SECANT MODES SUPERPOSITION: A SIMPLIFIED METHOD FOR SEISMIC ASSESSMENT OF RC FRAMES

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ABSTRACT :
Dealing with seismic demands at life safety and collapse prevention limit states implies the necessity of explicitly considering the nonlinear behavior of structures. The nonlinear time-history analysis is the most rigorous procedure to compute the inelastic seismic demand, unfortunately this method is still too complex for widespread professional use. Therefore, the use of nonlinear static procedures (NSP) is going to be favored by the engineering practice.
A simplified nonlinear assessment procedure, named Secant Modes Superposition (SMS) method, is presented herein focusing on bi-dimensional frames. The quantities of interest (floor displacements, inter-storey drifts and storey shears) are derived from the combination of the modal contributions as the modal response spectrum method does in the linear case. An iterative procedure is used to define the stiffness of an equivalent linear system and the spectral modification coefficients accounting for the hysteretic energy dissipation. Some response indices are used to check convergence of the method assuring the compatibility between the quantities called into play.
Comparing the seismic response of some plane frames as estimated by nonlinear dynamic analyses and by using the SMS method, the simplified procedure seems to be able to reasonably assess their nonlinear behavior.
Additionally, the plastic hinge distribution is obtained from the iterative procedure: an useful tool to locate potential local collapse mechanisms. Worth to mention is the extremely reduced computational effort.
KEYWORDS:
frame analysis, simplified assessment procedure, equivalent linear structure,
secant stiffness, equivalent system damping.

1. INTRODUCTION

During the last decades Performance Based Seismic Engineering (PBSE) kept acquiring importance. As a key point of this engineering approach, the estimation of the inelastic deformation demand has been the goal of various researches focusing on the development of simplified procedures, commonly referred to as nonlinear static procedures (NSP), able to replace impractical nonlinear time-history analyses. The Capacity Spectrum Method (ATC40), the N2 Method (EC8) and the coefficient method (FEMA 440) are some of the procedures that can be found in literature. Basically, all these simplified methods compare structural capacity, evaluated through pushover analysis, to seismic demand, represented with the appropriate response spectrum.
Unfortunately, pushover analysis is not free from limitations resulting in a partial loss of the effectiveness of the above mentioned NSP. When using a pushover to estimate the capacity, a model of the structure is subjected to a lateral load distribution that is increased step by step up to a target value or until a pre-determined limit state is achieved. A fundamental choice, influencing the evaluated structural capacity, is whether to use a fix load pattern or an adaptive procedure. The consequences of this choice have to be considered to appropriately select which way to follow in the determination of the structural capacity. In particular, when a fixed pattern is used no strength degradation can be reproduced. On the other hand, the adaptive procedures update step by step the load profile depending on the instantaneous modal characteristics being the only way to account for their change. Unfortunately, this increases the analysis complexity but it is only able to partially solve the above mentioned problem since when the structural behavior shows a softening branch the adaptive algorithm can no longer be applied and a fix load pattern has to be used. Beside to account for the changes in the stiffness distribution, the adaptive procedures have been developed in an attempt to capture higher mode effects. Unfortunately, the inclusion of such effects in the pushover procedure is still an open issue.
In the framework of developing NSP, an attempt at developing a method able to overcome the pushover deficiencies is presented herein. The idea of the proposed Secant Modes Superposition (SMS) method is to try to

extend the modal response spectrum analysis (MRSA) to the nonlinear case.

2. REVIEW OF THE MOST COMMON NSP

Simplified nonlinear analysis methods, or NSP, able to account for inelastic structural behavior and consider potential for progressive collapse without excessive computational effort are spreading in the professional community. The capacity spectrum method (e.g. ATC40), the N2 method (e.g. EC8) and the Displacement Coefficient method (e.g. FEMA440) are some of the most common NSPs and will be briefly discussed here.
The capacity spectrum method (CSM), firstly developed by Freeman (1978), requires the determination of two primary elements: capacity and demand. The capacity curve is determined through nonlinear pushover analysis to consider the strength and the deformation capacity beyond the elastic limit. This curve is then converted in Acceleration vs. Displacement Response Spectra (ADRS) format using the conversion factors available in literature (e.g. ATC40). The seismic demand is obtained plotting together the acceleration and displacement spectra. Over-damped spectra are used at this scope considering the system equivalent viscous damping evaluated as combination of the viscous damping, inherent in the structure, and the hysteretic damping, arising when the seismic motion drives the structure into the inelastic range. ATC40 suggests the relationship developed by Newmark & Hall (1982) among different formulations in the literature for the spectral reduction. Once the structural capacity and the seismic demand are known, the demand point, representing an estimate of the maximum expected response of the building during the ground motion, can be found iteratively as the intersection of the capacity and demand curves. Iterations are required since the demand curve reduction factor is unknown, being function of the displacement demand.
Another NSP is the N2 method developed at the University of Ljubljana in mid-eighties by Fajfar (1996). This method is quite similar to the previously described capacity spectrum method and like the former gives a graphic representation of the procedure comparing structural capacity and seismic demand. The basic difference between the N2 method and the CSM lies in the demand representation. Rather than by equivalent highly damped elastic spectra, the seismic input is defined in the N2 method by the inelastic demand spectra in the ADRS format. This allows advantage to be taken of both the visual representation of the CSM and the superior physical basis of the inelastic demand spectra (Fajfar, 1999).
A different approach is used by the Displacement Coefficient method, the NSP adopted by FEMA, that evaluates the displacement demand (target displacement) from the inelastic displacement spectrum. The last is obtained from the elastic spectrum by using a number of correction factors based on statistical analysis. These coefficients are used: (i) to convert the single degree of freedom spectral displacement to the multi degree of freedom roof displacement; (ii) to account for the inelastic behavior of the analyzed structure; (iii) to consider the shape of the structural hysteretic loops; (iv) to include second order effects. All the resulting internal deformations and forces are then determined through a pushover analysis. To displace the model at least two load distributions must be considered, but in any case this procedures should not be used for structures with significant contribution from the higher modes, unless a linear dynamic evaluation is also performed (FEMA 440).

3. SECANT MODES SUPERPOSITION METHOD

The scope of the present work is to develop a simplified iterative procedure able to predict the nonlinear seismic response of bi-dimensional reinforced concrete (RC) frames without the need for complex and impractical nonlinear time-history nor pushover analysis. The predicted quantities are obtained through the superposition of modal contributions. Each of those is evaluated scaling the modal shapes, determined through eigen-analysis, by means of both the modal participation factor and the displacement spectrum ordinate corresponding to each modal period. All the eigen-quantities are evaluated applying modal analysis to an equivalent linear structure, the elements of which are characterized by an appropriate secant stiffness, while the spectral ordinates come from an over-damped displacement spectrum. Thus, very low computational demand is assured since nonlinear analyses are only required for the determination of the moment-curvature relations used to represent the element capacity.

3.1 Input Data

A number of input data are required to define the structural capacity and the seismic input. For what concerns the capacity, two-nodes frame elements connected together constitute the bi-dimensional frame model. The elements must have a constant section so that their structural capacity can be represented by a moment-curvature relation evaluated considering the nonlinear behavior of the materials and the acting axial load. Clearly, during the seismic excitation the axial load is not constant implying that the section capacity curves should be evaluated at each iteration of the simplified procedure considering the actual value. Nevertheless an approximation has been introduced evaluating the sectional capacity curves corresponding to the initial axial load, i.e. the one due to gravity loads only. Lumped representation of the mass distribution have to be included in the geometric model. Furthermore, an initial estimate for both the target displacement profile, i.e. seismic induced displacement, and the corresponding secant stiffness at an element level are required to start the iterative process. For the sake of simplicity, the displacement profile and secant stiffness at yielding can be assumed as a starting point. It is worth mentioning that whatever the initial target values, just the number of iteration to achieve convergence will be affected, not the final results.
On the side of the demand, the seismic input is characterized by means of the 5% damped displacement spectrum. It is worth mentioning however, that the simplified method is not thought to be applied to predict the structural response due to a single earthquake. As a matter of fact, the method accuracy could be affected because of peculiarities of a single ground motion that are lost in its spectral representation. Furthermore, also the spectral modification coefficients used to account for hysteretic energy dissipation could in this case jeopardize the effectiveness of the method since none of these coefficients available in literature are developed to be applied to the spectrum of a single earthquake record.

3.2 Summary of the Procedure

Modal analysis applied to an equivalent linear frame with a stiffness distribution corresponding to a given displacement pattern is at the base of the simplified method developed herein. It is worth noting that, when a structure is pushed into the inelastic range, the local stiffness significantly decreases at plastic hinge locations. Hence, the possibility to consider linear elements with variable stiffness along their length is essential. For this reason the implemented procedure subdivides each element into a number (n_p) of sub-elements, the secant stiffnesses (k_{sec}) of which are determined at the respective checkpoint locations (one per each sub-element) depending on the acting bending moment, see the following Figure 1.

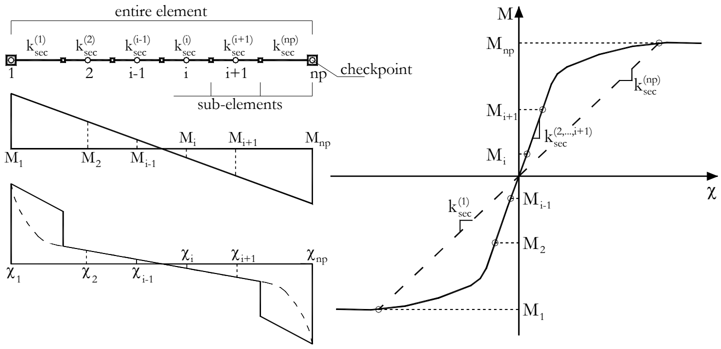

Figure 1 - Definition of secant stiffness for the variable stiffness element

Once the stiffness distribution is known, the element stiffness matrix can be assembled. Unfortunately, since the secant stiffness replaces here the traditional tangent stiffness, the usual formulation based on shape-functions can not be used to evaluate the stiffness matrix. This problem has been overcome by evaluating first the stiffness matrix of each sub-element (K_i), where the sub-element secant stiffness k_{sec} replaces the traditional flexural stiffness. These matrices are then assembled to constitute the element stiffness matrix: a N-by-N block diagonal matrix, with N equal to three times the number of sub-elements plus three. Reordering this element matrix and applying static condensation to eliminate all the dofs corresponding to the sub-elements ends, it is possible to derive the final element stiffness matrix, a symmetric matrix considering only the dofs at the two element ends.

These last matrices are then assembled in the global stiffness matrix (K) as usual for finite element procedures. The final stiffness matrix to be used for the modal analysis is then obtained by condensing out all of the mass-less dofs, in particular only one translational dof at each floor is considered coherently with the mass matrix (M), directly assembled knowing the mass distribution described by the input data.
Once the modal analysis has been performed determining all the quantities of interest, the next step towards the assessment of the structural seismic response is the evaluation of the spectral displacement corresponding to the vibration periods of the analyzed system. The 5% damped spectrum can be reasonably used when the structure behaves linearly (such as in the first iteration when the displacement pattern and the stiffness distribution at yielding are assumed as an initial guess for the target quantities). On the contrary, once the structure enters into the inelastic range with the consequent hysteretic energy dissipation, the displacement spectrum must be adequately modified. In particular, the procedure developed by Shibata & Sozen (1976) in their Substitute Structure Method has here been used to derived over-damped spectra ($S_{d,OD}$) depending on the system equivalent viscous damping. Shibata & Sozen approach assumes that "each element contributes to the modal damping in proportion to its relative flexural strain energy associated with the modal shape", hence the approach has to be applied separately to each considered vibration mode by evaluating the modal system damping.
Subsequently, the modal contributions to the method predicted displacement pattern are evaluated by scaling the modal shapes with a set of coefficients equal to the product of the modal participation factors and the spectral displacements, as shown in the following Eqn. 3.1:

$$\underline{\Delta}_{M,m} = \Gamma_m \cdot S_{d,OD}(T_m) \cdot \underline{\Phi}_m \tag{3.1}$$

where the subscript m indicates the vibration modes under consideration, $\underline{\Delta}_{M,m}$ is the modal contribution to the method displacement, Γ_m is the modal participation factor, $S_{d,OD}(T_m)$ is the spectral ordinate corresponding to the period of vibration (T_m) on the appropriate over-damped displacement spectrum and $\underline{\Phi}_m$ is the m-th modal shape. These modal contributions will then be combined to determine the method predicted displacement pattern ($\underline{\Delta}_M$ in the following) by using the square root of the sum of the squares (SRSS). The same combination rule is also used to obtain the bending moment distribution predicted by the method ($\underline{\mathrm{M}}_M$) from the corresponding modal contribution evaluated imposing the predicted modal displacement patterns on the equivalent linear structure.
Once the displacement pattern and the bending moment distribution predicted by the method have been evaluated, a check is required to ensure that the initial assumptions were correct. This check is performed using two response indices referred to as displacement and moment response indices in the following (R_D and R_M respectively). The first response index, R_D, works on the side of the displacement by verifying that the target and the method displacement profiles equal each other. Comparison of these two vectors is performed on all the considered degrees of freedom ($ndof$), i.e. three per each node of the mesh representing the RC frame, as in the following Eqn. 3.2.

$$R_D = 1 - \frac{\sqrt{\sum_{i=1}^{ndof} \left(\Delta_{T,i} - \Delta_{M,i}\right)^2}}{\sqrt{\sum_{i=1}^{ndof} \left(\Delta_{T,i}\right)^2} + \sqrt{\sum_{i=1}^{ndof} \left(\Delta_{M,i}\right)^2}} \tag{3.2}$$

On the side of the forces, the moment response index R_M checks the compatibility between the seismic demand and the capacities of the elements. To calculate this index, the target bending moment distribution (M_T) need to be computed across the frame (i.e. at each check point location). Their values are set equal to the moment capacity corresponding to the curvature predicted by the method (χ_M), evaluated as the ratio between the bending moment predicted by the method, M_M, and the secant stiffness (k_{sec}), see Figure 1. Knowing the method and the target moment distributions, they can be compared through the calculation of the moment response index as presented in the following Eqn. 3.2. This check is fundamental to assure that the adopted linearization (i.e. the secant stiffness at an element level) is compatible with the element capacity.

$$R_M = 1 - \frac{\sqrt{\sum_{i=1}^{np}\left(M_{T,i} - M_{M,i}\right)^2}}{\sqrt{\sum_{i=1}^{np}\left(M_{T,i}\right)^2} + \sqrt{\sum_{i=1}^{np}\left(M_{M,i}\right)^2}} \tag{3.3}$$

where *np* is the number of checkpoints across the frame.
The two response indices are then averaged to obtain a unique, synthetic response index *R*. The three response indices (R_D, R_M and *R*) behave in the same way with values varying between 0 and 1. In particular, these two bounding values respectively indicate no- and complete matching between the assumed target values and the quantities predicted by the method. In other words, when *R* equals or is close to unity within a certain tolerance, the method has reached convergence and the estimated quantities are assumed to represent the structural seismic response, otherwise a new iteration has to be performed.
When an additional iteration is required, prior to its beginning, the target values for both the displacement profile and the secant stiffnesses must be updated. The new target displacement profile is obtained by averaging the target and the method displacement at the previous cycle as shown in the following Eqn. 3.4

$$\underline{\Delta}_T^{(i+1)} = 0.5\left(\underline{\Delta}_T^{(i)} + \underline{\Delta}_M^{(i)}\right) \tag{3.4}$$

where the superscript indicates the iteration number. On the other hand, the new values of the moment-curvature secant stiffness are function of the ratio between the method and the target bending moment at each checkpoint as in Eqn. 3.5

$$k_{\sec,j}^{(i+1)} = k_{\sec,j}^{(i)} \cdot \frac{M_{T,j}^{(i)}}{M_{M,j}^{(i)}} \tag{3.5}$$

where *j* indicates the checkpoint under consideration. Practically, the secant stiffness at M_T is assumed as the new target value, see Figure 2.

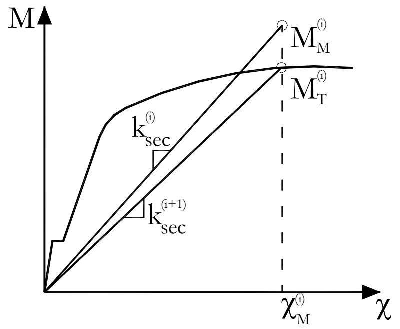

Figure 2 - Updating moment-curvature secant stiffness

Concluding, the proposed iterative procedure can be summarized as in the following:

1. Assume target displacement profile and secant stiffness distribution;
2. Assemble stiffness and mass matrices;
3. Evaluate modal quantities: vibration period, modal shapes and participation factors;
4. Estimate spectral ordinates entering the adequate over-damped displacement spectrum;
5. Evaluate the modal contribution to the displacement profile;
6. Evaluate the bending moment distribution corresponding to each modal displacement pattern;
7. Combine the modal contributions to obtain the displacement profile and the moment distribution predicted by the method;
8. Check for convergence calculating the response indices;
9. If required (response indices less than 1), update the target displacement profile and the secant stiffness distribution and iterate from step 2;

10. Once convergence has been achieved (response indices equal or close to 1), the quantities obtained at step 7 are the final prediction of the method.

4. APPLICATION OF THE SMS METHOD TO A CASE STUDY

A verification of the proposed method was clearly required to evaluate the accuracy of its predictions. For this reason, nonlinear time-history analyses were performed to assess the seismic response of several RC frame models. Two set of seven earthquakes records, artificial and natural respectively, have been used as seismic input for the dynamic analyses. Then, the median response of each set of time-history analyses has been compared to the prediction of the SMS method corresponding to the median displacement spectrum. The maximum floor displacements, inter-storey drift have been considered to assess the method in terms of global and local response respectively. Additionally, the maximum storey-shear profiles have been checked to verify the adequateness of the adopted linearization. Here is reported just an example of the performed calculations.
The ICONS frame tested at Ispra (Carvalho et al., 1999) was a four-storey three-bay frame characterized by columns having different column sections and T-section beams as shown in Figure 3. Although the geometry was quite simple, this frame introduces several difficulties in the seismic assessment since its structural behavior significantly depends on the direction of the excitation. The frame has been modeled and analyzed using the nonlinear finite element code SeismoStruct (SeismoSoft, 2006). A fiber modeling approach is used herein automatically accounting for both material inelasticity and geometrical nonlinearity. In particular, to consider the spread of plasticity across the section, the sectional stress-strain relation is derived integrating the uniaxial response of the single fibers. Bilinear stress-strain relation with kinematic hardening has been used for the steel, while the concrete behavior has been described by means of the well-known constitutive law proposed by Mander.

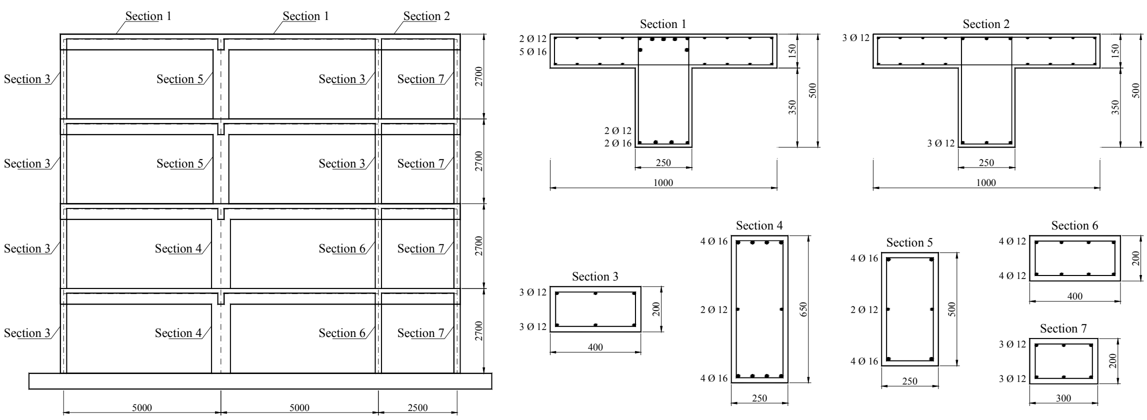

Figure 3 - Geometry and cross-sections of the considered frame

The next Figure 4 compares the maximum displacement profiles obtained by the nonlinear dynamic analyses and by the SMS method. Furthermore the obtained plastic hinge distribution is reported to illustrate the critical section as found by the simplified procedure proposed herein.
The following Table 4.1 summarizes the errors affecting the predicted quantities. Furthermore, it is possible to note that the maximum floor displacement is the quantity of easiest determination with a maximum error equal to 36%. Besides the displacement, the shear also has lower errors: they are below 20% in 65% of the cases. The most difficult parameter to determine is the inter-storey drift. It has to be considered, though, that this quantity is derived from the corresponding modal contributions calculated from the modal displacement patterns and it is not directly derived by the method. Furthermore, no response index directly checks their values unlike the other predicted quantities. These are probably two of the causes of the loss of accuracy for this parameter.
Besides the numerical values, it has to be recognized that the developed method is able to quite accurately predict the shape of the displacement pattern, localizing potential weaknesses (i.e. local mechanism) affecting the frame. The possibility of having an immediate visualization of the critical frame zones where plastic hinge would develop is also quite important. This is quite useful also because potential soft storys can be easily localized

looking at the spread of plasticity.

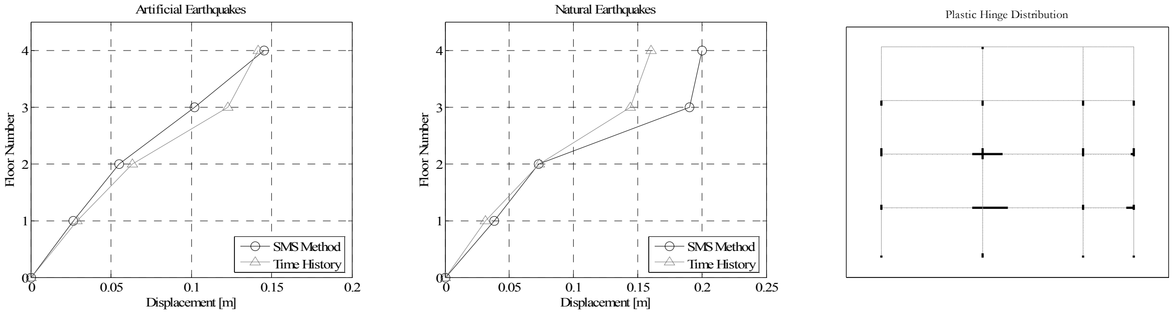

Figure 4 - Comparison between the SMS method and the nonlinear time-histories results

Table 4.1 Percentage error on the predicted quantities

	Storey displacement		Inter-story drift		Story shear	
Floor	Art EQ.	Nat. EQ.	Art EQ.	Nat. EQ.	Art EQ.	Nat. EQ.
1	-7 %	+22 %	-7 %	+22 %	-6 %	+2 %
2	-13 %	-1 %	-17 %	-19 %	-15 %	-15 %
3	-17 %	+32 %	-49 %	+20 %	+1 %	-10 %
4	+3 %	+25 %	+45 %	-63 %	+4 %	-40 %

5. CONCLUDING REMARKS AND FUTURE DEVELOPMENTS

It is important to stress that a static procedure will never be able to completely replace a dynamic analysis, nevertheless a methodology for the seismic assessment of bi-dimensional R.C. frames has been sought to obtain response information reasonably close to that predicted by the nonlinear time-history analyses. The proposed procedure, named Secant Modes Superposition (SMS) method, only performs modal analysis on an equivalent structure, the elements of which are characterized by their secant stiffness. All the quantities of interest are derived through the combination of the modal contributions. The hysteretic energy dissipation taking place during the seismic excitation of the nonlinear structure is accounted for deriving over-damped displacement spectra to be used as a representation of the seismic input. The only source of nonlinearity in the application of this method is the moment-curvature relation characteristic of each frame element. These curves are required as input data as well as the frame geometry, the mass and the gravity load distributions and the displacement spectrum describing the selected seismic input.
The SMS method has been implemented in an iterative procedure. Iterations are required to define the actual secant stiffness of each frame element. During each iteration, a target displacement pattern and the corresponding element secant stiffnesses are defined. Then the simplified method evaluates the resultant displacement pattern and the moment distribution by combining the corresponding modal contributions. Two response indices are used to check the convergence of the method ensuring the compatibility between all the quantities called into play. With a few iterations, the convergence of the two indices is normally achieved assuring that (i) the target displacement profile equals the one predicted by the method and (ii) the acting bending moments do not exceed the frame capacity at any location.
After the development of the SMS method, six plane frames were used to verify the accuracy of the its results. The output of the simplified procedure has been compared to the quantities estimated through nonlinear time-history analyses. Results show to be in quite good agreement especially from the point of view of the predicted maximum floor displacements (errors below 25% in 95% of the cases). The inter-storey drifts and the storey shears are affected by larger errors, but in any case the are of the same order of magnitude of those affecting the results of the N2 method, the NSP adopted by the EC8 and chosen as term of comparison. Besides the numerical values, the results of the assessment of the six analyzed frames seem to show that the SMS method is able to individuate potential local mechanisms, leading to less scattered results when compared to the N2 method. It is worth to mention that, once the convergence has been achieved, the SMS method shows the plastic

hinge distribution making easy to locate potential for partial collapses.
Finally, the reduced computational effort can not be neglected. For the analyzed cases, less than 10 seconds are enough to obtain the displacement pattern and the inter-storey drifts, the acting storey shears and the plastic hinges distribution relative to one displacement spectrum, with the program implemented in Matlab and running on a personal computer with a 3.2GHz processor. As already mentioned, the computational effectiveness of the SMS method is due to the fact that it just uses linear analysis and to the rapidity of convergence of the iterative process.
Further improvements are envisaged on several aspects of the simplified procedure developed herein. In particular, it could be interesting to include in the iterative process of a routine for the evaluation of the moment-curvature relation of each frame element in order to account for the effects of the varying axial load on the moment capacity. It is worth recalling that at the present time the capacity curves of the elements are relative to the initial axial load, i.e. the one due to gravity loads only. These moment-curvature relations were adopted assuming that the beams and the columns capacities were not significantly affected by the axial load variation taking place during the seismic excitation. Although this assumption has been found to be acceptable for the analyzed frames, it is clearly not generally valid. An other attractive development could also be the extrapolation of the SMS method to three-dimensional structures even if some problems will surely arise. Finally, the accuracy of the SMS method should be evaluated for a wide range of case studies varying the characteristics of both the frames and the seismic input. Work along these lines is in progress.

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