IMPROVING LATERAL STIFFNESS ESTIMATION IN THE DIAGONAL STRUT MODEL OF INFILLED FRAMES

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ABSTRACT :

In the diagonal-strut macromodel for infilled frames, the axial stiffness of the equivalent diagonal strut is usually calculated by semi-empirical formulas and application diagrams resulted from experimental results in typical one-bay infilled frames. Formulas and application diagrams proposed by Stafford Smith are among the first applied and most widely used. In these formulas and diagrams, the effects of several main modelling parameters on the system's behaviour, such as the relative beam to column stiffness and the friction coefficient at the frame-infill interface, have not been taken into consideration. In the present work a formation of application diagrams for calculating the effective width of the equivalent diagonal strut is attempted, taking into consideration all the above mentioned factors. To this purpose, the results of an extensive parametric investigation with finite element micromodels were used for the determination of the diagonal strut's axial stiffness, instead of using experimental results on which most of existing application formulas and diagrams are based.

KEYWORDS: Infilled frames, diagonal strut model, lateral stiffness.

1 INTRODUCTION

During pre-dimensioning or analysis of large structural systems, when the existence of infill panels must be taken into account, the additional lateral stiffness and strength of the building frames can be adequately estimated by using the equivalent diagonal-strut macromodel [1]. In this model the axial stiffness of the equivalent diagonal strut is mainly calculated with the aid of semi-empirical formulas and application diagrams [2], [3], [4], resulted from experimental results in typical one-bay infilled frames (Figure 1). Formulas and application diagrams proposed by Stafford Smith [1] stand out from the most frequently used.

The derivation of these formulas and diagrams was based on the evaluation of a rather limited number of experimental results, thus containing all the inevitable inaccuracies, approximations and restrictions introduced by the way the results were being interpreted. On the other hand, substantially more accurate results can be provided by properly formed micromodels of finite elements [5,] [6], which can take into consideration several types of boundary conditions at the frame-infill interface, various material laws and construction details. Therefore the existence of results' comparisons between the equivalent diagonal strut macromodel and the corresponding precise micromodels is very practical and desirable, thus giving rise to investigators to further study this subject [5], [6].

In the formulas and diagrams proposed so far for calculating the axial stiffness of the equivalent diagonal strut, the following parameters are primarily considered (Figure 1):

a) the heights h and h' of the frame and infill,
b) the ratio L/h of the width to the height of the infilled frame,
c) the ratio E_f/E_i of the elasticity modulus of the frame and infill,
d) the moment of inertia I_c of the column's section,

while the effects of certain main modelling parameters on the system's behaviour [6], such as:

e) the relative beam to column stiffness,
f) the friction coefficient at the frame-infill interface,
have not been taken into consideration.

In the present study a formation of application diagrams for calculating the axial stiffness of the equivalent diagonal strut is attempted, by taking into consideration all the above mentioned factors (a) to (f). Instead of using experimental results, the proposed application diagrams are based on the results of an extensive analytical parametric study on proper precise micromodels of finite elements. At the same time, the results obtained by the proposed methodology are directly comparable with those obtained by semi-empirical formulas and application diagrams based on existing experimental results.

2 METHOD OF ANALYSIS

The following analysis focuses on the calculation of the horizontal displacement $u_{x,P}$ at the top beam and the lateral stiffness K_s of the single-storey one-bay infilled frame shown in Figure 1a, by forming and using a precise micromodel. It also deals with the calculation of the section area F and the effective width w/d of the equivalent diagonal strut's model (Figure 1b). The shape of the frame is considered closed (instead of the conventional open shape) because it constitutes the smallest possible typical substructure of which a multi-storey infilled frame can be composed. Moreover the assumption of linear elastic behaviour of all the materials was made, together with Coulomb's friction law of unilateral contact at the frame-infill interface. This frame-infill interaction system was subjected to proportionally increasing quasi-static horizontal loads P acting at the ends of the top beam, thus providing results which are proportional to the loading magnitude [6].

2.1 Equivalent diagonal strut.

The equivalent diagonal strut model (braced frame) is shown in Figure 1b and the section area F of the strut is defined so that the horizontal displacement $u_{x,P}$ at the top beam of the infilled frame (Figure 1a), due to the action of the lateral load P, to be equal to the horizontal displacement of the braced frame (Figure 1b).
Denoting as
X_1 the axial force of the diagonal strut (Figure 1b)
f_{11} the relative displacement along the diagonal direction 1-4 of the bare frame, due to the forces X_1=1
f_{1P} the relative displacement along the diagonal direction 1-4 of the bare frame, due to the external force P
the compatibility condition of the Force Method of Structural Analysis is written as:

$$[f_{11} + d/(E_i \cdot F)] \cdot X_1 + f_{1P} = 0 \tag{2.1}$$

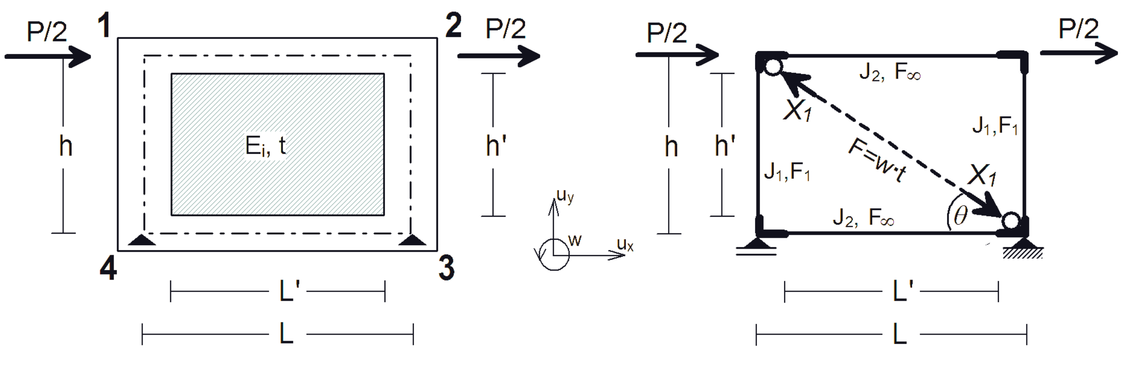


Figure 1: (a) A typical infilled frame, (b) the respective equivalent diagonal strut model

where F is the cross-section area, d is the length and E_i is the elasticity modulus of the diagonal strut (same with the infill panel).

The horizontal top displacement $u_{x,P}$ of the braced frame can be expressed as:

$$u_{x,P} = \hat{u}_{x,1} \cdot X_1 + \hat{u}_{x,P} \tag{2.2}$$

where
$\hat{u}_{x,1}$ the horizontal top displacement of the bare frame due to the forces X_1=1
$\hat{u}_{x,P}$ the horizontal top displacement of the bare frame due to the external force P.

The solution of equations (2.1) and (2.2) gives the values of X_1 and F:

$$X_1 = (u_{x,P} - \hat{u}_{x,P}) \,/\, \hat{u}_{x,1} \tag{2.3}$$

$$F = -d \cdot X_1 \,/\, E_i \cdot (f_{11} \cdot X_1 + f_{1P}) = 0 \tag{2.4}$$

while the effective width w/d is then obtained as:

$$w/d = -X_1 \,/\, [E_i \cdot t \cdot (f_{11} \cdot X_1 + f_{1P})] = 0 \tag{2.5}$$

where t is the thickness of the infill panel. It is reminded that the flexibility factors f_{11}, f_{1P}, $\hat{u}_{x,1}$ *and* $\hat{u}_{x,P}$ are calculated by analyzing the respective bare frame, while the horizontal top displacement $u_{x,P}$ is calculated by analyzing a proper precise micromodel of the infilled frame.

2.2 *Lateral stiffness of infilled frame.*

In order to calculate the lateral stiffness of the infilled frame the micromodel shown in Figure 2b was formed, by using 12 frame elements along each beam and column, 144 plane-stress elements for the infilling panel, 52 rigid zones connecting the nodes of the frame axis with the respective nodes on the internal face of the frame and 52 frictional gap elements at the interface contact area. Also rigid zones at the ends of the beams and columns were considered. The section properties of the columns (moment of inertia J_1 and axial area F_1) correspond to a square section 40×40 cm and remain constant during this study. The cross-section area F_2 of the beams is assumed to be large, in order to take into consideration the existence of slab-diaphragms at the levels of the beams. The moment of inertia J_2 of the beams can take several values during the analysis.

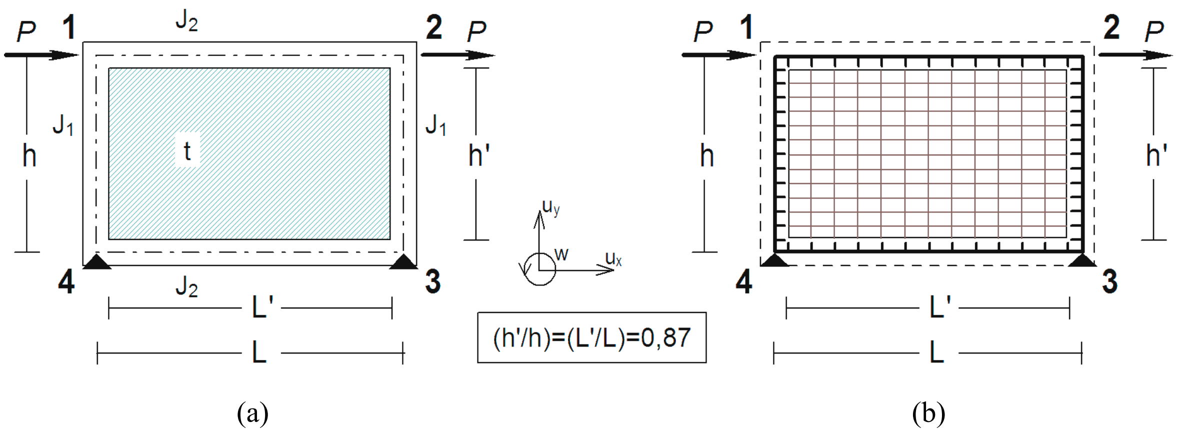


Figure 2: (a) The examined infilled frame, (b) The respective micromodel

This micromodel represents with sufficient accuracy the "exact" lateral response of the infilled frame, since the horizontal displacements at the corner nodes of an alternative model obtained by a twice finer mesh were almost identical with those of the examined micromodel (difference less than 0,5%).

The effects of certain critical modelling parameters on the micromodel's lateral stiffness K_s were further investigated through a detailed analytical parametric study. That is, for every different value of the parameters considered, a separate structural model was analyzed and the horizontal displacement $u_{x,P}$ at the top beam as well as the lateral system's stiffness $K_s=P/u_{x,P}$ were calculated. Thus several hundreds models were examined and in particular the influence of the variation of the following parameters was examined:

1. The ratio $L/h = L'/h'$ of the width to the height of the infilled frame and infilling panel respectively (Figure 2). The height h of the frame takes the constant value h=3,0 m while the length L of the frame can take the values L=3,0 m (L/h=1,0), L=4,50 m (L/h=1,50) and L=6,00 m (L/h=2,0).
2. The ratio J_2/J_1 of the inertia moments of the beams to the columns, which takes values from 0,25 (flexible girders) to 8,0 (stiff girders).
3. The ratio K_i/K_f of the lateral infill's stiffness K_i to the lateral stiffness K_f of the respective bare frame, which takes values between 0 and 100.
4. The size of the friction coefficient μ, which takes values between 0 and 1.

Relations (2.3)÷(2.5) are applied to the results of all the micromodels analyzed, thus giving the values of the effective width w/d of the respective diagonal strut models.

3 RESULTS INTERPRETATION

Using the results of the parametric analysis, application diagrams for calculating the effective width w/d as a function of the relative lateral stiffness $\lambda_D=K_i/K_f$ of the infill to frame were formed. The calculation of the lateral stiffness K_f of the bare frame can be achieved by analyzing the model of Figure 3b for a unit imposed horizontal top displacement u_x=1. Alternatively, the flexural part of this lateral stiffness can be calculated by the formula:

$$K_f = 24EJ_1k/(1+k)/(h')^3, \qquad k=(J_2h')/(J_1L') \tag{3.1}$$

Regarding the calculation of the lateral stiffness K_i of the infill panel alone, it is noted that it is proportional to the elasticity modulus E_i and to the thickness t of the infill panel and therefore it can be written as

$$K_i=a\cdot E_i\cdot t \tag{3.2}$$

The coefficient a can be calculated by forming and solving the finite element model of the pin-jointed infill panel shown in Figure 4b for imposed horizontal displacement $u_x=1$ at the left top corner. The results of such a modelling process, using a square mesh shape with 10 mesh spaces along the height of the panel, are presented in Figure 4a. It is noted that the coefficient a definitely depends on the mesh density of the model considered and therefore the diagram in Figure 4a strictly corresponds only to the mesh density shown in Figure 4b.

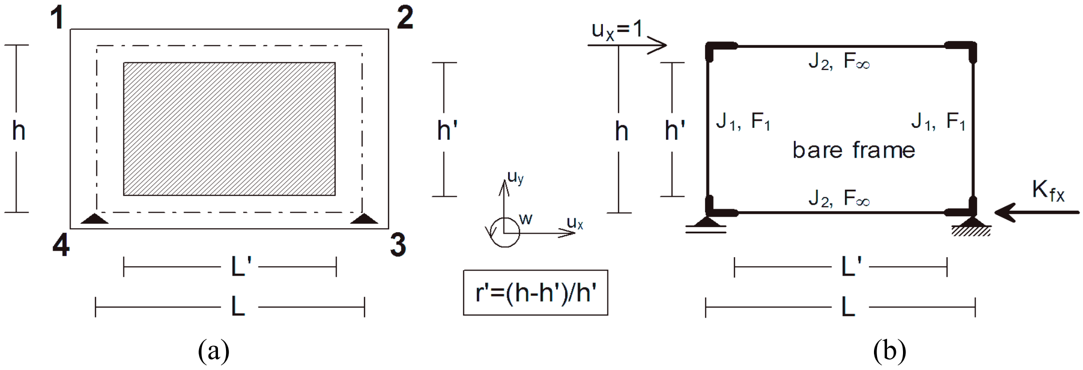
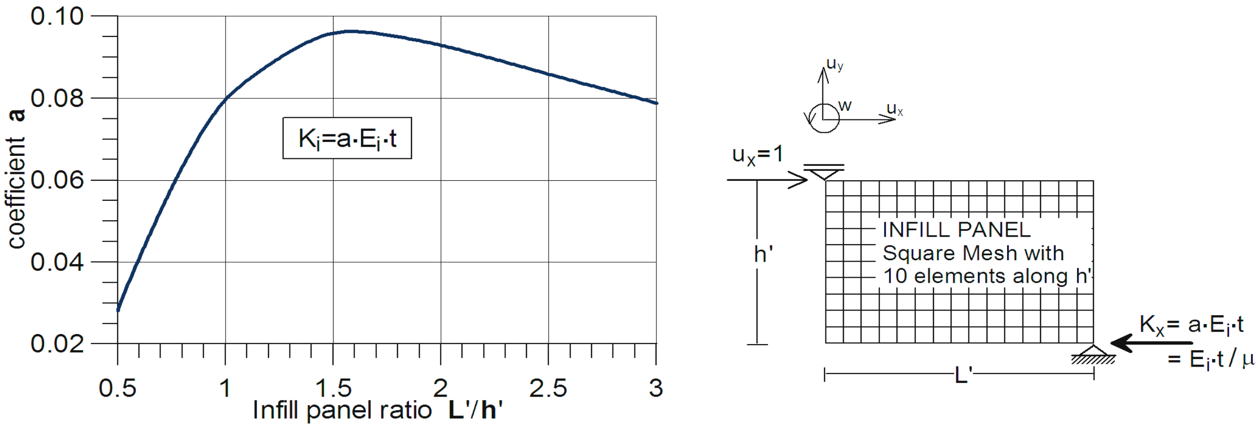


Figure 3: The examined infilled frame and the respective bare frame with rigid end-offsets



(a) (b)

Figure 4: (a) Values of coefficient "*a*" for calculating the infill's lateral stiffness. (b) Corresponding mesh density of the infill panel.

The relative lateral stiffness $\lambda_D=K_i/K_f$ defined here has a true physical meaning and is substantially different from the one defined by other researchers (Stafford Smith [1], Papia et al [5], etc.). For example, when multiplying the infill's elasticity modulus E_i by $2^4=16$ (while keeping constant all the other parameters of the infill and bare frame), it is obvious that the lateral infill's stiffness is multiplied by 16 and therefore the relative lateral stiffness $\lambda_D=K_i/K_f$ is also multiplied by 16. However, when considering the relative infill to frame lateral stiffness defined by the parameter $\lambda\cdot h$, as it is proposed by Stafford Smith:

$$\lambda\cdot h = h\cdot[(E_i\cdot t\cdot\sin 2\theta/(4E\cdot J_I\cdot h'))]^{1/4} \qquad (3.3)$$

this parameter $\lambda\cdot h$ is only multiplied by $(2^4)^{1/4}=2$, thus giving a serious underestimation of the true relative infill to frame lateral stiffness.

Applying all the above mentioned remarks, the calculation of the effective width *w/d* of the equivalent diagonal strut is presented in the diagrams of Figures 5, 6 and 7 as a function of the relative infill's to frame stiffness λ_D. Each of these figures corresponds to a constant value of the *L/h* ratio and includes 2 diagrams corresponding to 2 different values (μ=0 and μ=1) of the friction coefficient at the frame-infill interface. Each diagram includes 4 curves corresponding to 4 different values of the J_2/J_I ratio. For values of *L/h*, μ *and* J_2/J_I that are different from the values included in these diagrams, linear interpolations between the values obtained by the diagrams can be performed.

The diagrams in Figures 5, 6, 7, have resulted by considering in the calculation of the parameter $\lambda_D=K_i/K_f$ the exact lateral stiffness of the bare frame $K_f=P/\hat{u}_{x,P}$ (see equation 2.2), which takes into consideration the flexural as well as the axial deformations of the frame members. If the flexural part of this stiffness is used for simplicity reasons (see relation 3.1), the relative error in the calculation of λ_D is less than 2% in the diagrams of Figure 5 and less than 1% in the diagrams of Figures 6 and 7.

Observing the values of *w/d* given by the diagrams it becomes obvious that the influence of the different friction coefficient values μ=0 and μ=1 on the resulted values of *w/d* can be up to 50% in Figure 5, up to 55% in Figure 6 and up to 60% in Figure 7. Therefore the values of friction coefficient considerably affect the effective width *w/d* and must be taken into consideration.

It is also obvious that the different values of the J_2/J_I ratio shown on these diagrams, very little affect the value of *w/d* when the friction coefficient takes the value μ=0. However for the value μ=1 of the friction coefficient, the influence of the J_2/J_I ratio on the value of *w/d* can be as much as 15% .

Initially, the diagrams in Figures 5, 6 and 7 are strictly valid only when the columns' height is $h=3,0$ m ($h'=0,87h=2,60$ m) and the columns' cross-section is 40×40 cm, resulting in a cross-section inertia moment $J_1=0,002133$ m^4 and in a cross-section area $F=0,16$ m^2. In this case the relative axial to flexural stiffness of the columns $[(h')^2F_1/J_1]$ takes the constant value $[(h')^2F_1/J_1]=507$. In a first estimation of how the variation of the parameter $[(h')^2F_1/J_1]$ affects the effective width w/d, a square stiff infilled frame with a relative lateral stiffness $\lambda_D=K_i/K_f=7{,}90$ and $J_2/J_1=1$ was analyzed twice, by multiplying this parameter with the factors 1/3 and 3 respectively. The resulted effective width w/d changed less than 2%, which means that substantial changes of this parameter do not considerably affect the effective width w/d. Therefore all the diagrams in Figures 5, 6 and 7 can be used without any significant loss of accuracy when the relative axial to flexural stiffness of the columns $[(h')^2F_1/J_1]$ varies between 170 and 1500.

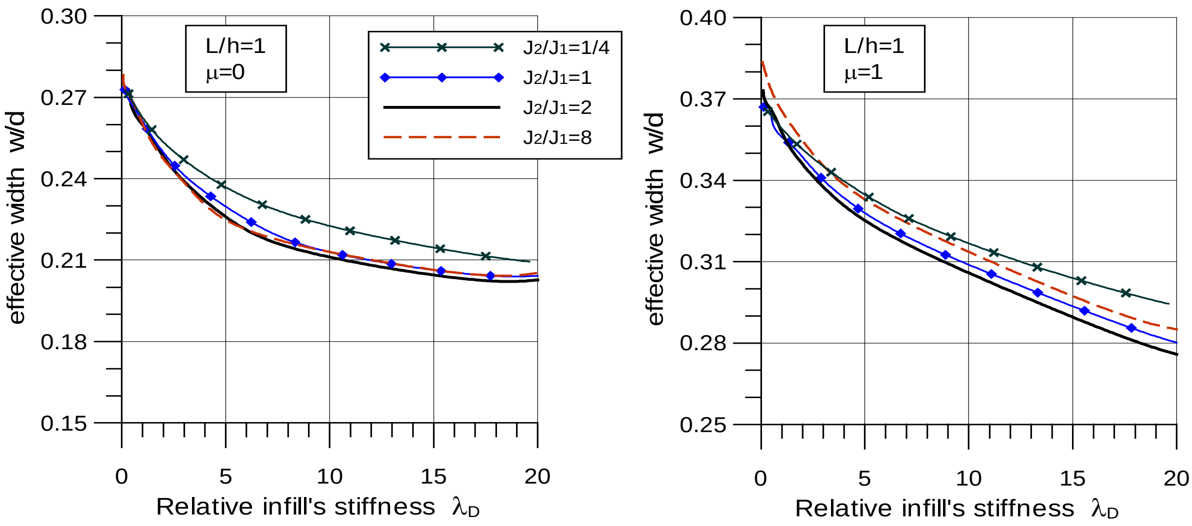


Figure 5: Effective width w/d versus relative infill's stiffness λ_D for square infilled frames (L/h=1).

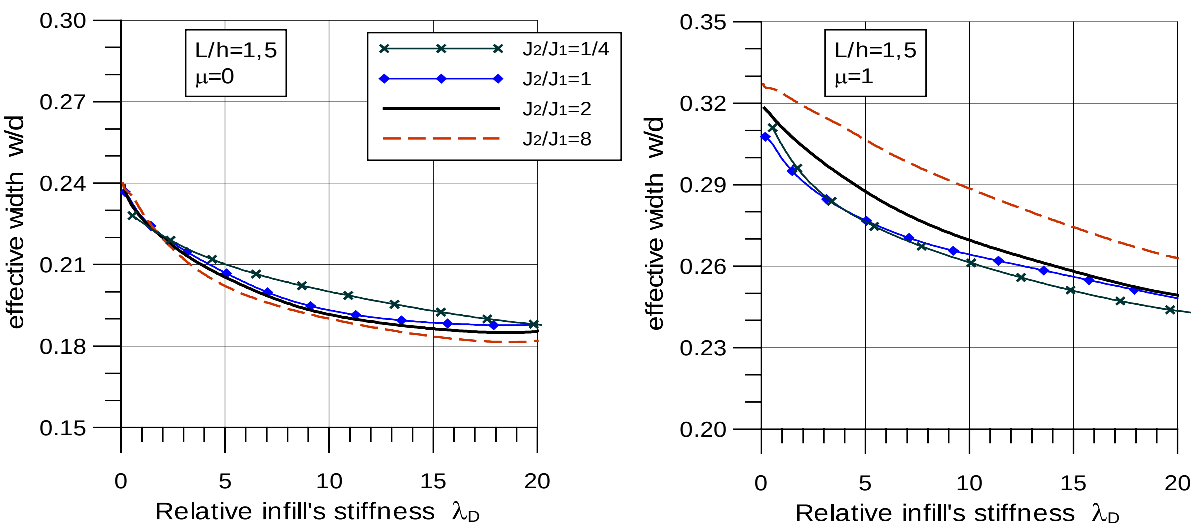


Figure 6: Effective width w/d versus relative infill's stiffness λ_D for rectangular infilled frames (L/h=1,5).

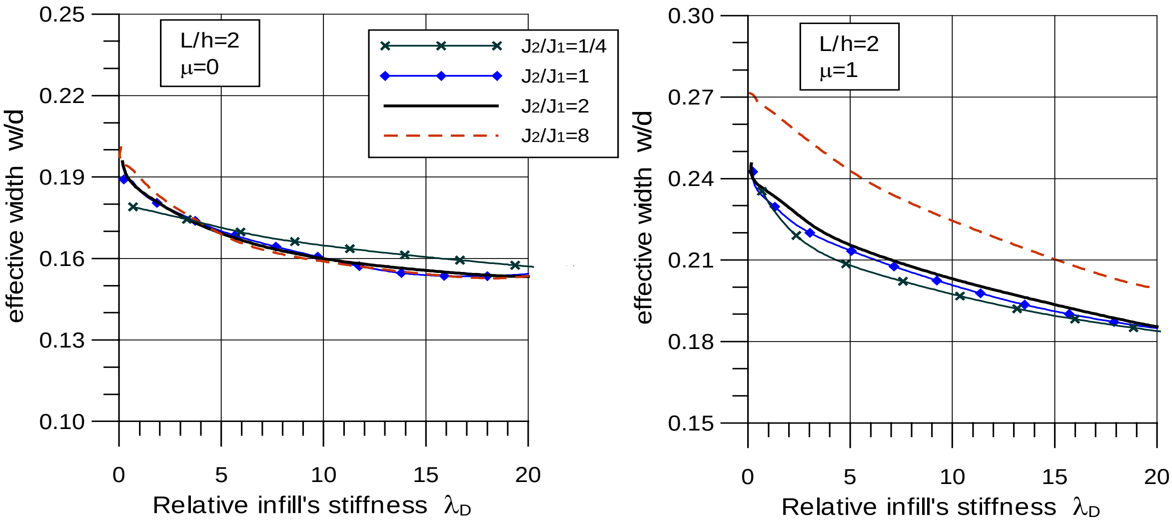


Figure 7: Effective width *w/d* versus relative infill's stiffness λ_D for rectangular infilled frames (*L/h*=2,0).

Figures 8 and 9 show detailed comparisons of the effective width *w/d* when it is calculated with the proposed procedure as well as with the semi-empirical formula (3.3) and the respective application diagrams suggested by Stafford Smith and Carter [1]. In the case of the square infilled frame we observe that a very close agreement exists in the resulted effective width *w/d* when μ=1, but a considerable difference up to 50% exists when μ=0. In the case of rectangular infilled frames with *L/h*=1,5 the procedure suggested by Stafford Smith gives results that differ from the proposed procedure up to 16% when μ=1 and up to 55% when μ=0. In the case of *L/h*=2,0 the procedure suggested by Stafford Smith gives results that differ from the proposed procedure up to 33% when μ=1 and up to 60% when μ=0. It is reminded that the semi-empirical formula and the respective application diagrams suggested by Stafford Smith cannot take into consideration the effects of the friction coefficient μ and of the J_2/J_1 ratio on the system's lateral stiffness, thus concluding that the accuracy and applicability range of this procedure is rather questionable in general.

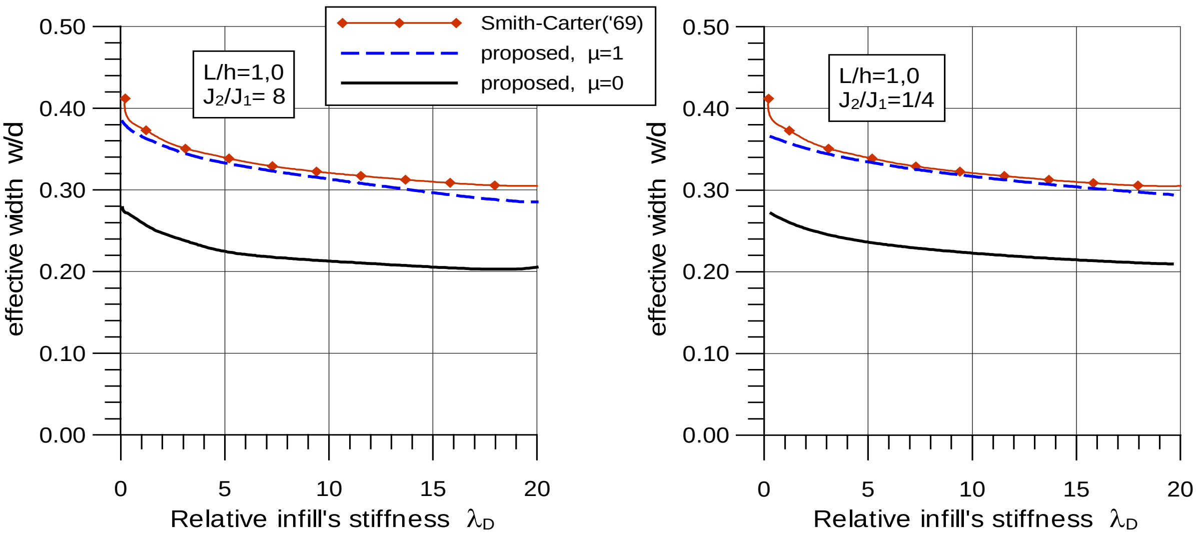


Figure 8: Effective width *w/d* versus relative infill's stiffness λ_D for rectangular infilled frames (*L/h*=2,0).

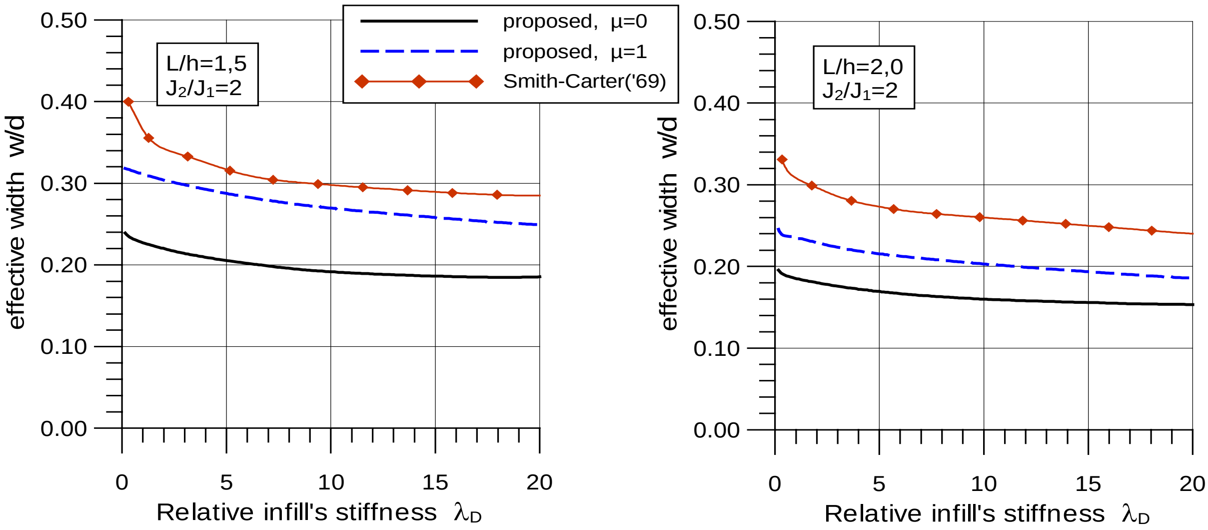


Figure 9: Effective width *w/d* versus relative infill's stiffness λ_D for rectangular infilled frames (*L/h*=2,0).

4 CONCLUSIONS

The proposed methodology of using the results of proper micromodels for calculating the effective width of the equivalent diagonal strut model is simple in application and provides results related with modelling parameters that had not been adequately taken into consideration until now. It was shown that the values of friction coefficient μ definitely affect the effective width *w/d* and must be taken into consideration. It was also shown that the values of the J_2/J_1 ratio may considerably affect the effective width *w/d*. Summarizing, the proposed application diagrams present several similarities, as well as substantial differences, to the application diagrams proposed by Stafford Smith, but in addition, the proposed diagrams are more complete and accurate.

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