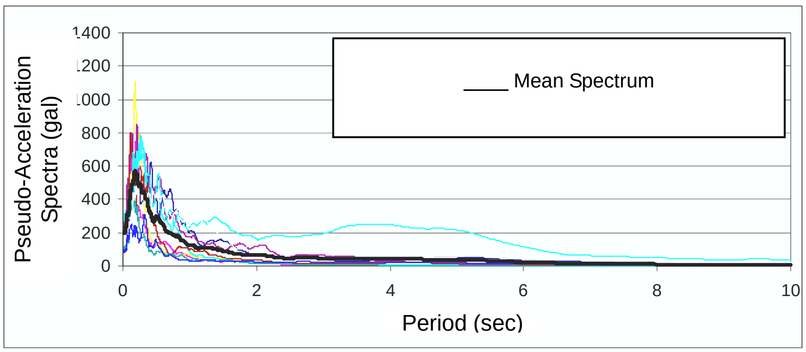


Figure 3 - Pseudo-acceleration spectra for minor components and mean spectrum (5% damping)

4. ANALYSES

Response History Analysis (RHA) and Response Spectrum Analysis (RSA) were performed for each structure considering: (a) the action of one horizontal component, the major component, and (b) two horizontal orthogonal components, the major and the minor components, for the selected ground motions.

4.1. Response History Analysis (RHA)

Critical responses (R_{RHA}) and corresponding critical incidence angles (θ_{RHA}) were obtained by integration in time of the structural responses. RHA results for each structure were obtained varying systematically the incidence angle (θ) for each ground motion, between 0° and 180°, with increments of $\Delta\theta$=10°. Incidence angles of 45° and 135° were also considered. Analyses were performed with commercial software SAP2000 Non Linear (Computers and Structures Inc., 1998).

4.2. Response Spectrum Analysis (RSA)

Critical incidence angles (θ_{cr} or θ_{RSA}) were obtained with Equation (1) using the mean spectra for the selected ground motions.

$$\theta_{cr} = \frac{1}{2}\tan^{-1}\left\{\frac{2\left(R_{2xy} - R_{1xy}\right)}{\left(R_{1y}^2 - R_{1x}^2\right) - \left(R_{2y}^2 - R_{2x}^2\right)}\right\} \tag{1}$$

Critical responses (R_{RSA}) were obtained with Equation (2) making $\theta = \theta_{cr}$. Equations (1) and (2) come from the well known CQC3 combination rule (see for instance López and Torres, 1997).

$$R(\theta) = \left\{\left(R_{1x}^2 + R_{2y}^2\right)\cos^2\theta + \left(R_{1y}^2 + R_{2x}^2\right)\text{sen}^2\theta + 2\,\text{sen}\theta\cos\theta\left(R_{1xy}^2 - R_{2xy}^2\right)\right\}^{1/2} \tag{2}$$

$$R_{mn} = \left\{\sum_i \sum_j \rho_{ij} R_{mni} R_{mnj}\right\}^{1/2} \qquad m = 1, 2;\ n = x, y \tag{3}$$

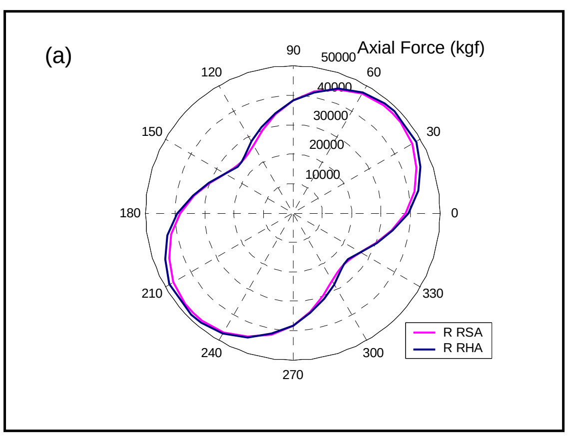
$$R_{mxy} = \left\{\sum_i \sum_j \rho_{ij} R_{mxi} R_{myj}\right\}^{1/2} \tag{4}$$

$$\rho_{ij} = \frac{8\xi^2 (1+\beta_{ij})\beta_{ij}^{3/2}}{(1-\beta_{ij}^2)^2 + 4\xi^2\beta_{ij}(1+\beta_{ij})^2} \tag{5}$$

Equation (3) is the well known CQC combination rule, Equation (4) evaluates the correlation between responses in orthogonal directions for a given ground motion component, and Equation (5) evaluates correlation coefficient between modal responses (see for instance Chopra, 2001).

5. RESULTS AND DISCUSSION

Structural response considered in this investigation was the absolute value of maximum axial load induced by seismic action in the shaded column indicated in Figure 1. RHA responses for each incidence angle were obtained as the mean values of maximum axial forces obtained for each ground motion. Responses obtained with RHA were considered as "exact responses" as they represented the best available approximation to "actual responses". RSA responses for each incidence angle were obtained as the axial forces obtained using mean spectra. Axial forces obtained with RHA and RSA for Structures #16 and #21 are shown in Figure 4 for each incidence angle, as examples of results obtained in this investigation.



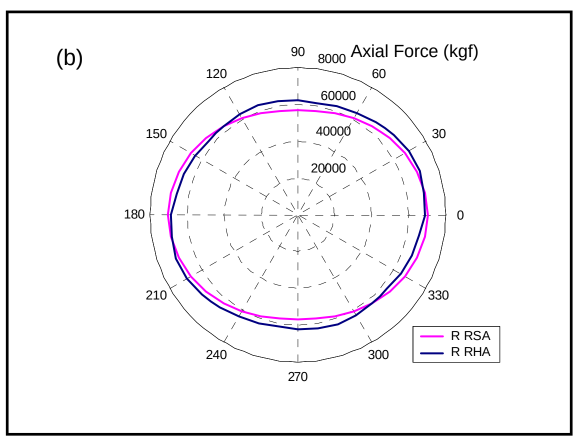


Figure 4 - Axial forces obtained with RHA and RSA: (a) Structure #16 and (b) Structure #21

5.1. One component

RHA were performed for each structure using the major component of the selected ground motions, according to the procedure previously described. RSA were performed using mean spectrum obtained for the major component of the selected ground motions.

5.1.1 Critical Responses

Ratio of responses obtained with RSA and RHA are presented in Figure 5. RSA responses showed a good agreement with responses obtained with RHA, with errors by underestimation not exceeding in general 10%. However, for short period structures it was found that errors may exceed 20%. Errors obtained in this investigation are very similar to errors reported by (Chopra *et al.*, 2001) for a near field ground motion. Errors obtained in this investigation are larger than errors obtained by (Biggs *et al.*, 1977). However, (Biggs *et al.*, 1977) considered an ensemble of ground motions which were recorded in different soil conditions and distances to the fault.

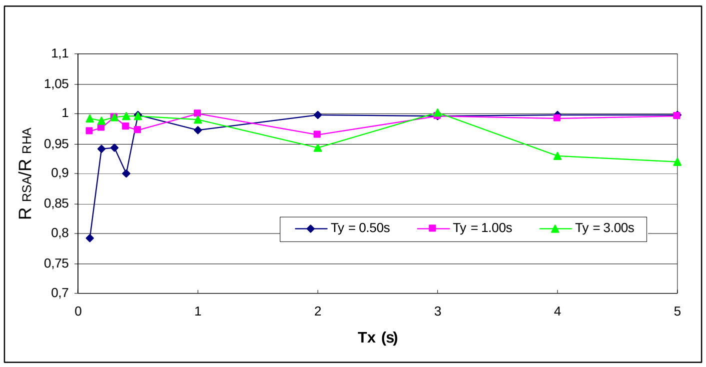


Figure 5 - Comparison between maximum critical responses obtained with RSA and RHA (one component)

5.1.2 Critical Incidence Angles

Errors in incidence angles obtained with RSA and RHA are shown in Figure 6. Incidence angles obtained with RSA showed, in general, a good agreement with incidence angles obtained with RHA, with errors not exceeding 10º. However, for short period structures or large period structures error may be as large as 26º.

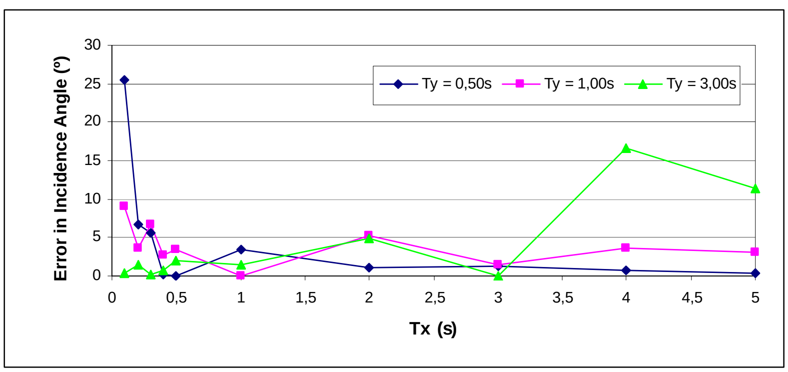


Figure 6 - Errors in critical incidence angles obtained with RSA and RHA (one component)

5.2. Two components

The RHA response to the mayor and the minor ground motion components was calculated according to the procedure previously described. The RSA response was obtained using mean spectra for the mayor and the minor components of the selected ground motions, according to the procedure previously described. The responses to the orthogonal seismic components were combined with the SRSS rule.

5.2.1 Critical Responses

Ratio of responses obtained with RSA and RHA are presented in Figure 7. RSA responses showed a good agreement with responses obtained with RHA, with errors not exceeding in general 10%. However, it was found that for short or large period structures errors may reach 20%. Errors were found to be by underestimation or by overestimation, without showing a direct dependence with structural period. Errors obtained in this investigation are very similar to errors reported by Chopra *et al*. (2001) and they are larger than errors obtained by Biggs *et al*. (1977). However, Biggs *et al*. (1977) and Chopra *et al*. (2001) considered only one component in their respective analyses.

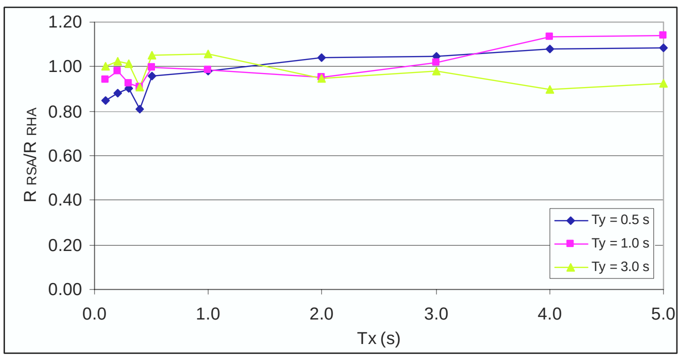


Figure 7 - Comparison between maximum critical responses obtained with RSA and RHA (two components)

5.2.2 Critical Incidence Angles

Errors in incidence angles obtained with RSA and RHA are shown in Figure 8. Error did not exceed 20º in most of the cases; however, in some cases errors of about 60º were found.

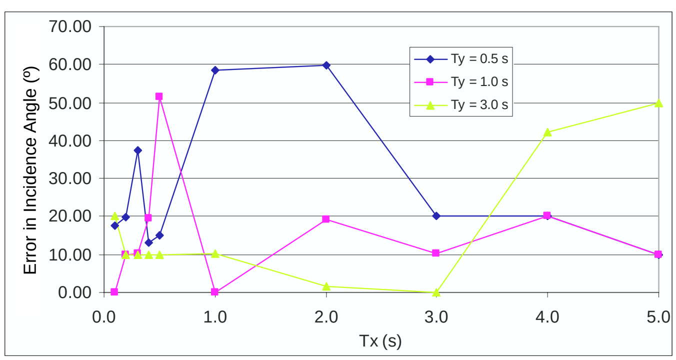


Figure 8 - Errors in critical incidence angles obtained with RSA and RHA (two components)

6. CONCLUSIONS

The results show that RSA is adequate to estimate the critical responses and corresponding critical incidence angles for design purposes. More refined values of the critical responses could be obtained using RHA in a reduced incidence angle range that is given by the RSA results thus reducing the numerical computations required. The results show that RSA is adequate to estimate the critical responses and corresponding critical incidence angles for design purposes. More refined values of the critical responses could be obtained using RHA in a reduced incidence angle range that is given by the RSA results thus reducing the numerical computations required.

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