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SIMPLIFIED NONLINEAR SEISMIC ASSESSMENT OF STRUCTURES USING APPROXIMATE SDOF-IDA CURVES

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ABSTRACT :

Simplified inelastic procedures used in seismic design and assessment combine the nonlinear static (pushover) analysis and the response spectrum approach or nonlinear dynamic analysis of a single-degree of freedom model (SDOF). One of such procedures is the N2 method which has been developed at the University of Ljubljana and implemented into the Eurocode 8 standard. The inelastic spectrum, which is prescribed by Eurocode 8 and used for the determination of the target displacement, allows only a rough bi-linear idealization of the pushover curve and assumes an unlimited ductility. In this study an attempt has been made to predict the target displacement by four-linear idealization of the pushover curve using the approximate SDOF-IDA curves. Instead of calculating the SDOF-IDA curve for particular input parameters that describe the equivalent SDOF system, a large database of SDOF-IDA curves, which correspond to uniformly distributed input parameters (i.e. periods, damping ratios, force-displacement envelopes) and different ground-motion records, was established. The prediction of the IDA curve for a specific structure can be made by combining the database of the SDOF-IDA curves with a simple approach, known as *n*-dimensional linear interpolation. The application of the proposed methodology is demonstrated using an example of a four-storey reinforced concrete structure. The results obtained by the simplified nonlinear seismic assessment method are compared with the results based on the IDA analysis.

KEYWORDS: simplified method, inelastic, N2 method, SDOF-IDA curve, *n*-dimensional linear interpolation

1. INTRODUCTION

A popular method for the determination of the seismic response parameters, which are of high interest in performance-based earthquake engineering (PBEE), is the incremental dynamic analysis (IDA) (Vamvatsikos and Cornell 2002). It is a parametric analysis method, which involves nonlinear dynamic analysis. Although it is extensively used for research purposes, its practical application is limited since the method is computationally demanding. In order to reduce computational effort, a number of different approximate methods have recently emerged. In most practical approximate methods for IDA analysis, the nonlinear dynamic analysis is replaced by a combination of a pushover analysis of a multi-degree of freedom system (MDOF) and a nonlinear dynamic analysis of a single-degree of freedom system (SDOF). For example, Vamvatsikos and Cornell (2006) developed SPO2IDA software tool that is capable of recreating the seismic behaviour of oscillators with complex quadrilinear backbones. Han and Chopra (2006) used modal pushover analysis (MPA) for the determination of approximate IDA curves. They have shown that accuracy of a MPA-based approximate procedure is satisfactory for estimating the structural capacities for different limit states although higher modes effects had important influence on the response of the buildings which were investigated. Recently also the N2 method (Fajfar 2000), which had been developed at the University of Ljubljana and implemented in Eurocode 8 standard, was used for determination of approximate IDA curves of infilled reinforced concrete frames (Dolšek and Fajfar, 2005).

In this paper a general approach, which combines a database of IDA curves for a SDOF system (SDOF-IDA) and the *n*-dimensional linear interpolation, is proposed. The database is defined in a way that it can be easily upgraded for additional ground motion records. The use of the approximate SDOF-IDA curves for the seismic assessment of structures is then demonstrated with an example of a four-storey reinforced concrete building which was pseudo-dynamically tested at ELSA Laboratory in Ispra. The results obtained by the simplified nonlinear seismic assessment method are compared with the results based on the IDA analysis.

2. METHODOLOGY

The methodology for the determination of the approximate SDOF-IDA curves combines the seismic response database of a single-degree of freedom (SDOF) system with the ***n*-dim**ensional **l**inear **in**terpolation (n-DimLIn). The database contains a certain number of the SDOF-IDA curves, which are determined only for discrete values of the input parameters. These parameters are related to the force-displacement relationship, damping, and hysteretic rules of the SDOF system. Since the SDOF-IDA curves are determined only for a selected number of the input parameters, the *n*-dimensional linear interpolation is employed for the prediction of the approximate SDOF-IDA curve for arbitrary values of the input parameters which are within the interval for which the database is established. The accuracy of the approximate SDOF-IDA curves depends on the size of the seismic response database of the SDOF system. It is therefore important how the discrete values of the input parameters of the SDOF system are selected. In the next Section both elements of the methodology for prediction of the approximate SDOF-IDA curves are briefly explained.

2.1. Basic parameters of the seismic response database of the SDOF system

The parameters of the seismic response database of the SDOF system are divided into the input parameters and the output parameters. The input parameters are further classified as the structural input parameters and the loading input parameters. The structural input parameters describe the force-displacement relationship, period, damping, and the hysteretic behaviour of the SDOF system, whereas the loading input parameters consist of a seismic intensity measure and the ground motion record. The ground motion record is intentionally treated separately from the other input parameters since it was decided that the database has to be upgradable for at least this parameter. Therefore, SDOF-IDA curves for additional ground motion records can be easily added to the seismic response database. The output parameters, also called engineering demand parameters, are used for the description of the structural response of the model. The usual output parameters are the displacement and/or ductility demand.

In the study, the seismic response database was established for the SDOF system, which is intended to simulate the seismic response of reinforced concrete buildings. For this purpose a piecewise four-linear backbone curve was chosen to mimic the static pushover curve of both the MDOF and the equivalent SDOF system. A typical four-linear backbone curve (see Figure 1a) starts elastically up to the cracking point (LS1), yields at ductility μ=1 (LS2), remains fully plastic up to the ductility μ_u (LS3), and then starts to degrade with a slope $\alpha\, k_0$ until the zero strength. Four parameters control the shape of the backbone curve. With a suitable variation of the four parameters the idealized curve can be fitted to almost any pushover curve. Additional structural input parameters are period and damping, which was assumed mass proportional. The parameter β, which describes the unloading stiffness of the Takeda's hysteretic rules (Takeda at al. 1970), was assumed constant (0.5).

The SDOF-IDA curves were calculated for eleven periods (0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75 and 2.0 seconds, respectively), for three different damping ratios (1, 3 and 5% mass proportional damping), for eleven different combinations of F_{cr}/F_y and μ_{cr}/μ_y (see Figure 1b), for seven different ductilities (2, 3, 4, 5, 6, 7 and 8), and for three different slopes $\alpha\, k_0$ of the degrading strength (-0.05, -0.25 and -0.5, respectively). Note that any of the relations $F_{cr}/F_y = \mu_{cr}/\mu_y$ (Figure 1) corresponds to the usual bi-linear idealization of the section 0 – LS3 which is commonly used in earthquake engineering. Using all combinations of the defined structural input parameter of the SDOF system, it is necessary to calculate 7623 SDOF-IDA curves for each selected ground motion record. Such database was established for 30 ground motion records used also by Vamvatsikos and Cornell (2006). All nonlinear dynamic analyses were performed by OpenSees (McKenna et al. 2000).

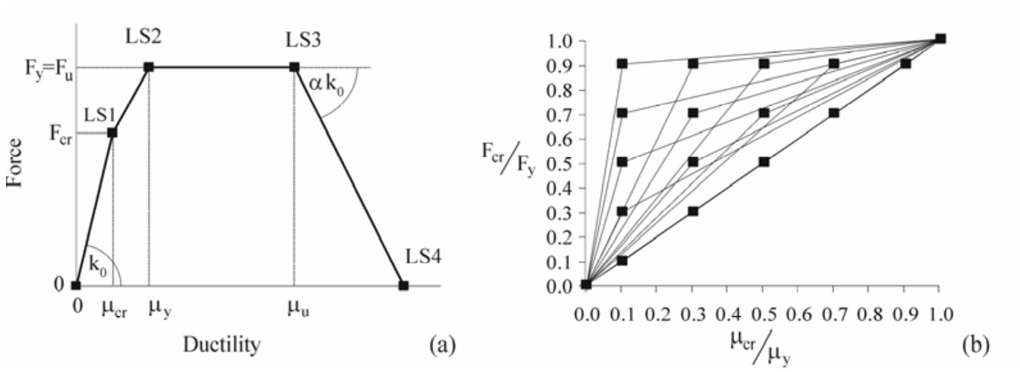

Figure 1 Definition of the backbone curve (a) and all different possible combination of the backbone curves for the ratios F_{cr}/F_y and μ_{cr}/μ_y (b), used in the study

2.2 n-dimensional linear interpolation (n-DimLIn)

Among the data fitting methods such as spline fitting or interpolation, which are commonly used to model sparsely sampled data, the linear interpolation is the simplest one. If the data are sampled on a rectangular grid, as in the case of the seismic response database described in Subsection 2.1, the solution of the problem is very easy. The *n*-dimensional linear (also known as *multi-linear*) interpolation is defined by applying one-dimensional linear interpolation in each separate coordinate dimension (Mathpages, 2008) by expression:

$$f\left(x_1, x_2, \ldots, x_n\right) = \sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{1}\left\{ f\left(i_1, i_2 \ldots, i_n\right)\cdot \prod_{j=1}^{n}\left[(1-i_k j)\, x_j + i_k j\,(1-x_j)\right]\right\} \qquad (2.1)$$

The application of Eqn. 2.1 requires the neighbouring data which are mapped into the unit hypercube $[0\ 1]^n$. x_j are (normalized) coordinates of the point in which we seek the interpolated value, and $f(i_1, i_2, \ldots, i_n)$ are the values at the corners of the unit hypercube. For the product $i_k j$ the following is valid:

$$i_k j = \begin{cases} 1 & \text{if } i_k = 0 \\ 0 & \text{if } i_k = 1 \end{cases} \qquad (2.2)$$

Eqn. 2.1 for two dimensional (x, y) linear interpolation can be written as:

$$f(x,y) = f(0,0)\,(1-x)(1-y) + f(0,1)\,(1-x)\,y + f(1,0)\,(x)\,(1-y) + f(1,1)\,x\,y \qquad (2.3)$$

$PGA(1, 1\%) = f(0,0) = 1.0$
$PGA(3, 1\%) = f(0,1) = 3.0$
$PGA(1, 4\%) = f(1,0) = 11.1$
$PGA(3, 4\%) = f(1,1) = 5.1$

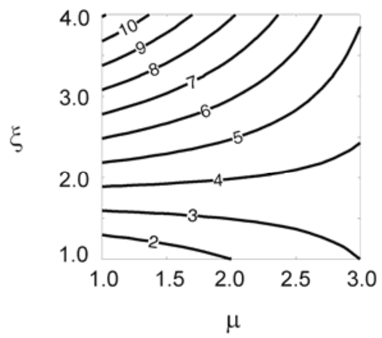

Figure 2 Application of Eqn. 2.3 for IDA curves as a function of two input parameters. Isolines connect the points of equal values of *PGA*

2-dimensional linear interpolation is demonstrated for case of two IDA curves (which correspond to 1% and 4% damping, respectively) in the range of ductilities from 1 to 3. The question is which are the interim values for engineering demand parameter, e.g. $PGA(\mu, \xi)$? By applying repeatedly the Eqn. 2.3 for each intermediate values of ductility and damping the result from Figure 2 is obtained. Note that values of *PGA* are intentionally selected as to emphasize the interesting property of the *n*-dimensional linear interpolation. Namely, it actually possesses intrinsic curvature, which is not linear with respect to arbitrary independent basis vectors.

3. CASE STUDY: A FOUR-STOREY REINFORCED CONCRETE FRAME

3.1 Description of four-storey RC frame building

A four-storey reinforced concrete structure was selected to demonstrate the proposed procedure. The elevation and the plan of the building, as well as the typical reinforcement in the columns and beams are shown in Figure 3. For this structure different pseudo-dynamic tests were performed at the European Laboratory for Structural Assessment (ELSA, Ispra) (Negro et al. 1996). The structure was designed according to early versions of Eurocodes 2 and 8 (Fardis (ed.) 1996). The design base shear versus the weight of the structure corresponded to about 16% (Fardis (ed.) 1996).

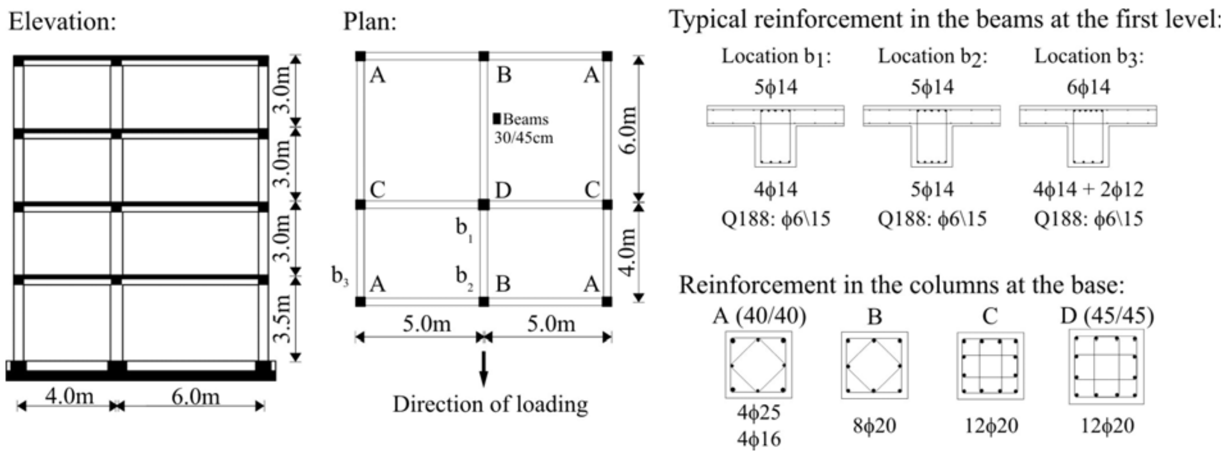

Figure 3 The elevation, the plan view and the typical reinforcement in the beams and in the columns at the base

Basically, the mathematical model of the test structure was developed in compliance with the Eurocode 8 (CEN 2004) requirements. The same modeling principles as in Fajfar et al. (2006) were employed in this study. Beam and column flexural behaviour was modeled by one-component lumped plasticity elements, composed of an elastic beam and two inelastic rotational hinges (defined by the moment-rotation relationship). The element formulation was based on the assumption of an inflexion point at the midpoint of the element. For beams, the plastic hinge was used for major axis bending only. Bilinear moment-rotation relationships were used for the first part of the moment-rotation relationship. In the second part of the moment-rotation relationship the linear strength degradation was assumed. The slope of the strength degradation in columns was predicted by CAE method (Peruš et al. 2006). For beams the slope was based on the assumed ratio of 3.5 between the rotation at the zero moment (total collapse) and maximum moment. The axial forces due to gravity loads were taken into account when determining the moment-rotation relationship for plastic hinges in the columns. All analyses were performed by OpenSees (McKenna et al. 2000).

3.2 Ground motion records

The group of 30 ground motion records, which were used also by Vamvatsikos and Cornell (2006), was selected to represent a scenario earthquake. The magnitude is within 6.5–6.9. All records were obtained on firm soil and show no directivity effects. The corresponding spectra are shown in Figure 4 and are normalized to PGA=0.3g for which the RC frame building was designed.

3.3 Determination of the equivalent SDOF system

The SDOF system has to be defined for the determination of the approximate SDOF-IDA curves. Firstly, the non-linear static (pushover) analysis of the MDOF system was performed in the positive and negative direction of loading as shown in Figure 3. The distribution of the lateral loads for the pushover analysis was determined as the product of the storey mass $\boldsymbol{m} = \{87, 86, 86, 83\}^T$ (in tons) and the first mode shape $\boldsymbol{\phi} = \{0.30, 0.60, 0.86, 1.00\}^T$. The resulting pushover curves are presented in Figure 5. The next step in determination of the SDOF system is the idealization of the pushover curve with the multi-linear force-displacement relationship (Figure 1a). Since the shape of the pushover curves are practically linear before the yielding of the columns at the base, a tri-linear idealization was used instead of more complex four-linear idealization. Very good agreement between the idealized and computed force-displacement relationship can be observed (Figure 5). For comparison, the bi-linear idealization of the pushover curve according to the principles of Eurocode 8 (CEN 2004) is also presented. It is clear that the procedure based on the Eurocode 8 principles fails to approximate the pushover curve in the strength-degrading region. This does not have a significant influence on design, since in the design process the ductility demand is limited. However, this discrepancy may underestimate the prediction of the ground motion intensity at the near collapse limit state, if this limit state corresponds to a displacement in the degrading part of the pushover curve.

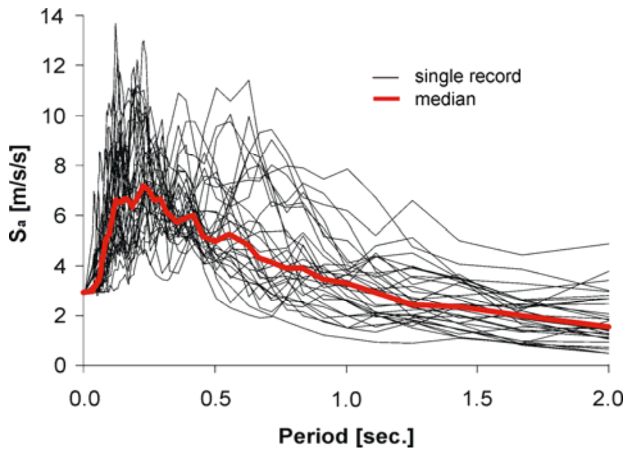

Figure 4 Spectra for the used ground motion records and median spectrum

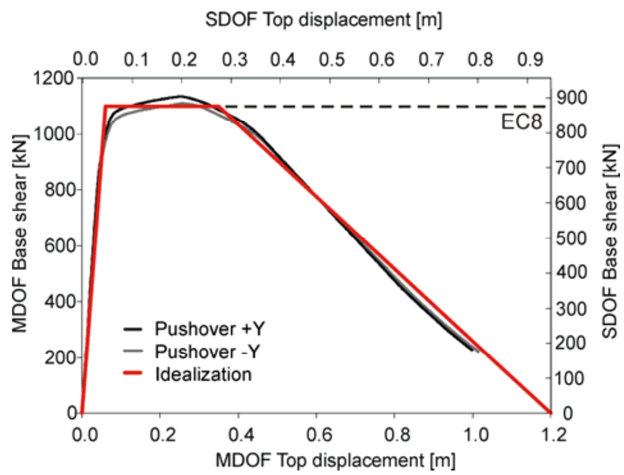

Figure 5 Pushover curves of the MDOF system and idealized force-displacement relationship. The scale is presented for the MDOF as well as for the SDOF system

Quantities of the SDOF system (denoted by asterisk *) are then determined as follows (Fajfar et al, 2000):

$$\Gamma = \frac{\sum m_i \phi_i}{\sum m_i \phi_i^2}, \quad k^* = \frac{F_y / \Gamma}{D_y / \Gamma}, \quad T^* = 2\pi \sqrt{m^*/k^*} \quad \text{where} \quad m^* = \sum m_i \phi_i \tag{3.1}$$

where m_i is the mass of the i-th floor, ϕ_i is in our case the component of the first mode shape at the i-th floor, F_y (1100 kN) and D_y (0.06 m) are the strength and the yield displacement of the idealized pushover curve of the MDOF system, Γ=1.266 is the transformation factor, used for the transformation of the MDOF system to the SDOF system, and the T^* (0.71 s) is the period of the SDOF system. The additional parameters defining the SDOF system are the ductility μ_u=5.83, which is related to the displacement (D_u=0.35 m) at the beginning of the strength degradation of the idealized force-displacement relationship, the parameter α=-0.07 defining the slope of the degrading part of the idealized pushover curve (Figure 1a), and the mass proportional damping, which was assumed to be 5%, the same value as assumed in the IDA analysis of the MDOF system.

3.4 Approximate SDOF-IDA curves

The approximate SDOF-IDA curves are determined for the defined SDOF system by using the proposed N-DimLIn procedure. The advantage of this procedure is a fast determination of the approximate SDOF-IDA curves for each ground motion record from the set of records, which are used in the database. The approximate SDOF-IDA curves are presented in Figure 6a. The selected intensity measure and the engineering demand parameter are the peak ground acceleration and ductility, respectively. The ductility demand is the output parameter of the seismic response database and is defined as the ratio between the displacement demand and the yield displacement of the SDOF system. In general, very good agreement between the approximate SDOF-IDA curves and calculated “exact” SDOF-IDA curves, presented in Figure 6b, can be observed. Note that the “exact” SDOF-IDA curves are for the defined SDOF system and are directly computed with nonlinear dynamic analyses, similarly as the SDOF-IDA curves in the database. In Figures 6a and 6b the summarized SDOF-IDA curves are also presented. Very good agreement can be observed also for the summarized curves (Figure 8a). However, some differences can be seen especially in the region near the global dynamic instability, where the IDA curve becomes horizontal. In this region the problem is highly nonlinear. Nevertheless, the results are still acceptable for practical applications since the maximum difference in the prediction of the PGA, which corresponds to global dynamic instability, is less than 13 %, for all three summarized IDA curves.

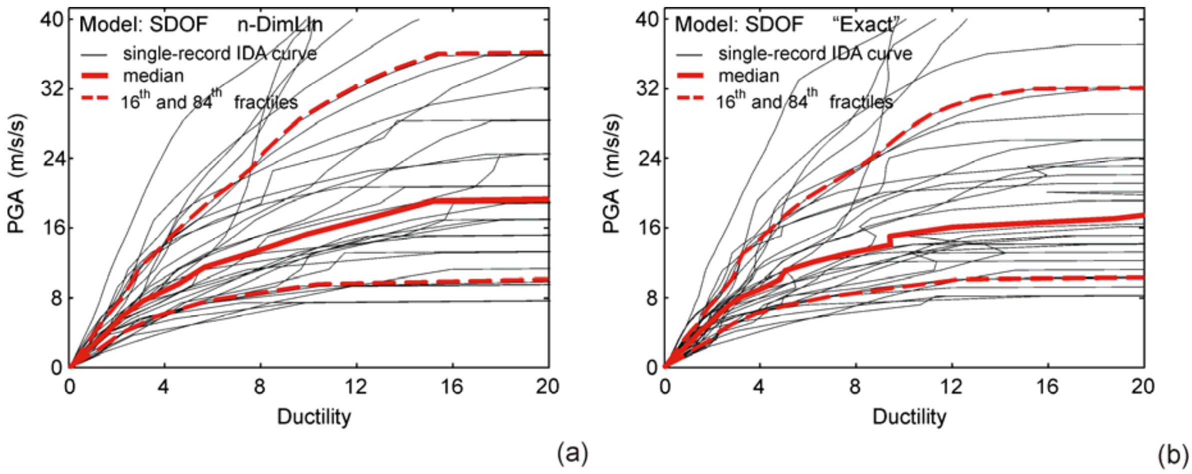

Figure 6(a) The approximate SDOF-IDA curves, obtained by the n-DimLIn procedure and (b) the “Exact” SDOF-IDA curves. In both cases the median, 16 and 84% fractiles curves are also presented

3.5 The IDA analysis of the MDOF system and comparison with the approximate IDA curves

The IDA curves were computed for the MDOF system for all ground motion records (Section 3.2) in order to validate the proposed procedure for the determination of the approximate IDA curves. The peak ground acceleration (PGA) and the top displacement were selected as the intensity measure and engineering demand parameter, respectively. The PGA, which corresponds to the dynamic instability, was determined with the tolerance of 0.01 g. The resulting IDA curves for the MDOF system together with the summarized IDA curves are presented in Figure 7. The determination of the approximate IDA curves for the MDOF quantities is straightforward. Only the ductility, which is the engineering demand parameter of the SDOF-IDA curve, has to be transformed to the top displacement, simply by multiplying the ductility with the yield displacement (D_y=0.06 m) of the idealized pushover curve.

A comparison between the computed summarized IDA curves and approximated summarized IDA curves is presented in Figure 8. Very good agreement can be observed. The radial line, also presented in Figure 8, corresponds to the so called "equal displacement rule", which is usually assumed for structures with moderate and long periods. It is obvious that this rule is acceptable for displacements less than the displacement at the beginning of the strength deterioration. However, displacement demand determined based on the "equal displacement rule" may be underestimated for high PGAs, which are usually important for the prediction of the probability of exceedance of the near collapse limit state.

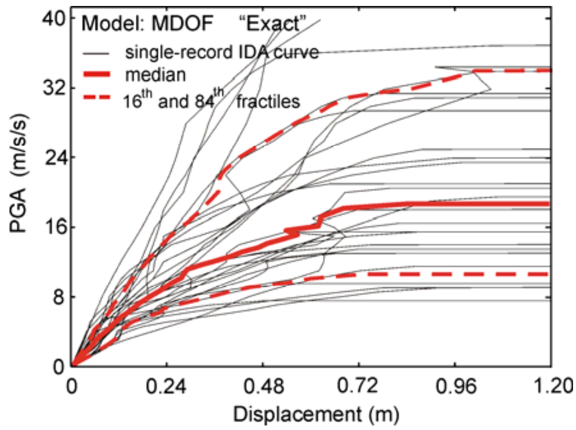

Figure 7 The computed MDOF-IDA curves. The median, 16 and 84% fractiles curves are also presented

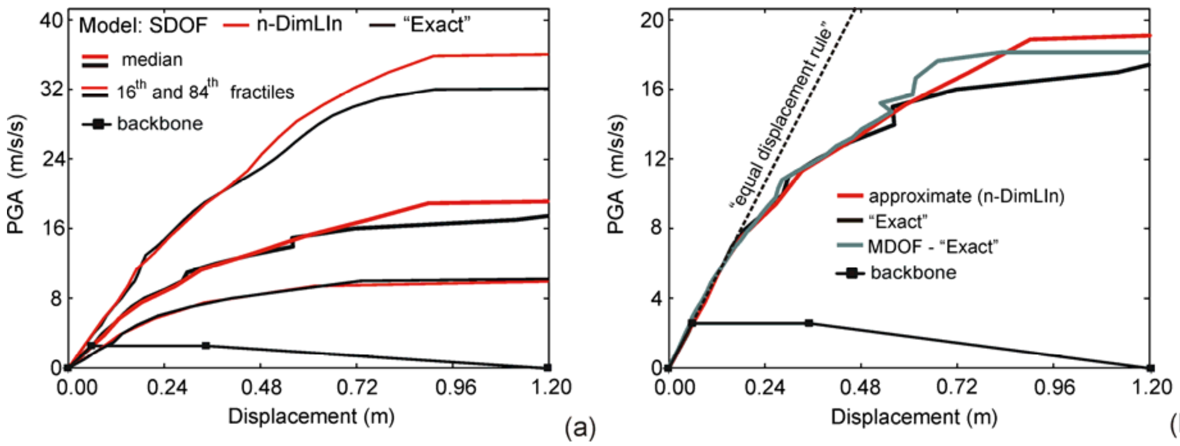

Figure 8 (a) Comparison of the "exact" and approximate results, obtained by the n-DimLIn procedure for the equivalent SDOF system, and (b) comparison of the median MDOF-IDA and equivalent SDOF-IDA curves

4. CONCLUSIONS

The procedure for determination of the approximate SDOF-IDA curves, which are intended to be used for the prediction of the approximate summarized IDA curves, is proposed. Instead of calculating the SDOF-IDA curve for particular input parameters that describe the equivalent SDOF system of the structure, a large database of SDOF-IDA curves, which correspond to uniformly distributed input parameters (i.e. periods, damping ratios, force-displacement envelopes) and different ground motion records, was established. The prediction of the IDA curve for specific structural parameters can be made by applying a simple approach, known as n-dimensional linear interpolation. The use of the approximate SDOF-IDA curves for the seismic assessment of structures is demonstrated with an example of a four-storey reinforced concrete building, which has been pseudo-dynamically tested at ELSA Laboratory in Ispra. The results obtained by the simplified nonlinear seismic assessment method were compared with the results based on the IDA analysis.

The proposed method for the determination of the approximate IDA curves is accurate enough for the practical application, as it was proven at least for the presented example. Very good agreement between approximate and computed summarized IDA curves was observed especially in the range of ductilities, which are the most important for assessment of a structure. A somewhat larger difference, but still acceptable for practical application, was observed in the range near dynamic instability, which is highly nonlinear and therefore more difficult to predict.

The proposed procedure for performance assessment of a structure allows a multi-linear idealization of the pushover curve including the strength degradation. This reduces the problems with the idealization of the pushover curves and enables an explicit determination of the global dynamic instability. The approximate IDA curves can be determined for each ground motion record from the seismic response database. Therefore the dispersions measures, which are needed for a probabilistic seismic assessment of structures, can be also determined, and the determination of the approximate IDA curves depends on the selected earthquake scenario. However, to achieve this goal the database has to be extended with additional ground motion records. Therefore the ground motion record was intentionally treated separately from other input parameters in order to have upgradable database for this parameter. In the future the database will be extend, and available online. It is foreseen that the user will have a possibility of adding the additional ground motion record to the seismic response database.

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