# MODAL COUPLING EFFECTS OF MID-STORY ISOLATED BUILDINGS

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**ABSTRACT :**

This paper describes vibration characteristics of mid-story isolated buildings based on calculation results by basic analyses of the eigenvalue analysis and the frequency response function. Especially, it is clarified that response amplification of upper structures of the isolation story is caused by modal coupling effects between modes of vibration on upper and lower structures. In this paper, results obtained by the eigenvalue analysis and random vibration analysis with frequency response function are as follows: 1) Modal coupling effects are caused by vibration modes on which isolation story is not deformed, and 2) Modal coupling effects amplify earthquake response of upper structure and decrease a effect of seismic isolation, however modal coupling effects do not greatly influence the story deformation of isolation story.

**KEYWORDS:** Seismic Isolation, Mid-story Isolation, Base Isolation, Eigenvalue Analysis, Modal Coupling Effect, Frequency Response Function

## 1. INTRODUCTION

Since the 1995 Hyogoken-Nanbu Earthquake, in Japan, seismic isolation has been esteemed for structural safety and maintaining of functional capacities. Mid-story isolated buildings have attracted attention for adaptability to diverse needs such as urban redevelopment projects and seismic retrofit in Japan (Murakami et al. 1999, Kuroda et al. 1997). The Mid-story isolation is generally recognized as a structural system with high adaptability in recent years.
The seismic design and the seismic performance evaluation of the mid-story isolated buildings are generally verified by a time history response analysis, because mid-story isolation buildings have complex structure system consist of upper and lower structures of the isolation story as shown in Fig.1. From numerical analysis (Kobayashi et al. 2002), the authors have found that the response amplification of upper structures of the isolation story is caused by modal coupling effects between modes of vibration on upper and lower structures. In this paper, we describe how modal coupling effects arise on mid-story isolated buildings based on calculation results by the eigenvalue analysis and the frequency response function.

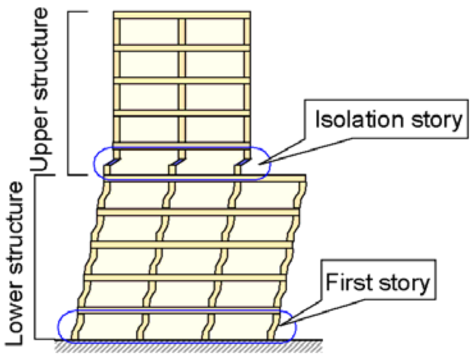


Figure 1 Mid-Story Isolated Building

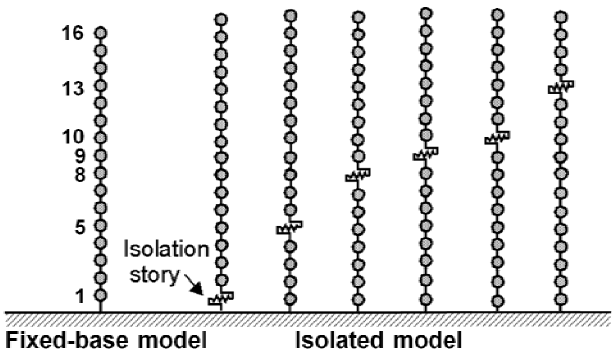


Figure 2 16-Story Shear Deformation Models

## 2. MODAL COUPLING EFFECTS OF MID-STORY ISOLATED BUILDINGS

This section shows analytical examples of modal coupling effects. 16-story shear deformation models as shown in Fig.2 are examined.

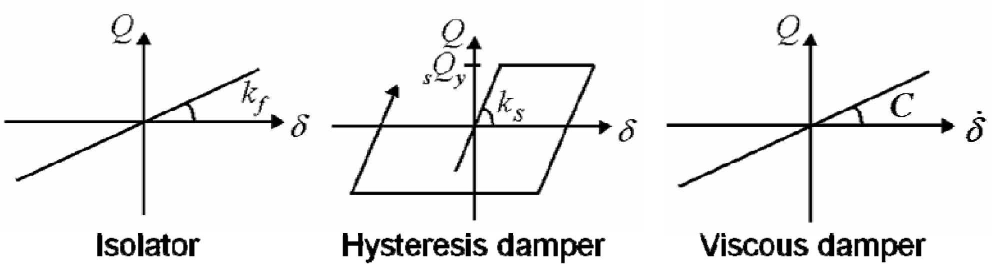



Figure 3 Resisting Force Characteristics of Isolator and Damper

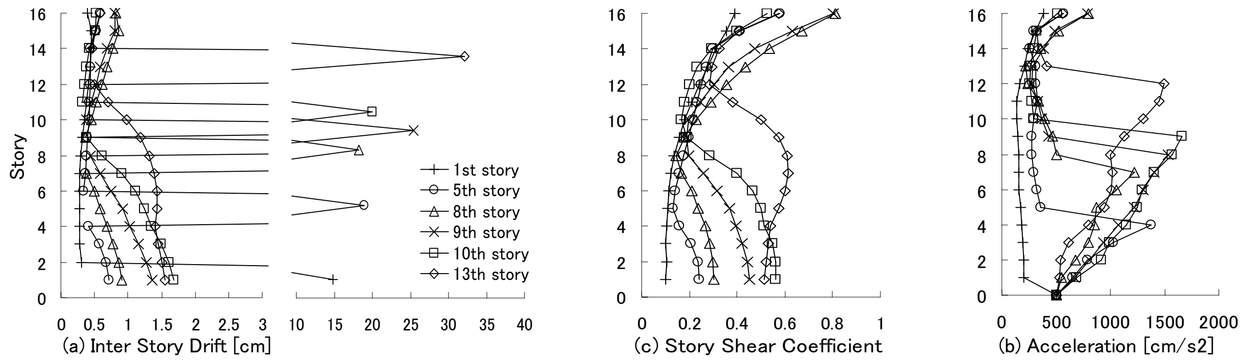



Figure 4 Maximum Response of Isolated Model with Hysteresis Damper

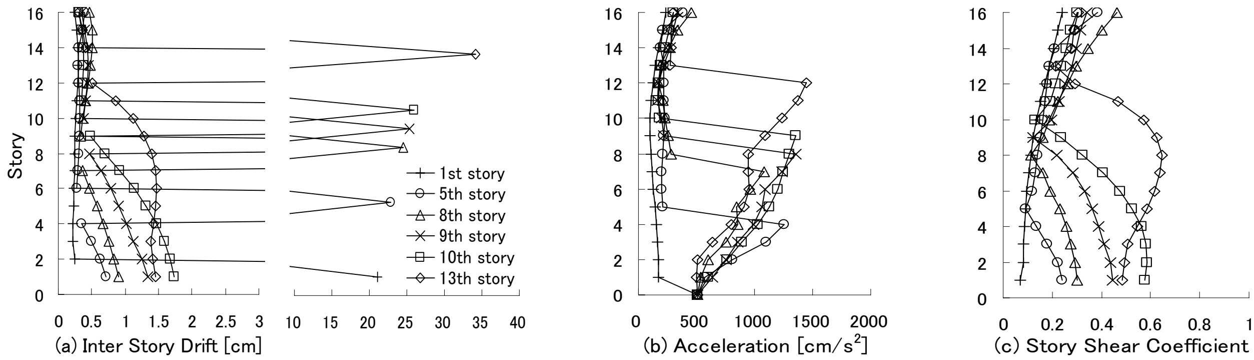



Figure 5 Maximum Response of Isolated Model with Viscous Damper

On Fixed-base model, mass for each story is 1,000 tons. *i*-th story stiffness, $k_i$ is decided so that a building shows uniformly distributed inter-story drift under the static lateral force given by Japanese seismic code, and the first natural period is 1.0sec. The resisting force Characteristic is linear. On Isolated model, the position of isolation story is set to 1st, 5th, 8th, 9th, 10th and 13th story as shown in fig.2. The mass and story stiffness are the same as Fixed-base model excluding an isolation story. The isolator and damper on the isolation story have resisting force $Q$ - inter story drift $\delta$ and velocity $\dot{\delta}$ Characteristics as shown in Fig.3. Lateral stiffness of isolator $k_f$ is adjusted to 1/100 of the lateral stiffness of the same story as Fixed-base model. Then, Natural vibration period of 1st, 5th, 8th, 9th, 10th and 13th story isolated model with the upper and lower structure assumed to be rigid are 3.4, 3.1, 2.9, 2.9, 2.8 and 2.5sec, respectively. These periods are called the isolation period $T_f$. The ${}_sQ_y$ as the yield strength of the hysteresis damper is given as story shear coefficient $\alpha_s$. The $\alpha_s$

of 1st, 5th, 8th 9th, 10th and 13th story isolated model are set to 0.05, 0.055, 0.06, 0.065, 0.065 and 0.09, respectively so that base shear coefficient is effectively decreased (Kobayashi et al. 2004). Lateral stiffness of the hysteresis damper is $k_s = {}_sQ_y / \delta_y$ , $\delta_y$ as the yield displacement of the hysteresis damper is 2cm. The damping coefficient of the viscous damper $C$ is given so that damping factor of isolation story $h$ is 20% for the isolation period $T_f$. The initial structural damping is applied by stiffness-proportional damping in which the first mode damping factor is set to 0.02 on the upper structure and lower structure, respectively.

Analysis results are show in Figs.4 and 5, when an observed earthquake (El Centro NS, Max.Acc. was amplified to 510.7 cm/sec2) was input to six isolated models. The acceleration response and story shear coefficient of the upper structure on 8th story isolation model are remarkably large compared with other models. This is caused by 'Modal Coupling Effects'. We describe how the modal coupling effects arise on mid-story isolated buildings from the next chapters.

## 3. ANALYTICAL MODEL AND ANALYSIS METHOD

### *3.1. Analytical Model*

Analytical Models are 3-mass-system as Mid-story isolated buildings and two systems (Base isolation model and Lower structure model) obtained dividing 3-mass-system as shown in Fig.6. Lumped mass and Story stiffness of Lower structure, $m_l$ and $k_l$ are decided so that the natural period of Lower structure is 0.5sec (2Hz). Lateral stiffness of isolator is decided so that the isolation period $T_f$ is 4.0 sec (0.25Hz). Story stiffness of upper structure is made to change as an analytical parameter. Two lumped masses of Upper structure are even and its total amount $m_u$ is given by mass ratio $\mu=m_u/m_l$ ($\mu$=0.5, 1.0, 3.0). The initial structural damping is applied by stiffness-proportional damping in which the first mode damping factor is set to 0.02 on the upper structure and lower structure, respectively. The damping coefficient of the viscous damper $C$ is given so that damping factor of isolation story $h$ is 20% for the isolation period.

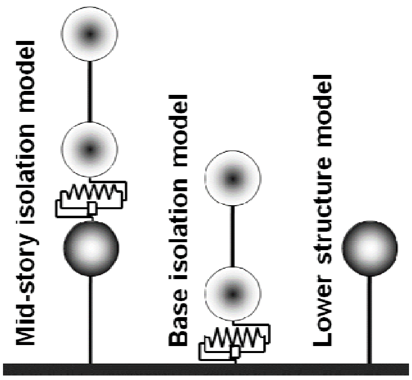


Figure 6 Analytical Model

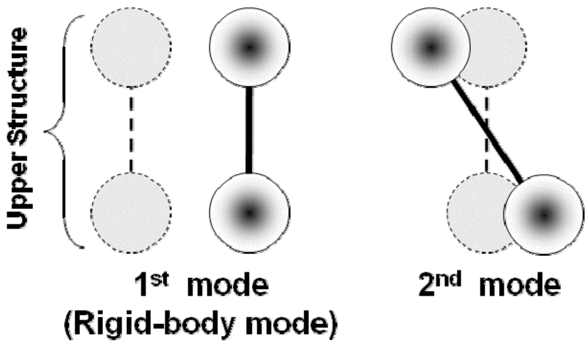


Figure 7 Free-Free Mode Shape Vector

### *3.2. Free-Free Mode of Vibration*

Skinner (1993) has revealed the contribution of the higher mode to earthquake response of base isolation buildings by sweeping the modal response with free-free mode shape vectors. In this study, the same method is applied to mid-story isolation buildings. When the stiffness of isolation story is zero, the mode shape vector $\mathbf{u}_0$ and natural circular frequency $\omega_0$ are as follows:

$$\mathbf{K}_0\mathbf{u}_0 = \omega_0^2\mathbf{M}\mathbf{u}_0 \tag{3.1}$$

Where $\mathbf{K}_0$ and $\mathbf{M}$ are the stiffness and mass matrix, respectively, as the stiffness of isolation story is zero. Two frequency equations obtained from Eqn.3.1 are as follows:

$$\det\left[{}_u\mathbf{K}_0 - {}_u\omega_{FF}^2\, {}_u\mathbf{M}\right] = 0, \quad \det\left[{}_l\mathbf{K}_0 - {}_l\omega^2\, {}_l\mathbf{M}\right] = 0 \tag{3.2a, b}$$

Where ${}_u\mathbf{K}_0$ and ${}_u\mathbf{M}$ are the mass and stiffness matrix of the upper structure, ${}_l\mathbf{K}_0$ and ${}_l\mathbf{M}$ are the mass and stiffness matrix of the lower structure. The modal matrix $\mathbf{U}$ is as follows:

$$\mathbf{U} = \left[\mathbf{u}_{0,s}\right] = \begin{bmatrix} \mathbf{u}_{FF,1} & \mathbf{u}_{FF,2} & \cdots & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \cdots & {}_l\mathbf{u}_1 & {}_l\mathbf{u}_2 & \cdots \end{bmatrix} \tag{3.3}$$

where $\mathbf{u}_{FF,i}$ is $i$-th mode shape vector obtained from Eqn.3.2a (i.e. the free-free mode shape vectors as shown in Fig.7) and ${}_l\mathbf{u}_j$ is $j$-th mode shape vector obtained from Eqn.3.2b. The ratio $\gamma$ of the ${}_u\omega_{FF,2}$ (as the 2nd natural circular frequency of free-free mode) and the ${}_l\omega_1$ (as the 1st natural circular frequency of the lower structure) is defined as Eqn.3.4. This is the analytical parameter in this paper.

$$\gamma = {}_u\omega_{FF,2} / {}_l\omega_1 \tag{3.4}$$

### *3.3. Eigenvalue Analysis*

Since mid-story isolation models are the non-classically damped system, the eigenvalue problem of Eq.(3.5) is solved to obtain the $i$-th modal parameters. $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{I}$ are mass, damping, stiffness and unit matrix, respectively.

$$\mathbf{A}\mathbf{u}_i = \lambda_i \mathbf{u}_i, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{3.5a, b}$$

where $\lambda_i$ and $\mathbf{u}_i$ are complex eigenvalue and eigenvector. The $i$-th natural frequency $f_i$ and damping factor $h_i$ are given by Eqn.3.6.

$$f_i = 2\pi\left|\lambda_i\right|, \quad h_i = -\mathbf{Re}(\lambda_i)/\left|\lambda_i\right| \tag{3.6a, b}$$

### *3.4. Frequency Response Function*

The $H_{y,j}(i\omega)$ as frequency response function of $j$-th story displacement under sinusoidal excitation $\ddot{y}_g = e^{i\omega t}$ is evaluated as follows:

$$\mathbf{H}_\mathbf{y}(i\omega) = -\left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1}\mathbf{M}\mathbf{r}, \quad \mathbf{r} = \begin{bmatrix}1 & 1 & \cdots\end{bmatrix}^T \tag{3.7a, b}$$

$$\mathbf{H}_\mathbf{y}(i\omega) = \begin{bmatrix}H_{y,1}(i\omega) & H_{y,2}(i\omega) & \cdots & H_{y,j}(i\omega) & \cdots\end{bmatrix}^T \tag{3.8}$$

The $H_{A,j}(i\omega)$ as frequency response function of $j$-th story absolute acceleration, and $H_{r,j}(i\omega)$ as that of inter-story drift of $j$-th story are evaluated by $H_{y,j}(i\omega)$ as follows:

$$H_{A,j}(i\omega) = 1 - \omega^2 H_{y,j}(i\omega) \tag{3.9}$$

$$H_{r,j}(i\omega) = H_{y,j}(i\omega) - H_{y,j-1}(i\omega), \quad H_{r,1}(i\omega) = H_{y,1}(i\omega) \tag{3.10a, b}$$

The $\sigma_X$ as root-mean-square (RMS) value of a random vibration response $X$ to $S(\omega)$ as power spectrum of ground acceleration is expressed by using frequency response function $H_X(i\omega)$ as Eqn.3.11. When the power spectrum $S(\omega)$ is assumed to be white noise which is constant value for $\omega$, the $\sigma_m/\sigma_n$ as RMS value ratio of random vibration response $m$ and $n$ is evaluated by Eqn.3.12.

$$\sigma_X{}^2 = \int_0^\infty |H_X(i\omega)|^2 S(\omega) d\omega \tag{3.11}$$

$$\sigma_m/\sigma_n = \sqrt{\int_0^\infty |H_m(i\omega)|^2 d\omega \Big/ \int_0^\infty |H_n(i\omega)|^2 d\omega} \tag{3.12}$$

## 4. ANALYSIS RESULTS AND DISCUSSIONS

### *4.1. Natural Frequency and Modal damping factor*

Figs.8 and 9 show the relationship between the analytical parameter $\gamma$ calculated by Eqn.3.4 and the natural frequency and modal damping factor calculated by Eqn.3.6 on 3-mass mid-story isolation model. Fig.8 shows that 2nd and 3rd natural frequencies reach a close value near $\gamma$ = 1. Moreover, Fig.9 shows that the 2nd and 3rd modal damping factors change greatly near $\gamma$ = 1. The 2nd damping factor decreases rapidly, and the 3rd one increases.

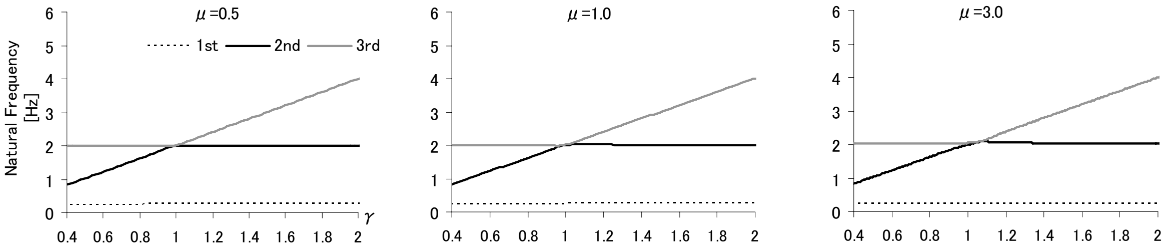



Figure 8 Relationship between $\gamma$ and Natural Frequency

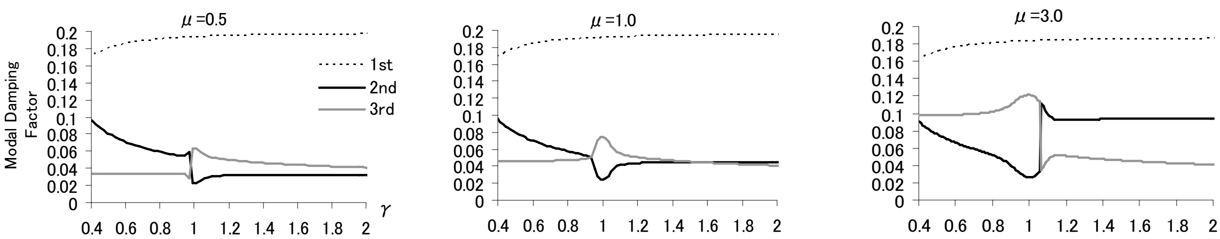



Figure 9 Relationship between $\gamma$ and Modal Damping Factor

### *4.2. Mode of Vibration*

Fig.10 shows the modal participation function $\beta u$ as classically mode vectors and the value of the natural frequency and modal damping factor calculated by Eqn.3.6. When $\gamma$ = 0.9, the direction of the 2nd and 3rd modal participation functions of the upper structure is opposite, and these amplitude is almost equal. The inter story drift of isolation story is large in the 2nd mode, and it is small in the 3rd mode. This tendency becomes the most remarkable at $\gamma$ = 1.0. The 2nd and 3rd modal participation function of upper structure become considerable amplitude. Moreover, the inter story drift of isolation story is extremely small in the 2nd mode. According to these tendencies, the change of the modal damping factor as shown in Fig.9 can be understood. when $\gamma$ = 1.0, the 2nd modal damping factor becomes 2.4%, because high damping of the isolation story cannot be taken in the 2nd mode. On the other hand, the 3rd modal damping factor becomes 7.5%, because high damping of the isolation story can be efficiently taken in the 3rd mode. The 2nd mode at $\gamma$ = 1.0 as shown in Fig.9 has slight damping factor as 2.4% and considerable amplitude participation function of upper structure. Such a mode of vibration causes modal coupling effect.

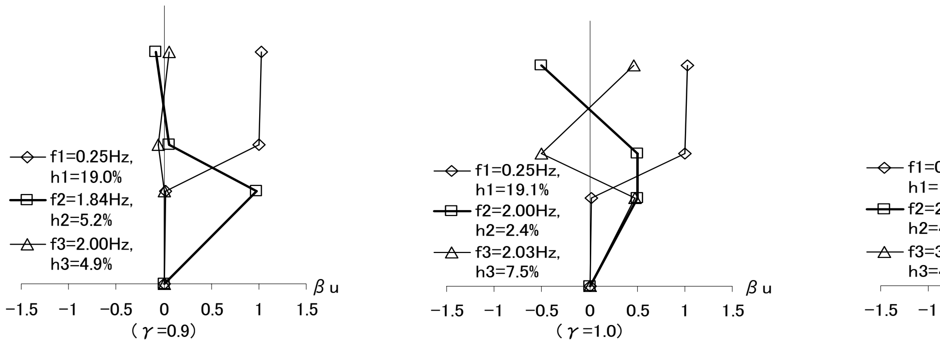



Figure 10 Modal Participation Function $\beta u$ ( $\mu$ = 1.0)

### *4.3 Frequency Response Function and Random Vibration Response*

Figs.11 to 14 show the frequency response function of absolute acceleration of the building top calculated by Eqn.3.9 and that of inter story drift of the isolation story by Eqn.3.10, respectively. Base isolation model described in these figures are these in Fig.6. These frequency response functions increase compared with Base isolation near 2Hz as natural frequency of Lower structure. This tendency becomes the most remarkable, when $\gamma$ = 1.0 on absolute acceleration in Fig.11.

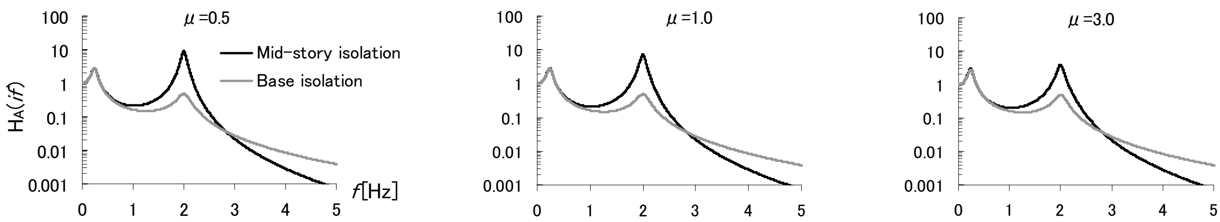



Figure 11 Frequency Response Function of Absolute Acceleration of Building Top (γ=1.0)

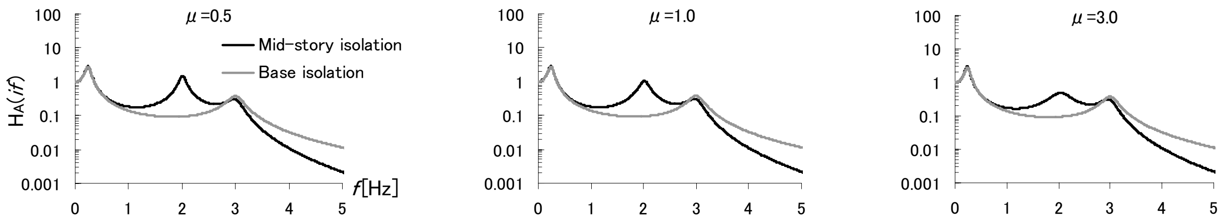



Figure 12 Frequency Response Function of Absolute Acceleration of Building Top (γ=1.5)

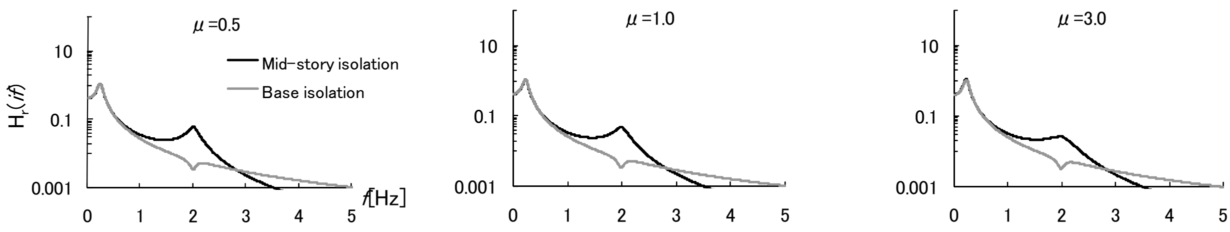



Figure 13 Frequency Response Function of Inter Story Drift of Isolation Story (γ=1.0)

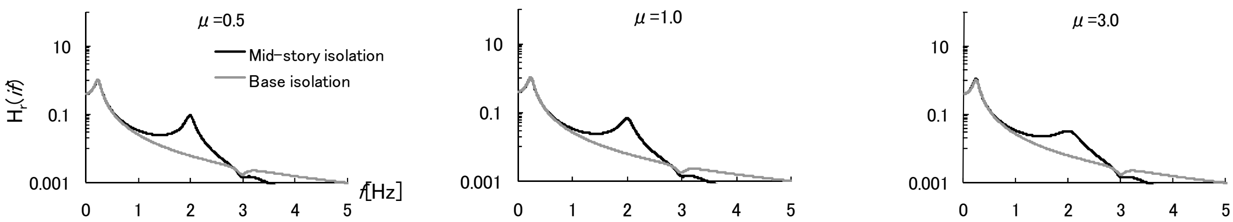



Figure 14 Frequency Response Function of Inter Story Drift of Isolation Story (γ=1.5)

Results of the RMS value ratio of random vibration response calculated by Eqns.4.1 and 4.2 are shown in Figs.15 and 16, respectively.

$$Acc.ratio = \frac{\sigma_{A,Mid-story-isolation}}{\sigma_{A,Base-isolation}} = \sqrt{\frac{\int_0^\infty \left|H_{A,Mid-story-isolation}(i\omega)\right|^2 d\omega}{\int_0^\infty \left|H_{A,Base-isolation}(i\omega)\right|^2 d\omega}} \tag{4.1}$$

$$Interstory\quad drift\quad ratio = \frac{\sigma_{r,Mid-story-isolation}}{\sigma_{r,Base-isolation}} = \sqrt{\frac{\int_0^\infty \left|H_{r,Mid-story-isolation}(i\omega)\right|^2 d\omega}{\int_0^\infty \left|H_{r,Base-isolation}(i\omega)\right|^2 d\omega}} \tag{4.2}$$

These figures show the response ratio of Mid-story isolation and Base isolation. That is, it is the response amplification to Base isolation. The RMS value ratio of absolute acceleration of the upper structure (2nd and 3rd story) shown in Fig.15 are greatly amplified by modal coupling effect at $\gamma$ = 1.0. However, this response amplification becomes small with increasing mass ratio $\mu$. The RMS value ratio of inter story drift of the upper structure (2nd and 3rd story) are shown in Fig.16. The RMS value ratio of 3rd story drift is greatly amplified by modal coupling effects at $\gamma$ = 1.0. However, That of 2nd story drift (isolation story drift) is fairly constant for $\gamma$. That is, the modal coupling effects do not influence the story drift of isolation story.

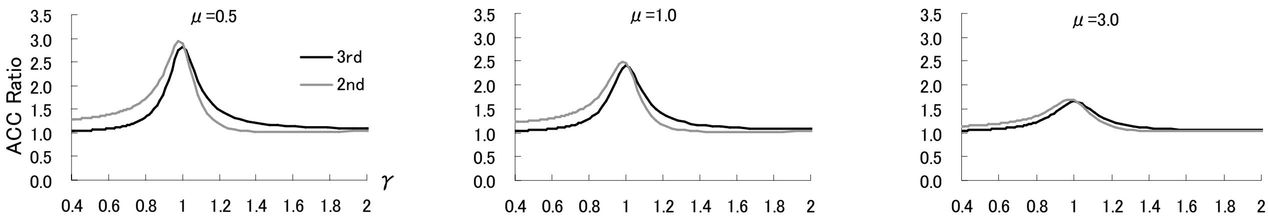



Figure 15 RMS Value Ratio of Acceleration Response of Upper Structure

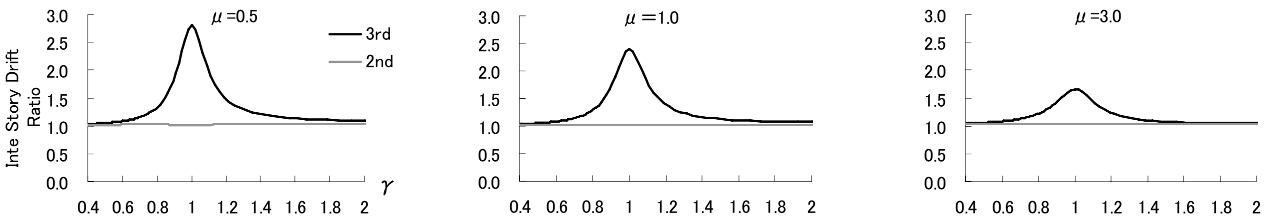
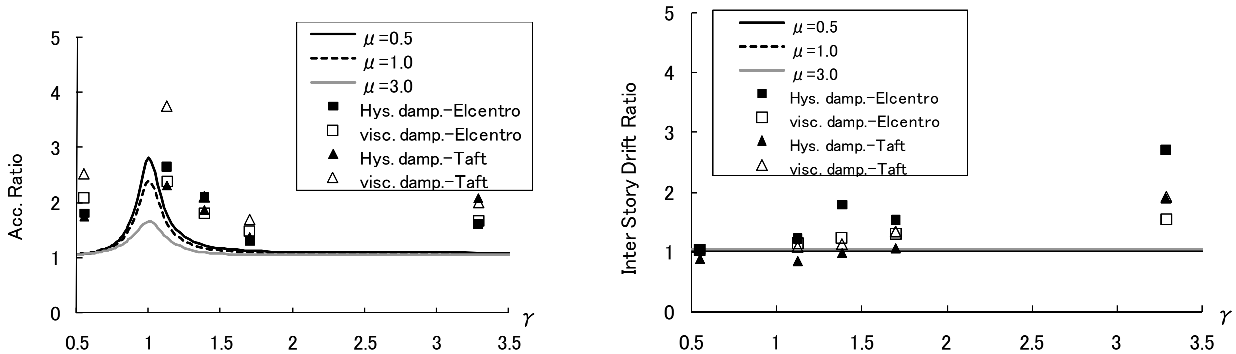



Figure 16 RMS Value Ratio of Inter Story Drift of Upper Structure




(a) Acceleration Response Ratio of Upper Structure (b) Inter Story Drift Ratio of Upper Structure

Figure 18 Correspondence of RMS Value Ratio and Results of time history analysis

Fig.18 shows the correspondence of RMS Value Ratio of 3-mass model and results of time history analysis of 16-mass model as shown in Fig.2. The $\gamma$ of 5th, 8th, 9th, 10th and 13th story isolation are 0.55, 1.13, 1.39, 1.70 and 3.29 calculated by Eqn.3.4, respectively. Fig.18(a) shows the absolute acceleration response ratio of the building top of Mid-story isolation and Base isolation model, and Fig.18(b) shows the inter-story drift ratio of isolation story of Mid-story isolation and Base isolation model. Response ratio of time history analysis is larger than RMS value ratio, however, these response ratios show the same tendency, that is, acceleration ratio is greatly amplified by modal coupling effects near $\gamma = 1.0$ and that of isolation story drift is largely-unaltered for $\gamma$ compared with acceleration ratio.

## 5. CONCLUSIONS

This paper describes vibration characteristics of mid-story isolation buildings based on the calculation result by the eigenvalue analysis and the frequency response function. Especially, it is clarified that the response amplification of upper structures of the isolation story is caused by modal coupling effects. Results obtained by analytical studies are as follows: 1) Modal coupling effects are caused by vibration modes on which isolation story is not deformed. 2) Modal coupling effects do not greatly influence the inter-story drift of isolation story, however The modal coupling effects amplify earthquake response of upper structure and decrease a effect of seismic isolation.

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