Study on Non-linear Responses of Eccentric Structure

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ABSTRACT :

The purpose of this research is to calculate the center of rigidity in the non-linear range of eccentric structures. It is necessary to calculate the equivalent stiffness matrix in the non-linear range. One of the ways to calculate the matrix is the equivalent linearization method proposed by T. K. Caughey. Previous studies on this method, however, focused on the equivalent single-degree-of-freedom system (referred to as SDOF, hereafter), in which the matrices can not be calculated directly. On the other hand, few studies have been reported on multi-degree-of-freedom system (referred to as MDOF, hereafter). It is necessary for the calculation of the center of rigidity in the non-linear range of eccentric structures to develop the equivalent linearization method for MDOF. In this paper, the equivalent linearization methods for the MDOF were developed to extend the applicable scope of the method reported by T. K. Caughey, which is one of the equivalent linearization methods for SDOF and based on the Least-squares method. Furthermore, a dynamic response analysis was conducted with one mass model with eccentric structures to evaluate the validity of the proposal methods.

KEYWORDS: Eccentric Structure, Center of Rigidity, Equivalent Linearization, Multi Degree of Freedom, Dynamic Response, Least-squares Method

1. INTRODUCTION

Recently, the capacity spectrum method was developed to evaluate the performance of buildings. For example, the Building Standard Law of Japan contains the method to evaluate the performance of buildings based on the seismic evaluation method of the response and the limit strength. In this method, non-linear responses of the structure can be evaluated to substitute the non-linear structure with the equivalent structure based on the equivalent linearization method. The equivalent linearization method uses the equivalent stiffness calculated with the maximum response displacement and the equivalent damping calculated with the backbone curve and the maximum response displacement. One of the advantages of this method is that non-linear response of the structure can be obtained without conducting a time history response analysis.

The torsional vibration of the eccentric structure is caused by the difference between the locations of the center of rigidity and the center of gravity. In the non-linear range, the center of rigidity of the eccentric structure can shifts according to the stiffness degradation of the structure. One way to evaluate non-linear responses of eccentric structures is to convert the non-linear eccentric structure into the equivalent eccentric structure using the equivalent linearization method. This equivalent eccentric structure is defined as the equivalent linear structure which has the center of rigidity in the same location as the non-linear range. In order to calculate this location, it is necessary to calculate equivalent stiffness matrix in the non-linear range using the equivalent linearization method. In order to calculate equivalent stiffness matrix in the non-linear range, it is necessary to propose equivalent linearization methods for the MDOF, because the equivalent linearization methods for the SDOF can not be calculated the matrix directly.

Various studies have been reported on the equivalent linearization method. Previous studies on this method, however, focused on the equivalent SDOF. Furthermore, few studies have been reported on a method for the MDOF. Thus, the aim of this paper is to propose equivalent linearization methods for the MDOF to find the location of the center of rigidity of the eccentric structure in the non-linear range. In this paper, the equivalent linearization methods for the MDOF were developed by extending the applicable scope of the method reported by T. K. Caughey, which is one of the equivalent linearization methods for the SDOF and based on the Least-squares method.

2. EQUIVALENT LINEARIZATION FOR SINGLE-DEGREE-OF-FREEDOM SYSTEMS

T. K. Caughey developed one of the equivalent linearization for the SDOF using the least-square method. Here, this method is named as the dynamic stiffness method (referred to as DSM, hereafter). This method covers the steady-state response of the non-linear SDOF systems. In this method, the equivalent stiffness and the equivalent damping coefficient are calculated using the least-square method to minimize the residual sum of squares for the difference between linear restoring force and nonlinear restoring force. Thus, the DSM for the SDOF, which have bi-linear hysteresis curve as shown in Figure 1, will be described.

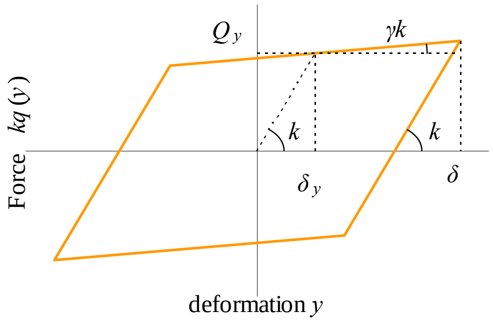


Figure 1 nonlinear restoring force characteristic

where, k is the initial stiffness of the system, γ is the stiffness degradation ratio, δ_Y is the yield deformation, δ is the maximum deformation, and Q_Y is the yield strength. The equation of motion for the SDOF is given as Eqn. (2.1). The non-linear hysteresis function of the system $q(y)$ is given as Eqn. (2.2)

$$M\ddot{y} + C\dot{y} + kq(y) = -\ddot{y}_0 M \quad (2.1)$$

$$q(y) = \begin{cases} y-(1-\gamma)(\delta-\delta_Y) & \delta-2\delta_Y \le y \le \delta \quad \text{and} \quad \dot{y} \le 0 \\ \gamma y-(1-\gamma)\delta_Y & -\delta \le y \le \delta-2\delta_Y \quad \text{and} \quad \dot{y} \le 0 \\ y+(1-\gamma)(\delta-\delta_Y) & -\delta \le y \le -(\delta-2\delta_Y) \quad \text{and} \quad \dot{y} \ge 0 \\ \gamma y+(1-\gamma)\delta_Y & -(\delta-2\delta_Y) \le y \le \delta \quad \text{and} \quad \dot{y} \ge 0 \end{cases} \quad (2.2)$$

where, M is the mass of the system, C is the damping coefficient of the system, , y is the deformation of the system, $\dot{y}$ is the velocity of the system, $\ddot{y}$ is the relative acceleration of the system, and $\ddot{y}_0$ is ground acceleration. The equivalent linearized equation of motion for the SDOF can be given as Eqn. (2.3) using the error term.

$$M\ddot{y} + C_e\dot{y} + K_e y + \varepsilon(\dot{y}, y, t) = -\ddot{y}_0 M \quad (2.3)$$

where, C_e is the equivalent damping coefficient, K_e is equivalent stiffness, and $\varepsilon(\dot{y}, y, t)$ is the error term defined as the difference between equivalent linearized restoring force ($C_e\dot{y} + K_e y$) and nonlinear restoring force ($C\dot{y} + kq(y)$). The purpose of the DSM for the SDOF is to calculate C_e and K_e. In the DSM for the SDOF, the C_e and K_e is calculated by minimizing the values of $\overline{\varepsilon^2}$. According to definition of the least-square method, both C_e and K_e reached extreme value is necessary conditions to minimize the values of the function $\overline{\varepsilon^2}$ .Where

$\overline{\varepsilon^2}$ represents the square mean of the error term $\varepsilon(\dot{y}, y, t)$. Thus, partial differential of $\overline{\varepsilon^2}$ by C_e and K_e become 0 as shown in the following equations.

$$\partial\overline{\varepsilon^2}/\partial K_e = 0, \ \partial\overline{\varepsilon^2}/\partial C_e = 0 \tag{2.4}$$

The ground acceleration is assumed as the harmonic ground motion as Eqn. (2.5). The steady-state response is only considered. Eqn. (2.6) can be approximated from Eqn. (2.3) using the assumption that $\overline{\varepsilon^2}$ can be ignored.

$$\ddot{y}_{0(t)} = a_0 \cos pt \tag{2.5}$$

$$y_{(t)} = \delta \cos(pt - \varphi) \tag{2.6}$$

where, a_0 is the amplitude, p is the angular frequency of the harmonic ground motion, and φ is the phase difference between the harmonic ground motion and the steady-state response. From Eqn. (2.3), Eqn. (2.4), and Eqn. (2.6), K_e is given as Eqn. (2.7). Similarly, C_e is given as Eqn. (2.8).

$$K_e = \frac{k}{\pi}\left\{(1-\gamma)\theta^* + \gamma\pi - \frac{(1-\gamma)}{2}\sin 2\theta^*\right\}, \cos\theta^* = 1 - 2\frac{\delta_Y}{\delta} = 1 - \frac{2}{\mu} \tag{2.7}$$

$$C_e = \frac{4k}{p\mu\pi}(1-\gamma)\left(1 - \frac{1}{\mu}\right) + c \tag{2.8}$$

where, μ is the ductility factor ($=\delta/\delta_y$). From Eqn. (2.7), the equivalent stiffness K_e is shown as the function of μ. From Eqn. (2.8), the equivalent damping coefficient C_e is the function of μ and p.

3. EQUIVALENT LINEARIZATION METHOD FOR MULTI-DEGREE-OF-FREEDOM

In this chapter, outline of the Multi-degree-of-freedom Equivalent Linearization Method (referred to as MELM, hereafter) and outline of the Simplified Multi-degree-of-freedom Equivalent Linearization Method (referred to as SMELM, hereafter) will be described. The MELM was developed using the DSM for the MDOF, which is based on the DSM for the SDOF proposed by T. K. Caughey. The SMELM was developed to simplify the MELM.

3.1. Modeling the Structures

The MELM covers all of structures which are allowable to model lumped mass systems. Thus, structures are modeled as follows. The structure has n degrees-of-freedom, and m inelastic springs. The displacement vector $\{u\}$ and mass matrix $[M]$ can be written as Eqn. (3.1) and Eqn. (3.2), respectively.

$$\{u\} = \{u_1 \quad u_2 \quad \ldots \quad u_n\}^T \tag{3.1}$$

$$[M] = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & m_n \end{bmatrix} \tag{3.2}$$

Hysteresis model of all of the inelastic springs is bi-linear type as shown in Figure 1. Thus, stiffness matrix in the inelastic range is given as Eqn. (3.3) using two factors ${}_sa_j$ ($s = 1\ldots m$, $j = 1\ldots n$) and ${}_sb_i$ ($s = 1\ldots m$, $j = 1\ldots n$).

$$[K]=\begin{bmatrix} \sum_{s=1}^{m} {}_s a_1 \cdot {}_s b_1 \cdot {}_s k \cdot {}_s q(y) & \sum_{s=1}^{m} {}_s a_2 \cdot {}_s b_1 \cdot {}_s k \cdot {}_s q(y) & \cdots & \sum_{s=1}^{m} {}_s a_n \cdot {}_s b_1 \cdot {}_s k \cdot {}_s q(y) \\ \sum_{s=1}^{m} {}_s a_1 \cdot {}_s b_2 \cdot {}_s k \cdot {}_s q(y) & \sum_{s=1}^{m} {}_s a_2 \cdot {}_s b_2 \cdot {}_s k \cdot {}_s q(y) & & \\ \vdots & & \ddots & \\ \sum_{s=1}^{m} {}_s a_1 \cdot {}_s b_n \cdot {}_s k \cdot {}_s q(y) & & & \sum_{s=1}^{m} {}_s a_n \cdot {}_s b_n \cdot {}_s k \cdot {}_s q(y) \end{bmatrix} \tag{3.3}$$

where, ${}_sk$, ${}_s\gamma$, ${}_s\delta_Y$, ${}_s\delta$, and ${}_sQ_Y$ for each inelastic spring named s were defined same as k, γ, δ_Y, δ, and Q_Y respectively, as shown in Figure 1. The ${}_sq_{(y)}$ is non-linear hysteresis function of each inelastic spring named s. The damping matrix is assumed proportional to the initial stiffness matrix. Thus, the damping matrix in both, elastic and inelastic range, is written as Eqn. (3.4).

$$[C]=\begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n1} \\ c_{12} & c_{22} & & \\ \vdots & & \ddots & \\ c_{1n} & & & c_{nn} \end{bmatrix} \tag{3.4}$$

3.2. The DSM for Multi-Degree-of-Freedom Systems

In this section, the DSM for the structure modeled in the previous section is described, based on the DSM for the SDOF. The equivalent stiffness matrix $[K_e]$ and the equivalent damping $[C_e]$ are defined as Eqn. (3.5). Therefore, the nonlinear equation of motion is given as Eqn. (3.6).

$$[K_e]=\begin{bmatrix} k_{e11} & k_{e21} & \cdots & k_{en1} \\ k_{e12} & k_{e22} & & \\ \vdots & & \ddots & \\ k_{e1n} & & & k_{enn} \end{bmatrix}, \quad [C_e]=\begin{bmatrix} c_{e11} & c_{e21} & \cdots & c_{en1} \\ c_{e12} & c_{e22} & & \\ \vdots & & \ddots & \\ c_{e1n} & & & c_{enn} \end{bmatrix} \tag{3.5}$$

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+\{Q\}=-\ddot{u}_0[M]\{\Phi\} \tag{3.6}$$

$$\{Q\}=\left\{ \sum_{s=1}^{m} {}_s k \cdot {}_s b_1 \cdot {}_s q\left(\sum_{j=1}^{n} {}_s a_j \cdot u_j\right) \quad \sum_{s=1}^{m} {}_s k \cdot {}_s b_2 \cdot {}_s q\left(\sum_{j=1}^{n} {}_s a_j \cdot u_j\right) \quad \cdots \quad \sum_{s=1}^{m} {}_s k \cdot {}_s b_n \cdot {}_s q\left(\sum_{j=1}^{n} {}_s a_j \cdot u_j\right) \right\}^T \tag{3.7}$$

where, $\ddot{u}_0$ is the ground acceleration, $\{\Phi\}$ is the ground acceleration vector, and $\{Q\}$ is the non-linear restoring force vector expressed as Eqn. (3.7). The equivalent linearized equation of motion on the structure is given as Eqn. (3.8).

$$[M]\{\ddot{u}\}+[C_e]\{\dot{u}\}+[K_e]\{u\}+\{\varepsilon\}(\{\dot{u}\},\{u\},t)=-\ddot{u}_0[M]\{\Phi\} \tag{3.8}$$

where, $\{\varepsilon\}(\{\dot{u}\},\{u\},t)$ is the error vector and expressed as Eqn. (3.9).

$$\begin{aligned} \{\varepsilon\}(\{\dot{u}\},\{u\},t) &= \{\varepsilon_1(\dot{u}_1,u_1,t) \quad \varepsilon_2(\dot{u}_2,u_2,t) \quad \cdots \quad \varepsilon_n(\dot{u}_n,u_n,t)\}^T \\ &= -[C_e]\{\dot{u}\}-[K_e]\{u\}+[C]\{\dot{u}\}+\{Q\} \end{aligned} \tag{3.9}$$

According to definition of the least-square method, each element of both $[K_e]$ and $[C_e]$ reached extreme value is necessary conditions to minimize the values of the function each element $\overline{\{\varepsilon\}^2}$ .Where $\overline{\{\varepsilon\}^2}$ represents the square mean of the error vector $\{\varepsilon\}(\{\dot{u}\},\{u\},t)$. Thus, partial differential of each element $\overline{\{\varepsilon\}^2}$ by each element of both $[K_e]$ and $[C_e]$ become 0 as shown in the following equations.

$$\frac{\partial \overline{\varepsilon_i^2}}{\partial k_{ei1}}=\cdots=\frac{\partial \overline{\varepsilon_i^2}}{\partial k_{ein}}=0\,,\quad \frac{\partial \overline{\varepsilon_i^2}}{\partial c_{ei1}}=\cdots=\frac{\partial \overline{\varepsilon_i^2}}{\partial c_{ein}}=0 \quad (i=1\ldots n) \tag{3.10}$$

The ground acceleration is assumed as harmonic ground motion as Eqn. (3.11). The steady-state response is only considered. Eqn. (3.12) can be approximated from Eqn. (3.8) using the assumption that $\overline{\{\varepsilon\}^2}$ is can be ignored.

$$\ddot{u}_{0(t)}=a_0\cos pt \tag{3.11}$$

$$\{u_{(t)}\}=\{\delta_1\cos(pt-\varphi_1)\quad \delta_2\cos(pt-\varphi_2)\quad \cdots \quad \delta_n\cos(pt-\varphi_n)\}^T \tag{3.12}$$

where, a_0 is maximum amplitude, p is angular frequency of the harmonic ground motion, δ_i is the maximum deformation at each degree of freedom, and φ_i is the phase difference at each degree of freedom between the harmonic ground motion and the steady-state response. From Eqn. (3.9), Eqn. (3.10), and Eqn. (3.12), $[K_e]$ and $[C_e]$ are given as Eqn. (3.13) and Eqn. (3.14).

$$[K_e]=\begin{bmatrix} \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_1\cdot{}_sk_e & \sum_{s=1}^{m} {}_sa_2\cdot{}_sb_1\cdot{}_sk_e & \cdots & \sum_{s=1}^{m} {}_sa_n\cdot{}_sb_1\cdot{}_sk_e \\ \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_2\cdot{}_sk_e & \sum_{s=1}^{m} {}_sa_2\cdot{}_sb_2\cdot{}_sk_e & & \\ \vdots & & \ddots & \\ \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_n\cdot{}_sk_e & & & \sum_{s=1}^{m} {}_sa_n\cdot{}_sb_n\cdot{}_sk_e \end{bmatrix} \tag{3.13}$$

$$[C_e]=\begin{bmatrix} \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_1\cdot{}_sc_e & \sum_{s=1}^{m} {}_sa_2\cdot{}_sb_1\cdot{}_sc_e & \cdots & \sum_{s=1}^{m} {}_sa_n\cdot{}_sb_1\cdot{}_sc_e \\ \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_2\cdot{}_sc_e & \sum_{s=1}^{m} {}_sa_2\cdot{}_sb_2\cdot{}_sc_e & & \\ \vdots & & \ddots & \\ \sum_{s=1}^{m} {}_sa_1\cdot{}_sb_n\cdot{}_sc_e & & & \sum_{s=1}^{m} {}_sa_n\cdot{}_sb_n\cdot{}_sc_e \end{bmatrix}+[C] \tag{3.14}$$

$${}_sk_e=\frac{{}_sk}{\pi}\left\{(1-{}_s\gamma)\;{}_s\theta^*+{}_s\gamma\pi-\frac{(1-{}_s\gamma)}{2}\sin 2{}_s\theta^*\right\},\quad \cos{}_s\theta^*=1-2\frac{{}_s\delta_Y}{{}_s\delta}=1-\frac{2}{{}_s\mu} \tag{3.15}$$

$$C_e=\frac{4{}_sk}{p{}_s\mu\pi}(1-{}_s\gamma)\left(1-\frac{1}{{}_s\mu}\right)+c \tag{3.16}$$

where, ${}_sk_e$ is equivalent stiffness of each spring expressed as Eqn. (3.15), ${}_sc_e$ is equivalent damping coefficient of each spring expressed as Eqn. (3.16), ${}_s\gamma$ is the stiffness degradation ratio of each spring, ${}_s\mu$ is ductility factor of each spring ($={}_s\delta/{}_s\delta_Y$), and ${}_s\delta$ is maximum deformation at each spring expressed as Eqn. (3.17).

$$ {}_s\delta = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} {}_s a_i \cdot {}_s a_j \cdot \delta_i \cdot \delta_j \cos(\varphi_i - \varphi_j)} \tag{3.17} $$

Eqn. (3.13) represents the equivalent stiffness matrix $[K_e]$ which can be calculated by replacing ${}_sk_e$ of Eqn. (3.3) by ${}_sk\ {}_sq(y)$. Similarly, Eqn. (3.14) represents The equivalent damping matrix $[C_e]$ which can be calculated by replacing ${}_sc_e$ of Eqn. (3.3) by ${}_sk\ {}_sq(y)$. Therefore, in the DSM for the MDOF, the equivalent stiffness matrix and the equivalent damping matrix can be calculated using equivalent stiffness and equivalent damping coefficient calculated for each spring. Thus, the center of rigidity in the non-linear range of eccentric structures can be calculated as Eqn. (3.18), using the equivalent stiffness for each direction and each story.

$$ e_x = \frac{\sum {}_s k_{ey} \cdot {}_s l_x}{\sum {}_s k_{ey}}, \quad e_y = \frac{\sum {}_s k_{ex} \cdot {}_s l_y}{\sum {}_s k_{ex}} \tag{3.18} $$

where, ${}_sk_{ex}$ and ${}_sk_{ey}$ is equivalent stiffness of x and y direction for each story, ${}_sl_x$ and ${}_sl_y$ are x-coordinate and y-coordinate of each spring for each story, and ${}_ie_x$ and ${}_ie_y$ are eccentricity of x and y direction for each story.

3.3. Outline of the MELM

In this section, the MELM, which is based on random vibration, is proposed. According to the report by Shibata, when SDOF vibrate during earthquake, the equivalent stiffness can be calculated approximately as the slope of the maximum displacement point to the origin. And, the equivalent damping factor h_e can be calculated statistically as Eqn. (3.19).

$$ h_e = 0.2\left(1 - \frac{1}{\sqrt{\mu}}\right) + h \tag{3.19} $$

where, μ is ductility factor of inelastic spring, and h is the initial damping factor. The method is named the geometrical stiffness method (referred to as GSM, hereafter). The DSM for MDOF adopted the theory of the GSM, is the MELM. Thus, the equivalent stiffness of each spring ${}_sk_e$ is calculated as Eqn. (3.20) and the equivalent damping coefficient of each spring ${}_sc_e$ is calculated as Eqn. (3.20). However, in the MELM, ${}_s\delta$ cannot be calculated as Eqn. (3.17), because φ_i cannot be estimated at the random vibration. Therefore, in the MELM, ${}_s\delta$ is calculated by observing the deformation of the inelastic spring directly.

$$ {}_sk_e = {}_sk\left\{\frac{(1 - {}_s\gamma)}{{}_s\mu} + {}_s\gamma\right\}, \quad {}_sc_e = 0.4\left(1 - \frac{1}{\sqrt{{}_s\mu}}\right)\sqrt{m_i \cdot {}_sk_e} \tag{3.20} $$

3.4. Outline of the SMELM

In this section, the SMELM was developed to simplify the MELM. In the SMELM, ${}_s\delta$ is calculated by the δ_i as shown in Eqn. (3.21). It is assumed that nonlinear response of the structure shows a fundamental mode, then φ_i and φ_j correspond to each other, because each degree of freedom vibrates in the same phase. Thus, Eqn. (3.21) is calculated by replacing the φ_i equal by the φ_j in Eqn. (3.17)

$$ {}_s\delta = \sum_{i=1}^{n} {}_s a_i \cdot \delta_i \tag{3.21} $$

4. NUMERICAL ANALYSES ON THE ECCENTRIC STRUCTURE

In order to validate of the MELM and SMELM proposed in the previous chapter, dynamic response analyses were conducted with an eccentric structure. Dynamic response analyses were conducted with SDOF using the GSM, to compare the accuracy with the GSM for SDOF. In this paper, the accuracy of the methods was evaluated by focusing on the accuracy of the displacement of each degree of freedom.

4.1. Analytical Model of the Eccentric Structures

The structures in this study are one-story one-axis-eccentric structures. The analytical model of the eccentric structures is shown in Figure 2. The structure has four inelastic springs, three degrees of freedom in lateral, transvers and rotational direction, and rigid floor. The hysteresis of the inelastic springs is bi-linear type.

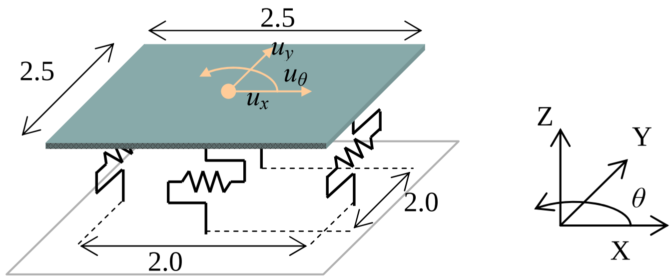


Figure 2 Analytical Model of Eccentric Structure

In this study, time history analyses were conducted using two thousand four hundred analytical models. The parameters of the models are period of X direction T_x, damping factor h, yield strength Q_Y, and eccentricity of Y direction e_Y, as shown in Table 1. The yield strength Q_Y is calculated by reduction percentage of the maximum strength of exploratory linear analyses.

Table 1 Details of parameters

period of X direction (sec)	T_x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
damping factor	h	0.01	0.02	0.03	0.05						
yield strength	Q_Y	40%	50%	60%	70%	80%	90%				
eccentricity of Y direction (m)	e_Y	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50

4.2. Process of the Analysis and Analysis Assumption

Process of the analysis is as follows:

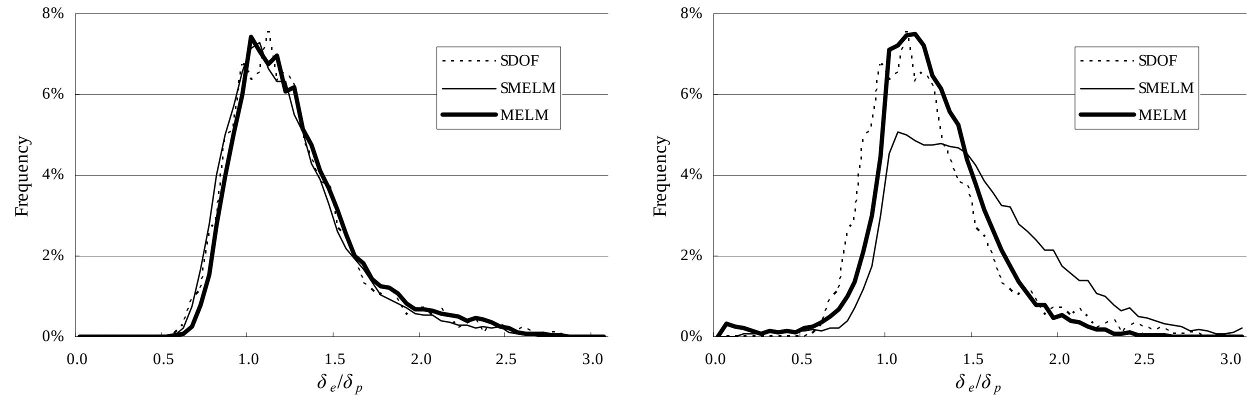
(1) Conduct nonlinear time history analysis, to obtain the nonlinear maximum displacement at each degree of freedom δ_p in the response of the nonlinear analysis.
(2) Calculate the equivalent stiffness matrix and the equivalent damping matrix by each of the MELM and the SMELM using the δ_p, which is obtained in the process (1).
(3) Conduct linear time history analysis, to obtain the equivalent linear maximum displacement at each degree of freedom δ_e in the response of the linear analysis using the equivalent stiffness matrix and the equivalent damping matrix, which is calculated in the process (2).
(4) Calculate the accuracy of the methods by the dimensionless value δ_e/δ_p, at the each degree of freedom.

Four earthquake acceleration time history records (El Centro NS, Hachinohe EW, Kobe NS, Kokuji) were inputted. Peak grand accelerations of all earthquake data were normalized as 100 gal and were inputted only in the X direction. Damping is assumed proportional to the initial stiffness matrix. The average acceleration method was used for the integral.

4.3. Results

The frequency distribution table of the accuracy about X degree of freedom is shown in Figure 3 (a). The frequency distribution table of the accuracy about θ degree of freedom is shown in Figure 3 (b). Vertical axis in Figure 3 shows the frequency. Horizontal axis in Figure 3 shows the accuracy (δ_e/δ_p); for each method, where

δ_e/δ_p=1.0 represents the most accurate point.



(a) X degree of freedom (b) θ degree of freedom

Figure 3 Frequency table

It can be seen from Figure 3 (a) that the accuracy of the MELM and the SMELM is almost the same with that of the SDOF. The accuracy of the SMELM is, however, more inaccurate than that of the MELM and the SDOF in Figure 3 (b). This is due to the fact that ${}_s\delta$ is calculated as Eqn. (3.17) in the SMELM. Thus, in the SMELM, ${}_s\delta$ calculated as Eqn. (3.17) for the assumption that nonlinear response of the structure can be shown the fundamental mode. However the assumption is inaccurate in eccentric structures because the structures do not ignore the higher mode of structures.

5. CONCLUSIONS

The MELM and the SMELM which is Equivalent linearization method for the MDOF were proposed using the least-square method for the calculation of the center of rigidity in the non-linear range of eccentric structures. Thus, the numerical analyses were conducted with the eccentric structures to evaluate the validity of the MELM and The SMELM. Results from the studies are as follows;

- The MELM was proposed to expand the applicable scope of the DSM developed by T. K. Caughey.
- In the MELM, the equivalent stiffness matrix can be calculated replacing the initial stiffness by equivalent stiffness at each spring in the linear stiffness matrix. Similarly, the equivalent damping matrix can be calculated replacing the initial stiffness by equivalent damping coefficient at each spring in the linear stiffness matrix.
- The center of rigidity in the non-linear range of eccentric structures can be calculated using the MELM.
- The accuracy of the MELM is almost the same with that of the GSM for the SDOF, which is the established equivalent linearization method.
- The accuracy of the SMELM is more inaccurate than that of the GSM for the SDOF about θ degree of freedom, because higher mode of the structure is ignored in the SMELM.

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