ACCOUNTING FOR P-DELTA EFFECTS IN STRUCTURES WHEN USING DIRECT DISPLACEMENT-BASED DESIGN

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ABSTRACT:

As part of the development of Direct Displacement-Based Design, a method for explicitly including P-Δ in the design process is presented. The differences in sensitivity to P-Δ of both elasto-plastic (approximating steel response) and stiffness degrading (reinforced concrete) hysteresis are discussed, from which a proposed multiplicative factor is derived to account for the enhanced performance of reinforced concrete structures.

The derivation and an example the parametric behaviour of the proposed factor is presented here, following which a four-storey frame designed for both reinforced concrete and steel response is tested using non-linear response history analysis with a suite of seven spectrum-compatible 'massaged' real records. It is found that the proposed account for P-Δ satisfactorily reduces the storey drift amplifications, such that the design performance targets are maintained at the original level when second-order effects are not included in the analyses. Design recommendations are also presented based on simplifications to the derived equation through the assumption of reasonable values for typical new structures.

KEYWORDS: P-Delta, non-linear response, reinforced concrete, hysteresis, displacement-based design

1. INTRODUCTION

The P-Delta effect and the influence it has on structural response has been the subject of significant research in recent decades. Many of these investigations have focussed on the displacement amplifications that result from the second-order actions, using both single- (SDOF) and multi-degree-of-freedom (MDOF) systems. Such displacement-based studies have become of more interest in recent years as a result of the shift towards performance-based design and assessment procedures. While a substantial body of investigative work on P-Δ effects exists, there is not a consistent approach or requirement to applying code-based provisions [Bernal, 1987; Paulay and Priestley, 1992]. The types of approaches to accounting and designing for P-Δ are numerous, to which further discussion on is provided in Pettinga and Priestley [2007].

The study presented here looks at the issue of accounting for, and designing to mitigate, P-Δ effects within the Direct Displacement-Based Design (DDBD) approach for seismic design [Priestley et al., 2007]. The DDBD method is itself then briefly summarised, and the inclusion of P-Δ within the overall context is considered. Considering reinforced concrete and structural steel as the two main structural materials used with seismic design, the conceptual development and closed form analytical derivation for the proposed solution is presented, along with a an application to a four-storey frame building.

2. THE INFLUENCE OF P-DELTA ON NON-LINEAR RESPONSE

A description of the influence of P-Δ action on system inelastic behaviour can be conceptually depicted (Figure 1) considering the backbone curve, or hysteretic response, of a SDOF. For simplicity the non-linear behaviour is assumed to be bilinear with zero post-yield stiffness (i.e. elasto-perfectly-plastic), and has equal strength and stiffness in each direction of deformation.

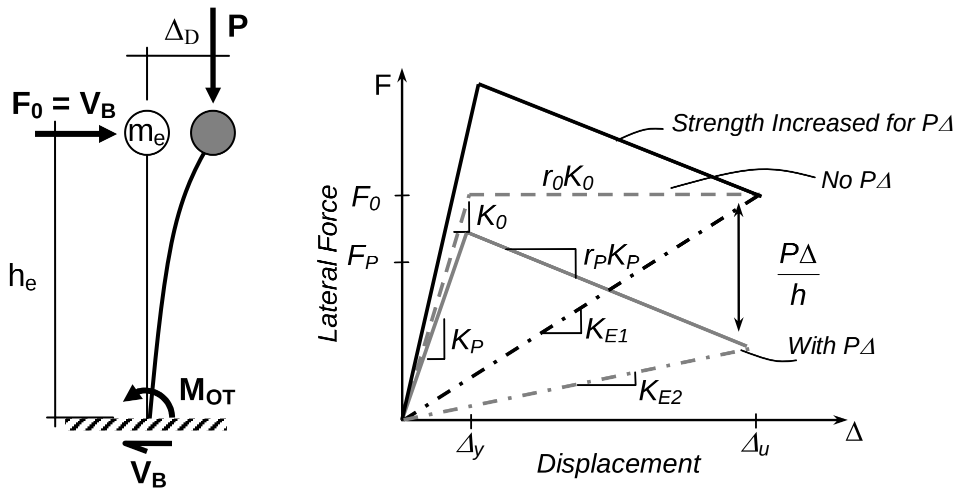


Figure 1. Effect of P-Δ on a Cantilever SDOF system (a) Structure and actions (b) Force-displacement response showing targeted effective stiffness

In Figure 1 a single-mass supported as a cantilever with height *h* is shown in (a), on which a downwards gravity load *P* and an equivalent lateral force *F* are acting. Due to the lateral force (equal to the base-shear strength V_B) there is a displacement Δ_D, thus providing the secondary moment $P{\cdot}\Delta_D$, and an overturning moment $F{\cdot}h$ which make up the overturning capacity M_{OT}. In part (b) the lateral force without second-order effects F_0 is shown with corresponding initial stiffness K_0. With the inclusion of P-Δ effects these are reduced to F_P and K_P respectively. The post-yield stiffness ratio of the unaffected system, r_0, is similarly reduced by P-Δ effects to become r_P. This ratio (either as a flexural or force-displacement stiffness parameter) has been identified by previous investigators [MacRae et al., 1993] to be particularly significant in determining the impact of P-Δ on the response of both SDOF and multi-degree-of-freedom (MDOF) systems.

Based on the parameters defined in Figure 1 the following fundamental relationships for SDOF response to P-Δ are described below. The principal parameter used to identify the level of significance of the P-Δ action is the so-called Stability Ratio, $\theta_{P\Delta}$, which represents the magnitude of the second-order moment to the system overturning capacity:

$$\theta_{P\Delta} = \frac{P\Delta}{V_B h} = \frac{P\Delta}{F_0 h} = \frac{P}{K_0 h} \tag{1.1}$$

From which the following equations for overturning moment, reduced lateral force capacity and initial stiffness can be found:

$$M_{OT} = F_P h + P\Delta \tag{1.2}$$

$$F_P = \frac{(M_{OT} - P\Delta)}{h} = F_0(1 - \theta_{P\Delta}) \tag{1.3}$$

$$K_P = K_0 - \frac{P}{h} = K_0(1 - \theta_{P\Delta}) \tag{1.4}$$

It is emphasised that the effect of P-Δ is always to lower the bilinear stiffness, as shown in Eq. (1.5).

$$r_P = \frac{(r_0 - \theta_{P\Delta})}{1 - \theta_{P\Delta}} \tag{1.5}$$

3. P-DELTA IN DIRECT DISPLACEMENT-BASED DESIGN

The general DDBD method is given in detail by Priestley et al. [2007] and is briefly summarised here for convenience. Principally the method is based on the secant stiffness to peak displacement, with this effective stiffness given by:

$$K_e = 4\pi^2 \frac{m_e}{T_e^2} \tag{2.2}$$

With a known effective stiffness and target design displacement, the design base-shear is found from:

$$V_B = K_e \Delta_D \tag{2.3}$$

With consideration of Figure 1 when P-Δ effects are significant, the total overturning moment is made up of two components as defined in Eq.(2.4):

$$M_B = K_e \Delta_D h_e + \chi \cdot P\Delta_D \tag{2.4}$$

The second term is the moment from the first-order lateral force, while the third term is that due to the gravity load acting to destabilise the system.

4. DEVELOPMENT OF AN EXPLICIT EVALUATION OF HYSTERETIC INFLUENCES ON P-DELTA

Initial consideration of Eq.(2.4) suggests that the value of χ should be unity. However from perspectives both energy-based and of self-centring ability, there is evidence to suggest that reinforced concrete response is not as susceptible to the destabilising effects of P-Δ, implying that a value of χ less than unity may be more appropriate.

To develop a suitable closed-form equation for χ the first step is to define the components of the bilinear (elasto-plastic) and Modifed Takeda hysteresis rules. This is done assuming stabilised response of equal force demand in each direction of loading and peak displacements of equal amplitude.

With reference to Figure 2 the equations relating the principal force and displacement points for both elasto-plastic and stiffness degrading Modified Takeda hysteresis are given below. In the figure, F_y and F_m are the yield and maximum lateral forces, Δ_y and Δ_m are the associated displacements, Δ_l is the displacement at unloading to zero force and K_i is the initial loading stiffness. For the TK loop, γ and β are specific unloading and reloading stiffness parameters (ranging from 0 to 0.5 and 0 to 1.0 respectively), while K_u is the unloading stiffness which reduces with increasing maximum ductility demand, μ.

Considering the P-Δ affected loop in Figure 2b the re-yield force A can be found from:

$$A = F_y - rK_i\left(\Delta_m - \Delta_y\right) \tag{2.5}$$

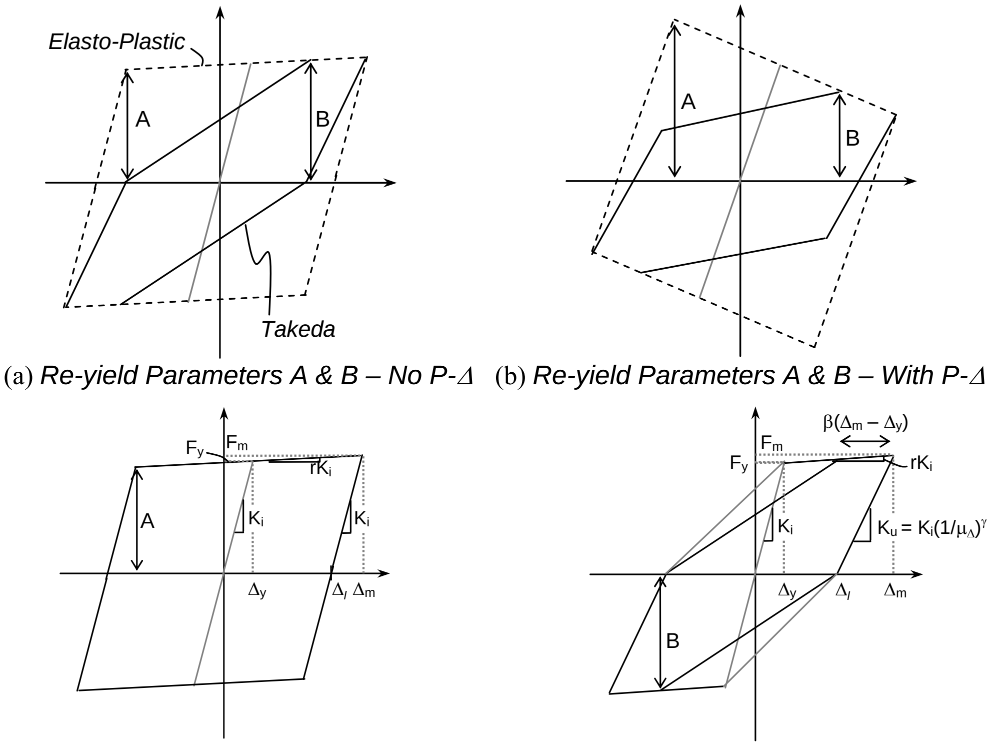
It should be noted that the assumption is made here to have to the yield force F_y unaffected by P-Δ to simplify subsequent derivations.

The following relations define the changes to the stable hysteretic Modified Takeda loop.

$$\Delta_l = \Delta_m - \frac{F_m}{K_u} = \Delta_m - \frac{F_m}{K_i\left(1/\mu_\Delta\right)^\gamma} = \Delta_m - \frac{F_y + rK_i\left(\Delta_m - \Delta_y\right)\left(\mu_\Delta\right)^\gamma}{K_i} \tag{2.6}$$

Considering the P-Δ affected loop in Figure 2b the re-yield force B can be found from:

$$B = F_y + rK_i\left(\Delta_m - \beta\left(\Delta_m - \Delta_y\right) - \Delta_y\right) = F_y + rK_i\left(\Delta_m(1-\beta) + \Delta_y(\beta - 1)\right) \tag{2.7}$$



(a) *Re-yield Parameters A & B – No P-Δ* (b) *Re-yield Parameters A & B – With P-Δ*

(c) *Hysteretic Parameters – Elasto-plastic* (d) *Hysteretic Parameters – Takeda*

Figure 2. Hysteretic parameters used for derivation of χ

It was noted by MacRae [1994] that "…single degree of freedom oscillators subjected to earthquake-type motion tend to oscillate with approximately the same magnitude of acceleration in both the positive and negative direction independently of the shape of hysteresis loop". This implies that the lower force required to re-yield the TK system is the dominant reason for the greater stability exhibited by such behaviour. By quantifying the relative amplitudes of the re-yield force for the TK and EP systems it is possible to assess the enhanced TK behaviour.

The TK factor χ_{TK} is to be less than χ_{EP}, therefore the functional ratio to consider would be *B/A*, however it must reflect the fact that the influence of P-Δ and the possibility of instability becomes more significant as ductility increases. Using the EP response as a base-line from which the improved TK response is quantified, the actual value of χ_{TK} to be used is $\chi_{EP} - B/A$, giving a relative measure of the improved reinforced concrete performance with respect to steel.

With the ratio for χ_{TK}, the requirement of needing more strength increase to maintain stability is met i.e. as the TK system tends to collapse (*B* tends to zero) more of the moment $P{\cdot}\Delta$ is required to be included in the strength enhancement. Assuming $\chi_{EP} = 1.0$, the formulation for χ_{TK} is thus:

$$\chi_{TK} = 1 - B/A = r_P\left[\frac{2(1-\mu_\Delta)+\beta(\mu_\Delta - 1)}{1 - r_P(\mu_\Delta - 1)}\right] \qquad (2.8)$$

To demonstrate the behaviour of Eq.(2.8), the following plot in Figure 3 is provided to show the variation in χ_{TK} as a function ductility, β, r and $\theta_{P\Delta}$.

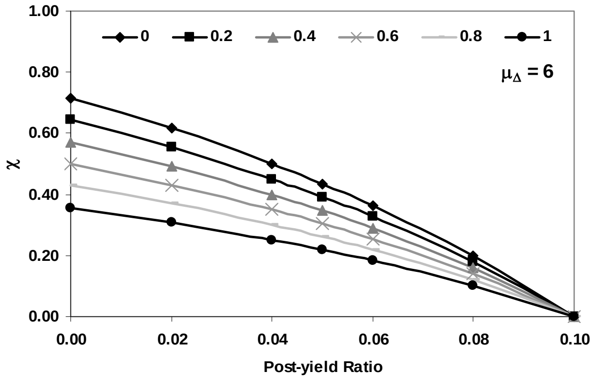


Figure 3. Parametric behaviour of χ_{TK} as a function of post-yield stiffness ratio and Takeda reloading factor β for values 0 to 1.0 shown by each separate line ($\theta_{P\Delta}$ = 0.10).

5. VERIFICATION OF THE PROPOSED DESIGN APPROACH

Using the proposed value of χ_{EP} and the design equation (2.8) for χ_{TK}, a simple four-storey frame building was developed using DDBD and modelled in *Ruaumoko 2D* (Carr, 2006). For the numerical investigation using non-linear time-history analyses a suite of seven real earthquake records compiled for a previous study by the Port of Los Angeles (POLA) authority was used. Each record in the suite had been adjusted using iterative modifications to the acceleration time-history, so as to provide a close match to the site-specific elastic design displacement spectrum with 5% damping. For this study only one component of each record pair was used. Full details of the record suite and modelling approach are given by Pettinga and Priestley [2007].

The structure considered is a vertically regular, two-bay, four-storey frame with equal spans of 5m and constant storey heights of 3.5m. Such frame geometry can be considered representative of a perimeter-frame system. A design seismic coefficient (C_h) equal to 10% was again used, to ensure that the second-order effects could be made sufficiently significant relative to the basic design strength. Of note was the low design ductility values that resulted from the DDBD method when applied to low seismic intensity designs. For the reinforced concrete frame a value of μ_Δ = 2.40 was found for a design drift of 2.25%, while for the structural steel frame μ_Δ = 1.56 was calculated at the design drift of 1.75%. The associated effective periods of the two frames were 2.70 seconds and 2.35 seconds respectively.

A range of P-Δ stability factors ($\theta_{P\Delta}$ = 0.05 to 0.30) were considered. The change in stability factor was implemented by increasing the vertical gravity load in the numerical models. For the designs accounting for the additional second-order demand, the appropriate vertical load was included and the updated base shear as a function of Eq.(2.8) was included in the design. Further design details are provided in Pettinga and Priestley (2007).

6. RESULTS

The comparative amplification and displacement response of the case-study four storey frame model is now considered. In each of the plots in Figure 4 the amplification of storey drift is shown for the three cases of "No PD" (no second-order effects in the analyses), "With PD" (no P-Δ design but second-order effects included in the analyses) and "With Design" (explicit design account for P-Δ included and second-order effects included in the analyses). It should be noted that it was assumed an inter-storey drift greater that 5% at any level in the structure would result in collapse. In such cases the maximum drift used for the particular records was 5%.

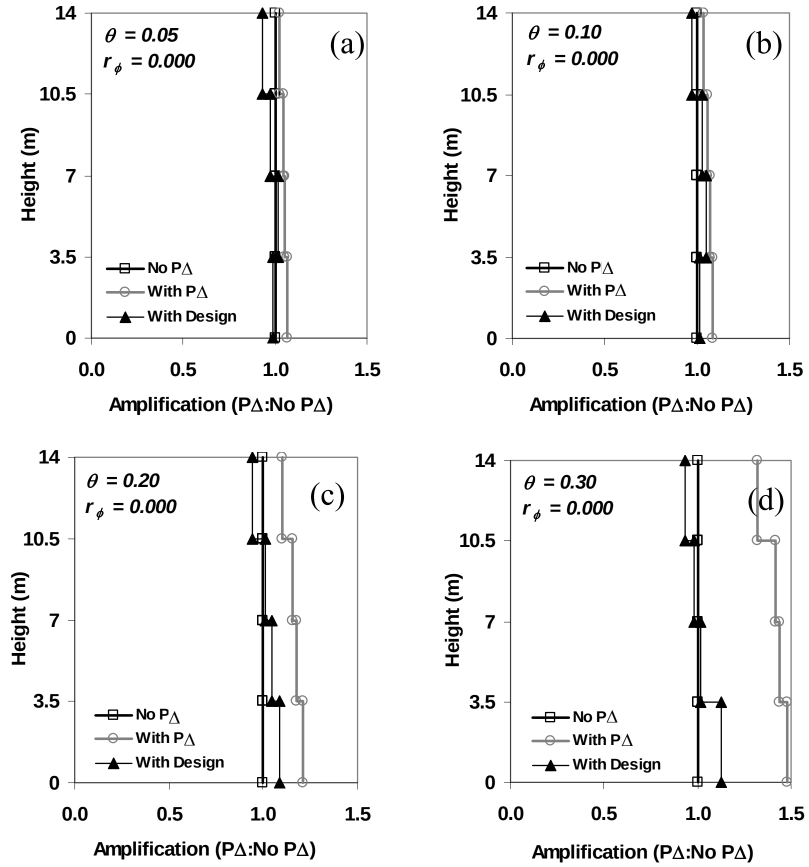


Figure 4. Elasto-plastic (EP) four-storey frame model drift amplification profiles at ground motion intensity 1.0x design (μ_Δ = 1.56) and r_ϕ = 0.0%.

The frames with EP hysteresis and zero post-yield stiffness show a consistent amplification due to P-Δ that becomes more significant with increasing values of stability ratio and intensity (or ductility demand). It is also observed that the proposed design approach leads to buildings that can consistently maintain the original "No P-Δ" drift with exceedance that is generally less than 10%, and therefore considered insignificant for design purposes.

For the TK structural models, the P-Δ amplification response of the buildings is similar to the EP models, however the proposed design approach is seen to become more effect as the stability ratio increases. Exceedance of the "No P-Δ" drift is again within 10%, and therefore considered insignificant. The amplification results are shown in Figure 5 for the same range of structure parameters as the EP case. It is noted that the strength (and therefore stiffness) of the reinforced concrete structure can be sufficiently increased through the proposed design method for all values of stability ratio, post-yield stiffness ratio and ductility. This does not hold for the EP results where it was found parametrically that combinations of high stability ratio and high ductility would lead to structures whose increased strength and stiffness from design could not sufficiently control the drift amplification due to the second-order effects.

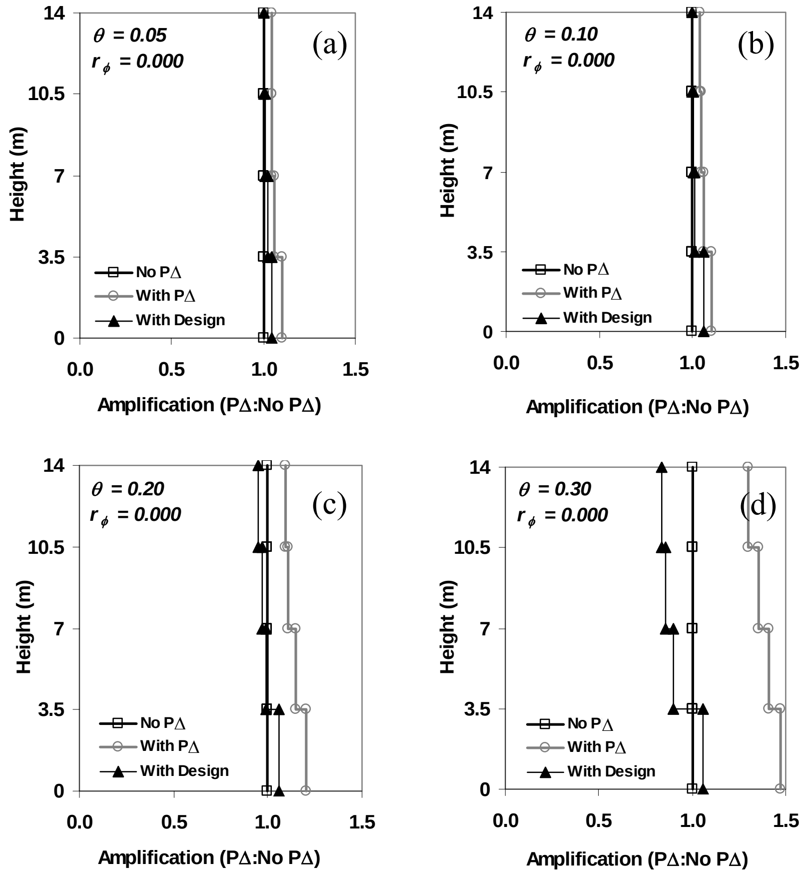


Figure 5. Takeda (TK) four-storey frame model drift amplification profiles at ground motion intensity 1.0x design (μ_Δ = 2.40) and r_ϕ = 0.0%

7. DESIGN RECOMMENDATIONS

From the results presented here, and the parametric studies carried out and presented by Pettinga and Priestley [2007], some simplifications and design limits can be made to the proposed approach of Section 4. Firstly, Eq.(2.8) for χ_{TK} can be simplified by assuming β equals zero which is in keeping with the conservative "thin" equivalent viscous damping equations proposed by Blandon and Priestley (2005). Thus Eq.(2.8) becomes:

$$\chi_{TK} = r_P\left[\frac{2(1-\mu_\Delta)}{1-r_P(\mu_\Delta-1)}\right] \tag{7.1}$$

By assuming a value of r_o equal to 0.01, Eq.(7.1) can be plotted as a function of design displacement ductility and stability ratio, giving the design curves in Figure 6. For likely values of ductility and stability ratio when using DDBD, a value of χ_{TK} = 0.5 has been suggested as satisfactory for design purposes (Priestley et al., 2007).

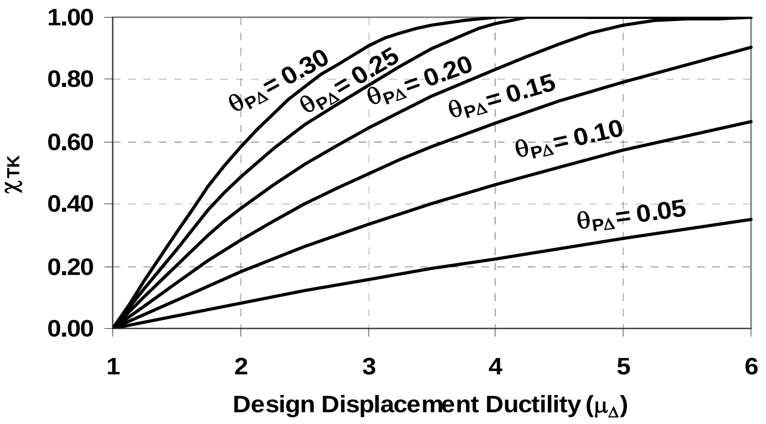


Figure 6. Design chart for χ_{TK} to be used with DDBD

8. CONCLUSIONS

A review of the P-Δ effect on inelastic systems has been presented and application within the overall Direct Displacement-Based Design context discussed. Within this the importance of allowing for different hysteretic behaviour and stability characteristics has been highlighted, from which a derivation has been presented to allow for the improved stability of well-detailed reinforced concrete behaviour.

The proposed design account for P-Δ amplification, in which the system strength (and therefore stiffness) has been tested, has been tested here using a parametric study of a four-storey in-plane frame structure designed to allow for either structural steel or reinforced concrete non-linear response. The proposed method has been shown to be effective at reducing displacement amplification to within allowable design limits, relative to the original response without second-order effects. Finally simplifications and a design chart have been presented, from which final recommendations for application can been drawn.

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