SEISMIC PERFORMANCE UNCERTAINTY OF A 9-STORY STEEL FRAME WITH NON-DETERMINISTIC BEAM-HINGE PROPERTIES

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ABSTRACT :
The variability in the seismic demand and capacity of a steel frame having beam hinges with uncertain properties is investigated through Incremental Dynamic Analysis (IDA) using both Monte Carlo simulation and approximating techniques. The 9-story steel moment-resisting frame is modeled using parameterized moment-rotation relationships with quadrilinear backbones for the beam plastic-hinges. The uncertain properties of the backbones include the yield moment, the post-yield hardening ratio, the end-of-hardening rotation, the slope of the descending branch, the residual moment capacity and the ultimate rotation reached. IDA is employed to accurately assess the seismic performance of the model for any combination of the parameters by performing multiple nonlinear time history analyses for a suite of ground motion records. IDA sensitivity analysis reveals the yield moment and the two rotational-ductility parameters to be the most influential for the frame behavior. To propagate the parametric uncertainty to the actual seismic performance we employ a) Monte Carlo simulation with latin hypercube sampling, b) point-estimate and c) first-order second-moment techniques, thus offering competing methodologies that represent different compromises between speed and accuracy. The final results provide firm ground for challenging current assumptions in seismic guidelines on using a mean-parameter model to estimate the mean seismic performance and employing the well-known square-root-sum-of-squares rule to combine aleatory randomness and epistemic uncertainty.

KEYWORDS: earthquake engineering, incremental dynamic analysis, epistemic uncertainty, monte carlo, latin hypercube sampling, point estimate

1. INTRODUCTION

The accurate estimation of the seismic demand and capacity of structures stands at the core of performance-based earthquake engineering. Still, seismic performance is heavily influenced by both aleatory randomness, e.g. due to natural ground motion record variability, and epistemic uncertainty, owing to modeling assumptions, omissions or errors. Ignoring their effect means that structures are being designed and built without solid data or even adequate understanding of the expected range of behavior. While guidelines have emerged (e.g., SAC/FEMA-350) that recognize the need for assessing epistemic uncertainties by explicitly including them in estimating seismic performance, this role is usually left to ad hoc safety factors or at best standardized dispersion values that often serve as placeholders. So, if one wanted to actually compute the variability in the seismic behavior due to parameter uncertainty, the question still remains: What would be a good way to do so?

In our present work we will use Incremental Dynamic Analysis (IDA, Vamvatsikos and Cornell, 2002) to answer this issue as best as possible. IDA being a resource-intensive method, we will attempt to economically tap into its power through computation-saving methods. Efficient Monte Carlo simulation and moment-estimation techniques will be employed to propagate the uncertainty from parameters to the IDA-evaluated seismic performance offering different compromises in speed and accuracy. Using a well-studied steel moment-resisting frame as a testbed and focusing on the plastic-hinge modeling uncertainties, we will nevertheless present a general methodology that is applicable to a wide range of structures.

2. MODEL DESCRIPTION

The structure selected is a 9-story steel moment resisting frame with a single-story basement (Figure 1) that has been designed for Los Angeles, following the 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions (Foutch and Yun, 2002). A centerline model with fracturing connections was formed using OpenSEES (McKenna and Fenves, 2001). It allows for plastic hinge formation at the beam ends while the columns remain elastic. P-Δ effects were included while the internal gravity frames have been directly incorporated. The fundamental period of the reference frame is T_1 = 2.35 sec and accounts for approximately 84% of the total mass. Essentially this is a first-mode dominated structure that still allows for significant sensitivity to higher modes. Previous studies (e.g., Fragiadakis et al., 2006) have identified the yield strength of the hinges as the most influential parameter in a steel frame, compared to mass and stiffness. Thus the influence of their properties on the seismic performance of the structure will be our focus.

The beam-hinges are modeled as rotational springs with moderately pinching hysteresis and a quadrilinear moment-rotation backbone (Figure 2) that is symmetric for positive and negative rotations (Ibarra, 2003). The backbone hardens after a yield moment of a_{My} times the nominal, having a non-negative slope of a_h up to a normalized rotation μ_c where the negative stiffness segment starts. The drop, at a slope of a_c, is arrested by the residual plateau appearing at normalized height r that abruptly ends at the ultimate normalized rotation μ_u. This complex backbone is versatile enough to simulate the behavior of numerous moment-connections, from ductile down to outright fracturing. Using a "base" hinge with properties a_{My} = 1, a_h = 10%, μ_c = 3, a_c = -50%, r = 50% and μ_u = 6, we have formed a reference frame that will serve as the basis for comparing all modified models.

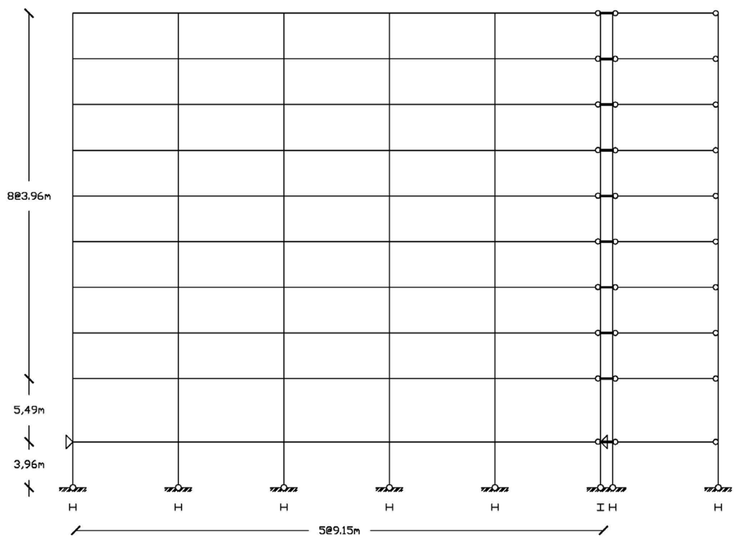


Figure 1 The LA9 steel moment-resisting frame

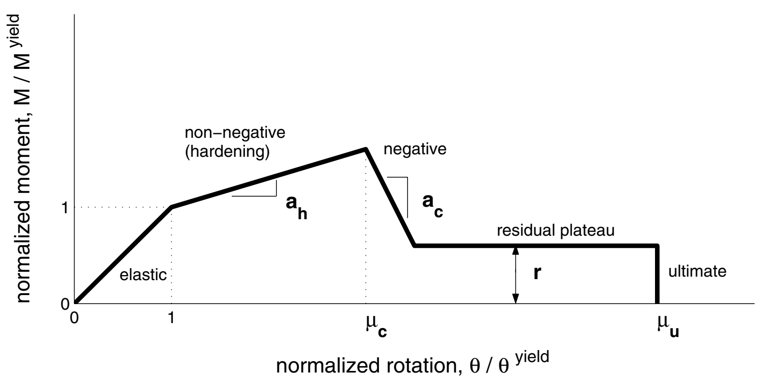


Figure 2 The moment-rotation relationship of the beam point-hinge in normalized coordinates

Table 3.1 The suite of thirty "ordinary" ground motions

No.	Event	Station	φ^{o} [1]	Soil [2]	M [3]	R [4] (km)	PGA (g)
1	Loma Prieta, 1989	Agnews State Hospital	090	C,D	6.9	28.2	0.159
2	Northridge, 1994	LA, Baldwin Hills	090	B,B	6.7	31.3	0.239
3	Imperial Valley, 1979	Compuertas	285	C,D	6.5	32.6	0.147
4	Imperial Valley, 1979	Plaster City	135	C,D	6.5	31.7	0.057
5	Loma Prieta, 1989	Hollister Diff. Array	255	–,D	6.9	25.8	0.279
6	San Fernando, 1971	LA, Hollywood Stor. Lot	180	C,D	6.6	21.2	0.174
7	Loma Prieta, 1989	Anderson Dam Downstrm	270	B,D	6.9	21.4	0.244
8	Loma Prieta, 1989	Coyote Lake Dam Downstrm	285	B,D	6.9	22.3	0.179
9	Imperial Valley, 1979	El Centro Array #12	140	C,D	6.5	18.2	0.143
10	Imperial Valley, 1979	Cucapah	085	C,D	6.5	23.6	0.309
11	Northridge, 1994	LA Hollywood Storage FF	360	C,D	6.7	25.5	0.358
12	Loma Prieta, 1989	Sunnyvale Colton Ave	270	C,D	6.9	28.8	0.207
13	Loma Prieta, 1989	Anderson Dam Downstrm	360	B,D	6.9	21.4	0.24
14	Imperial Valley, 1979	Chihuahua	012	C,D	6.5	28.7	0.27
15	Imperial Valley, 1979	El Centro Array #13	140	C,D	6.5	21.9	0.117
16	Imperial Valley, 1979	Westmoreland Fire Station	090	C,D	6.5	15.1	0.074
17	Loma Prieta, 1989	Hollister South & Pine	000	–,D	6.9	28.8	0.371
18	Loma Prieta, 1989	Sunnyvale Colton Ave	360	C,D	6.9	28.8	0.209
19	Superstition Hills, 1987	Wildlife Liquefaction Array	090	C,D	6.7	24.4	0.180
20	Imperial Valley, 1979	Chihuahua	282	C,D	6.5	28.7	0.254
21	Imperial Valley, 1979	El Centro Array #13	230	C,D	6.5	21.9	0.139
22	Imperial Valley, 1979	Westmoreland Fire Station	180	C,D	6.5	15.1	0.11
23	Loma Prieta, 1989	Halls Valley	090	C,C	6.9	31.6	0.103
24	Loma Prieta, 1989	WAHO	000	–,D	6.9	16.9	0.37
25	Superstition Hills, 1987	Wildlife Liquefaction Array	360	C,D	6.7	24.4	0.2
26	Imperial Valley, 1979	Compuertas	015	C,D	6.5	32.6	0.186
27	Imperial Valley, 1979	Plaster City	045	C,D	6.5	31.7	0.042
28	Loma Prieta, 1989	Hollister Diff. Array	165	–,D	6.9	25.8	0.269
29	San Fernando, 1971	LA, Hollywood Stor. Lot	090	C,D	6.6	21.2	0.21
30	Loma Prieta, 1989	WAHO	090	–,D	6.9	16.9	0.638

[1] Component [2] USGS, Geomatrix soil class [3] Moment magnitude [4] Closest distance to fault rupture

3. PERFORMANCE EVALUATION

Incremental Dynamic Analysis (IDA, Vamvatsikos and Cornell, 2002) is a powerful analysis method that can provide accurate estimates of the complete range of the model's response, from elastic to yielding, then to nonlinear inelastic and finally to global dynamic instability. To perform IDA we will use a suite of thirty ground motion records (Table 1) representing a scenario earthquake. These belong to a bin of relatively large magnitudes of 6.5–6.9 and moderate distances, all recorded on firm soil and bearing no marks of directivity. IDA involves performing a series of nonlinear dynamic analyses for each record by scaling it to multiple levels of intensity. Each dynamic analysis is characterized by two scalars, an Intensity Measure (IM), which represents the scaling factor of the record, and an Engineering Demand Parameter (EDP) (according to current Pacific Earthquake Engineering Research Center terminology), which monitors the structural response of the model. An appropriate choice for the IM for moderate period structures with no near-fault activity is the 5%-damped first-mode spectral acceleration $S_a(T_1,5\%)$, while the maximum interstory drift $\theta_{\max}$ of the structure is a good candidate for the EDP. Using the hunt-and-fill algorithm (Vamvatsikos and Cornell, 2004) allows the use of only twelve runs per record to capture each IDA curve. Appropriate interpolation techniques allow the generation of a continuous IDA curve in the IM-EDP plane from the discrete points obtained from the dynamic analyses. Such results are in turn summarized to produce the median and the 16%, 84% IDA curves.

Having such a powerful, albeit resource-intensive tool at our disposal, we are left with the selection of the alternate models to evaluate. There is obviously an inexhaustible number of variations one could try with the six parameters of the adopted plastic hinge, not including the possibility of having different hinge models in each story, or even for each individual connection. In the course of this study we chose to vary all six backbone parameters, namely a_h, μ_c, a_c, r, μ_u and a_{My}, uniformly throughout the structure. These parameters were varied individually to perform sensitivity analysis and then all together for uncertainty analysis. The results, evaluated using IDA, appear in the following sections.

4. SENSITIVITY ANALYSIS

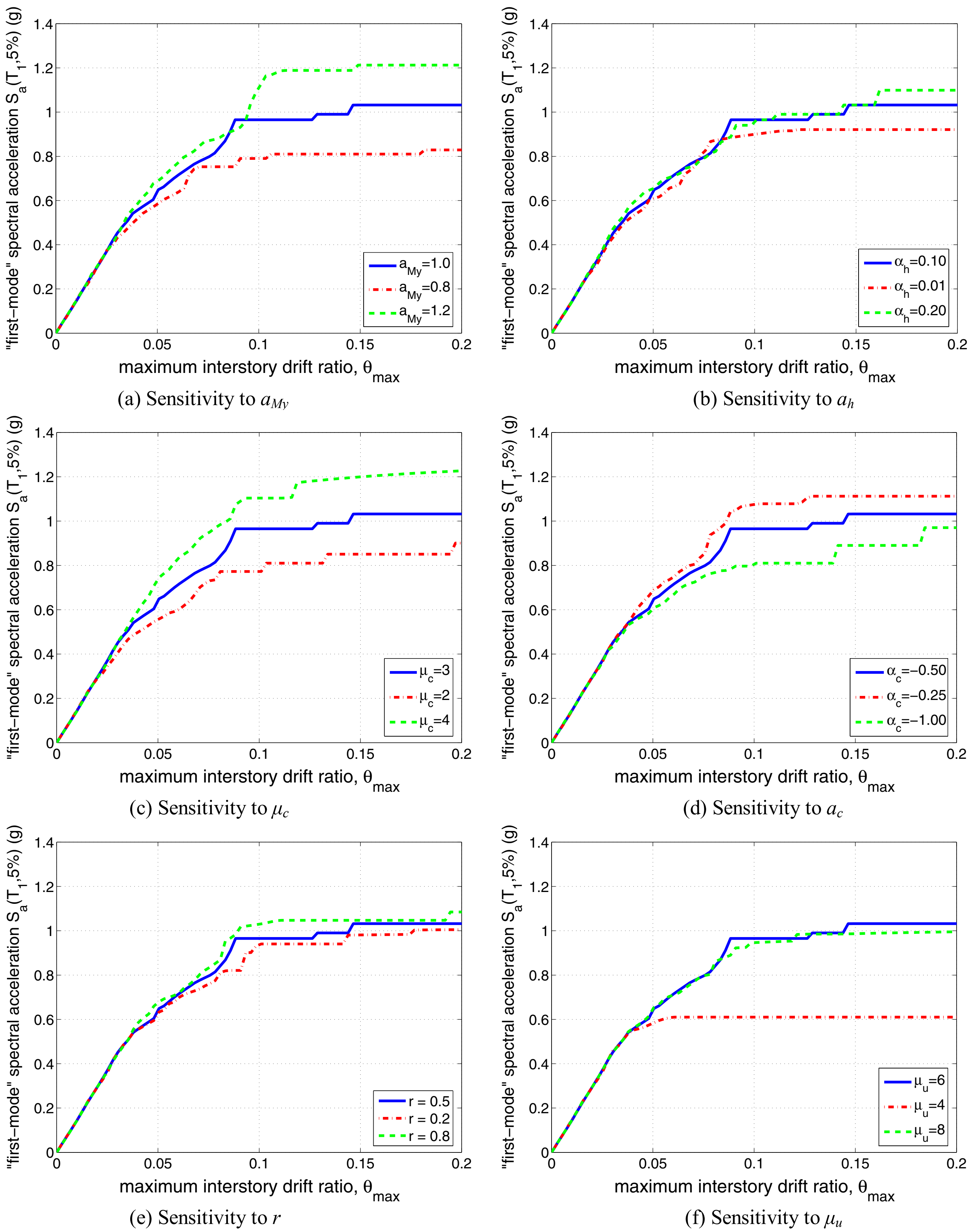
To evaluate the behavior of our model we performed a sensitivity study by perturbing each of the six backbone parameters independently of each other and only one at a time, by pushing it above or below its central value. The values a_{My} = {0.8,1.2}, a_h = {1%, 20%}, μ_c = {2, 4}, a_c = {-100%, -25%}, r = {20%, 80%}, and μ_u = {4, 8} were used. In each case IDA was performed to evaluate the sensitivity of the seismic performance which we chose to express by comparing in IM-terms the median IDA curves of the base case versus the modified ones appearing in Figures 3a-f. Keeping in mind that only thirty records were used to trace the median IDA curves shown, we should discount small differences as statistically insignificant. Thus we can safely state that a modified structure is better or worse-performing than the base case only when its median IDA appears at a reasonable distance higher or lower (in IM-terms) than the base case median.

In view of the above, Figure 3a is clear cut: Increasing or decreasing the yield strength of the plastic hinges through a_{My} does indeed cause an almost equal increase or decrease, respectively, of the seismic capacity of most post-yield limit-states. Actually this is the only parameter whose variability is propagated practically unchanged through the model while the other five parameters generally show much reduced effectiveness. Figure 3b shows one such case where both a large increase and a decrease of the hardening slope a_h seem to offer only a 10% respective change in global collapse capacity. On the other hand, accelerating or delaying the occurrence of the strength drop is of decisive importance (Figure 3c). Increasing μ_c to 4 has produced an almost 20% improvement practically everywhere in the median capacities after 3% interstory drift. Reducing μ_c to 2 has a -20% impact on the structural capacity as the accumulation of serious damage begins much earlier in the point hinges. The impact of a_c is shown in Figure 3d where, as expected, reducing (in absolute terms) the negative slope provides benefits up to 10% while making it steeper has a 15-20% detrimental effect. The relatively low value of these sensitivities is a direct result of the relatively high default residual plateau; at r = 50% it tends to trim down the effect of the negative drop, thus reducing its importance. Figure 3e shows the effect of r, where it appears that for a given negative drop and a relatively short plateau (μ_u = 6), the residual moment of the plastic hinge has little influence on the predicted performance of the LA9 structure. However, different default settings on a_c and μ_u can easily change such results; therefore no general conclusions should be drawn just yet. On the other hand, for μ_u there can be no objection that the median IDAs are greatly influenced by its reduction but not significantly by its increase. A 33% ultimate ductility decrease cost the structure a 40% reduction in collapse capacity, while an equal improvement made no difference statistically. It seems that the strength loss caused by a brittle and fracturing connection will dominate the response of the building. On the other hand, even a substantial increase in the rotational ductility does not make much difference for this building, perhaps because of other effects, e.g., P-Δ, taking the lead to cause collapse. In other words, even letting μ_u go to infinity, as is typically assumed by most existing models, we would not see much improvement as the building has already benefited from ultimate rotational ductility as much as it could.

5. UNCERTAINTY ANALYSIS

In order to evaluate the effect of uncertainties on the seismic performance of the structure we chose to vary the base beam-hinge backbone by assigning realistic distributions to its six parameters. Each parameter is assumed to be independently normally distributed with a mean equal to its default value and a coefficient of variation (c.o.v) equal to 0.2 for a_{My} (due to its overwhelming effect) and 0.4 for the remaining five parameters. Since the normal distribution assigns non-zero probabilities even for physically impossible values of the parameters, e.g., $r < 0$, or $a_h > 1$ we have truncated the distribution of each parameter within a reasonable minimum and maximum that satisfies the physical limits. We chose to do so by setting hard limits at 1.5 standard deviations away from the central value, thus cutting off only the most extreme cases.

Given the parameter distributions, Monte Carlo simulation was performed using latin hypercube sampling (LHS, McKay et al., 1979) for N = 200 realizations of the frame. LHS is a special case of stratified sampling that allows efficient estimation of the quantity of interest by reducing the variance of classic Monte Carlo. The Iman and Conover (1982) algorithm was employed to reduce any spurious correlation appearing between the samples.



(a) Sensitivity to a_{My} (b) Sensitivity to a_h

(c) Sensitivity to μ_c (d) Sensitivity to a_c

(e) Sensitivity to r (f) Sensitivity to μ_u

Figure 3 Sensitivity of the median IDA results to the beam-hinge backbone parameters

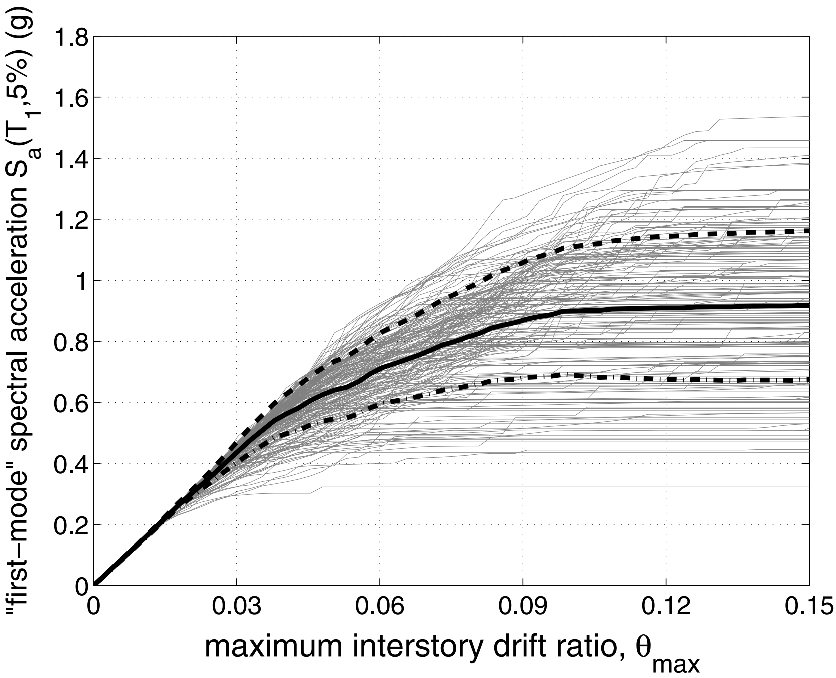


Figure 4 The 200 median IDAs shown against their mean and +/- one standard deviation curves

By performing IDA on each of the N samples we have obtained 30×N IDA curves and N corresponding median IDAs shown in Figure 4. These allow us to provide unbiased estimates of the mean and the variance of the median IDA curve due to the uncertainty in the parameters of the structure. Actually, as Figure 4 reveals, the mean of all sample medians shows a collapse capacity of 0.9g which is 0.1g lower than the base case median of almost 1.0g. Given the dispersion shown this difference becomes statistically significant, thus casting considerable doubt on the typical assumption that the mean parameter model will produce the mean seismic performance (e.g., Cornell et al., 2002).

The N IDA curves can also be used to estimate the variability caused by the parameter uncertainties in the median capacity for each limit-state. As proposed by Cornell et al. (2002), such dispersion caused by the uncertainty in the median capacity will be characterized by its β-value, β_U, which can be calculated directly as the standard deviation of the natural logarithm of the estimates of the median capacities,

$$\beta_U = \sqrt{\frac{\sum\left(\ln S_{a,i}^{50\%} - \overline{\ln S_a^{50\%}}\right)^2}{N-1}} \tag{5.1}$$

where $S_{a,i}^{50\%}$ (i = 1,2,…,N) are the estimates of the median S_a-value of capacity for a given limit-state from each model realization and $\overline{\ln S_a^{50\%}}$ is the mean of the natural logarithm of the median S_a-values of capacity.

A simpler alternative to performing Monte Carlo simulation, even with the efficient LHS, is the use of moment-estimation methods to approximate the variability in the IDA results. These are typically based on the use of only a handful of runs for appropriately perturbed versions of the base case. Using functional approximations or moment-matching such schemes manage to propagate uncertainty from the parameters to the final results using only a few IDA runs. Specifically we used the point estimate method (PEM) of Rosenblueth (1981) and the first-order-second-moment method (FOSM, e.g. Baker and Cornell, 2003). For uncorrelated and unskewed random variables, both methods need only two IDA evaluations per parameter, practically resembling the sensitivity results (Figure 3) spaced one standard deviation away from the mean value of each parameter.

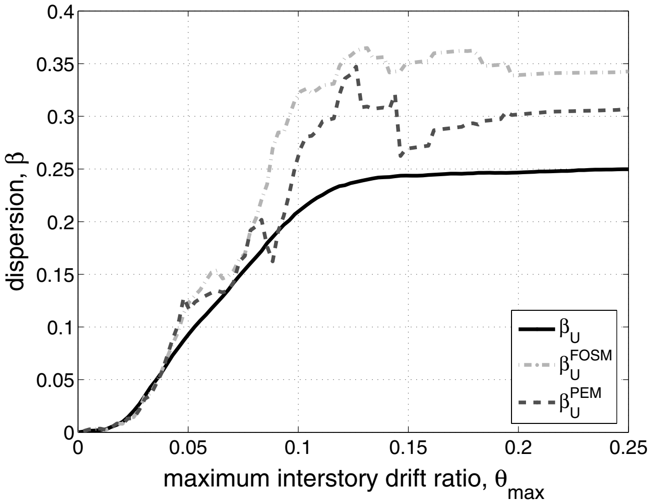


Figure 5 The values of β_U estimated using Monte Carlo simulation, FOSM and Rosenblueth's PEM given $\theta_{\max}$

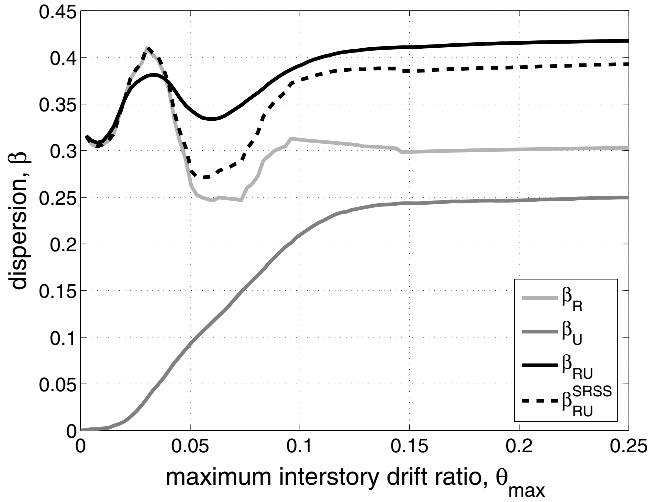


Figure 6 The values of β_U, β_R, β_{RU} compared against the SAC/FEMA approximation β_{RU}^{SRSS} given $\theta_{\max}$

The estimates of β_U obtained by the three methods appear in Figure 5. In all cases the epistemic uncertainty smoothly rises from a zero value in the elastic range (reasonable, as all modifications to the plastic hinges are post-yield) and slowly increases up to its collapse-level value. This is estimated to be 0.25 by Monte Carlo, while both PEM and FOSM manage to get quite close, slightly overpredicting the dispersion at collapse at 0.30 and 0.35 respectively, showing errors of 20-40%. Obviously, at least Rosenblueth's PEM can provide a reasonable estimate to β_U using only 2x6+1 = 13 sample points, rather than 200 for LHS. That is almost a 15-fold reduction in computations at the cost of only 20% error. A further attempt at reducing the computational load can be found in Fragiadakis and Vamvatsikos (2008), using static pushover analyses rather than IDA.

The epistemic uncertainty β_U is competing against the dispersion due to record-to-record aleatory randomness of $S_a(T_1,5\%)$ given the EDP $\theta_{\max}$. This dispersion is also important for the performance evaluation of structures and similarly represented by its β-value (SAC/FEMA 2000), i.e. by the standard deviation the natural logarithm of the IM given the EDP, which can also be calculated from the fractile IDAs as

$$\beta_R = \ln S_a^{50\%} - \ln S_a^{16\%} \tag{5.2}$$

where $S_a^{50\%}$ and $S_a^{16\%}$ are the 50% (median) and 16% values of $S_a(T_1,5\%)$-capacity. Since we are interested in the lower values of capacity, it makes sense to estimate any β-value using the median and the lower fractile (16%) rather than the higher one (84%).

Both the epistemic uncertainty β_U and the aleatory randomness β_R contribute to the value of the total dispersion β_{RU} caused by the record-to-record randomness and the model uncertainty. This is often directly used, e.g., in the SAC/FEMA framework, to assess performance in the presence of uncertainty (Cornell et al. 2002). Since we have available the full IDA data from the Monte Carlo simulation, we can estimate β_{RU} directly from the 200 samples times the 30 IDA curves computed. Alternatively, SAC/FEMA approximates β_{RU} as the square-root-sum-of-squares (SRSS) of β_R and β_U, an approximation which is usually taken for granted:

$$\beta_{RU}^{SRSS} = \sqrt{\beta_R^{\;2} + \beta_U^{\;2}} \tag{5.3}$$

Such a value for every limit-state, or value of $\theta_{\max}$, serves as a useful comparison of the relative contribution of randomness and uncertainty to the total dispersion as shown in Figure 6. In general, the high β_R overshadows the lower β_U, especially since the latter is produced by a c.o.v of only 0.2 to 0.4 in the parameter values resulting to a maximal value of 0.25 for β_U. The record-to-record variability is higher for any limit-state, ranging from 0.30 up to 0.40. Finally we see that the SRSS estimate of β_{RU} is very close to its value estimated by LHS. On average

Eqn 3.3 underpredicts the actual β_{RU} the error is in the order of 5% or less, except for drifts within 0.05 to 0.08 where the error grows to almost 20%. For all practical purposes, the SRSS rule for combining aleatory randomness and epistemic uncertainty can be considered accurate for such a structure.

6. CONCLUSIONS

The epistemic uncertainty in the seismic demand and capacity of a 9-story steel moment-resisting frame with non-deterministic beam-hinges has been estimated using IDA. Monte Carlo simulation with latin hypercube sampling has been employed as the primary means to propagate the uncertainty from the model parameters to the seismic performance, while simplified methods based on point-estimate methods and first-order second-moment techniques have also been proven to allow accurate estimation at a fraction of the cost of simulation. All in all the epistemic uncertainty in beam-hinges is shown to be an important contributor to the overall dispersion in the performance estimation as well as a key point that raises issues regarding the validity of current assumptions in performance evaluation. The classic notion that the mean-parameter model produces the mean seismic demand and capacity has been disproved. Additionally the simple square-root-sum-of-squares rule for the combination of aleatory randomness with epistemic uncertainty has been proven to be accurate enough for some limit-states but significantly off the mark for others. However, as a general conclusion, both assumptions are still reasonable accurate for practical applications.

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