On the earthquake input model with multiple support excitations

D.S. Li[1], H.N. Li[2], S.Y. Xiao[3] and G.X. Wang[3]

[1] *Lecturer, Dept. of Civil Engineering. Dalian University of Technology, 116023 China*
[2] *Professor, Dept. of Civil Engineering. Dalian University of Technology, 116023 China*
[3] *Associate Professor, Dept. of Civil Engineering. Dalian University of Technology, 116023 China*
Email: dsli@dlut.edu.cn

ABSTRACT :

The paper discusses the earthquake input model with multiple support excitations and its apparent discrepancy with the earthquake input model with uniform ground motion. When the excitations at the multiple supports of a structure are identical, both models agree with each other exactly when expressed in absolute displacement form despite their superfluous inconsistency. The key factors are embedded in the partitioned stiffness and damping matrices, in which the superstructure related terms can be mapped to the support related terms. Therefore, the effective earthquake forces in both models can be derived to be equivalent and the apparent discrepancy existing in both models is resolved. Furthermore, the earthquake force transferring mechanism from the foundation to the upper structure is exposed in details.

KEYWORDS: Earthquake input model, multiple support excitations, uniform ground motion.

1. INTRODUCTION

It is well known that the governing equation of motion for a structure with Multiple Degrees of Freedom (MDOFs) due to a uniform ground motion as followings (Clough, 1995),

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\ddot{\mathbf{u}}_g \tag{1.1}$$

where $\mathbf{m}$, $\mathbf{c}$ and $\mathbf{k}$ are mass, damping and stiffness matrices, respectively. $\mathbf{u}$ is a displacement vector of the structure components and the dot operator above the $\mathbf{u}$ represents the first derivative ($\dot{\mathbf{u}}$) and the second derivative ($\ddot{\mathbf{u}}$). $\ddot{\mathbf{u}}_g$ denotes the uniform ground motion acceleration.

Eqn.1.1 is the earthquake input model for a structure with uniform ground motion and noted hereinafter as Model 1. It seems that all the masses experience the earthquake accelerations at the same time in Model 1, and there is no term allowing for the consideration of the earthquake wave transferring from the base of a structure to its upper parts, which is not reasonable at first sight. On the other hand, the governing equation of motion for a large extended structure subjected to non-uniform ground motion can be written as (Chopra, 2001),

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{r}\ddot{\mathbf{u}}_g - (\mathbf{c}\mathbf{r} + \mathbf{c}_g)\dot{\mathbf{u}}_g \tag{1.2}$$

where $\mathbf{r}$ is the influence matrix describing the influence of support displacements on the structural displacements, $\mathbf{r} = -\mathbf{k}\mathbf{k}_g^{-1}$. In addition, the variables in Eqn.1.2 are defined in partitioned form for a general MDOF structure as,

$$\begin{bmatrix} \mathbf{m} & \mathbf{m}_g \\ \mathbf{m}_g^T & \mathbf{m}_{gg} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}^t \\ \ddot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c}_g \\ \mathbf{c}_g^T & \mathbf{c}_{gg} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}^t \\ \dot{\mathbf{u}}_g \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^t \\ \mathbf{u}_g \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{p}_g(t) \end{Bmatrix} \tag{2.3}$$

where the displacement vector is divided into two parts, (1) $\mathbf{u}^t$ contains the *m* DOFs of the superstructure; and (2) $\mathbf{u}_g$ contains the *s* components of the support displacement as illustrated in Figure 1. The variables with subscript $_g$ represent those terms associated with ground support motion. For instance, $\mathbf{c}_g$ is the partitioned damping term connected to the ground.

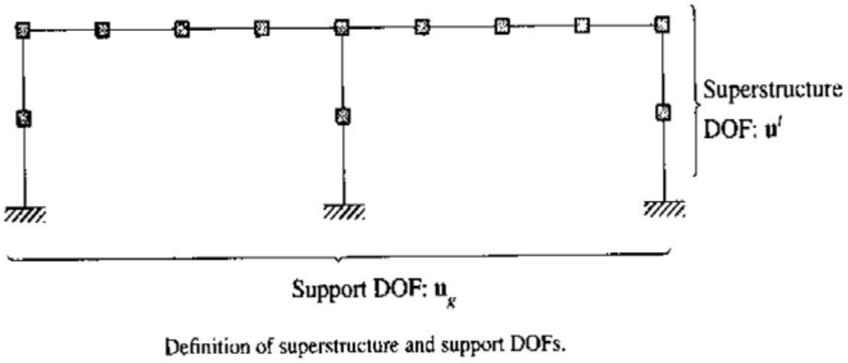

Figure 1 A general MDOF structure subjected to non-uniform ground motion

Eqn.1.2 is the earthquake input mode1 for a structure subjected to non-uniform ground motion and termed as Model 2 afterwards for the convenience of notation. Moreover, the damping related terms in the r.h.s. of Eqn.1.2 is usually neglected. For damping matrices proportional to the stiffness matrices, such simplification is exact although the proportionality assumption is not realistic. Even for arbitrary damping, the simplification is permissible since the damping terms are generally relative small compared to the inertia terms.

Therefore, both Model 1 and Model 2 are available to compute the responses of a structure undergoing uniform ground motion. It is naturally necessary that Model 1 and Model 2 agree with each other. Otherwise, there must be one model that is wrong. In other words, Eqn.1.2 should be reduced to Eqn.1.1 when the excitations are identical at the supports if both models are correct. At first sight, it appears that Eqn.1.2 differs a great deal from Eqn.1.1, especially in the fact that a velocity related term in the r.h.s. of Eqn.1.2 is included. Even one considers that the damping term is relatively insignificant compared to the inertia term in the r.h.s. of Eqn.1.2 and can be ignored, both equations are apparently inconsistent.

The paper deals with this apparent discrepancy. In section 1, a single DOF structure is considered and the agreement of Model 1 and Model 2 in the application to this simple example is demonstrated. Section 2 provides three typical structures illustrating the agreement of Model 1 and Model 2 in three different cases. The intuition gleaned from Section 2 is applied to a general structure subjected to uniform support excitation and is extended to prove the consistency of both models. Furthermore, a latent relationship between the superstructure related terms and the support related terms in the stiffness matrix are revealed, which is critical in verifying the consistency of Model 1 and Model 2. In the remainder of the paper, uniform ground motion is treated, if not explicitly stated otherwise.

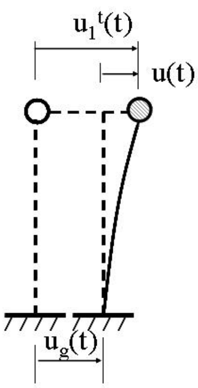

Figure 2 Uniform ground motion model

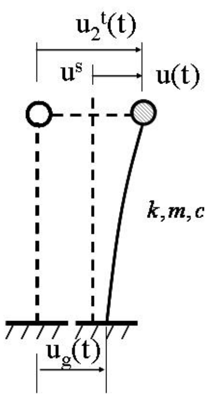

Figure 3 Non-uniform ground motion model

2. AGREEMENT OF BOTH MODELS IN A SIMPLE SDOF STRUCTURE

The apparent discrepancy between Model 1 and Model 2 is superfluous when one examines the background and the physical significance of the $\mathbf{u}$ in both models in more details. In Eqn.1.1, $\mathbf{u}$ denotes the displacement of a supported mass relative to the ground as shown in Figure 2, and the total displacement of the mass can be expressed as,

$$\mathbf{u}_1^t = \mathbf{u} + \mathbf{u}_g \tag{2.1}$$

On the other hand, the $\mathbf{u}$ in Eqn. 1.2 is the dynamic displacement besides a pseudo-static part, $\mathbf{u}_s$, and the total displacement of the mass is formed by,

$$\mathbf{u}_2^t = \mathbf{u} + \mathbf{u}^s \tag{2.2}$$

where, the pseudo-static part, $\mathbf{u}^s$, represents the vector of structural displacements due to static application of the support displacement $\mathbf{u}_g$ at each time instant and is related to the support displacement $\mathbf{u}_g$ through the relationship, $\mathbf{u}^s = \mathbf{r}\mathbf{u}_g = -\mathbf{k}\mathbf{k}_g^{-1}\mathbf{u}_g$, as shown in Figure 3. In addition, the velocity related terms appeared in the r.h.s of Eqn.1.2 is due to the fact that the pseudo-static displacement, $\mathbf{u}_s$, does not equal to the ground acceleration, $\mathbf{u}_g$, and there is a coupling effect in the damping and stiffness mechanism transferring the earthquake force to the upper structure.

A single spring-mass structure can be employed to illustrate the basic concepts and assumptions in both models. Both Figure 2 and 3 illustrate such a simple SDOF structure. In Model 2, The stiffness and damping matrices can be partitioned as a superstructure part and a support part as follows,

$$\mathbf{k} = \begin{bmatrix} \mathbf{k} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}, \mathbf{c} = \begin{bmatrix} \mathbf{c} & \mathbf{c}_g \\ \mathbf{c}_g^T & \mathbf{c}_{gg} \end{bmatrix} = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \quad (2.3)$$

The influence matrix in Model 2 is then simplified as, $\mathbf{r} = -\mathbf{k}\mathbf{k}_g^{-1} = -k(-k)^{-1} = \mathbf{I}$. Consequently, the effective force in the r.h.s of Eqn.1.2 can be reduced to, $\mathbf{p}_{eff\,2}(t) = -\mathbf{m}\mathbf{I}\ddot{\mathbf{u}}_g - (\mathbf{c}\mathbf{I} + (-\mathbf{c}))\dot{\mathbf{u}}_g = -\mathbf{m}\ddot{\mathbf{u}}_g$. Moreover, the absolute or total displacement of the mass can be written as, $\mathbf{u}^s = \mathbf{r}\mathbf{u}_g = \mathbf{u}_g, \mathbf{u}_2^t = \mathbf{u}^s + \mathbf{u} = \mathbf{u}_g + \mathbf{u}$.

As the same time, the effective force in the r.h.s of Eqn.1.1 is still $\mathbf{p}_{eff\,1}(t) = -\mathbf{m}\ddot{\mathbf{u}}_g$, and the total displacement of the mass is $\mathbf{u}_1^t = \mathbf{u}_g + \mathbf{u}$. Equivalent effective forces in Model 1 and Model 2 lead to the same absolute total displacements of the mass computed by both models, i.e., $\mathbf{u}_2^t = \mathbf{u}_1^t$. Therefore, Model 2 agrees with Model 1 at least for this SDOF structure subjected to uniform ground motion.

3. AGREEMENT OF BOTH MODELS IN TWO TYPICAL MDOF STRUCTURES

Section 2 clarifies the basic concepts and assumptions made in Model 1 and Model 2, and has arrived at a primary conclusion that both models agree with each other for a simple SDOF structure shown in Figure 2 and 3. Naturally, a question arises. Is such consistency still true for more complicated structures? In the following, the problem is discussed and treated for three different cases.

Figure 4 A 2-DOF structure with one support

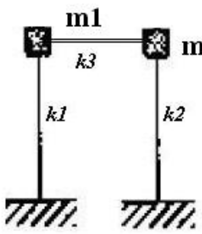

Figure 5 A 2-DOF structure with two supports

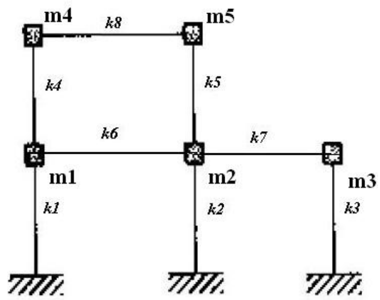

Figure 6 A 5-DOF structure with 3 supports

3.1. Governing equation of motion in absolute displacement form in both models

For the convenience of our analysis, both Eqn.1.1 and Eqn.1.2 are first expressed in the follows in absolute displacements with reference to the same global static reference framework, for instance, a remote static earth block. Although there is no absolute static reference framework in the world, one can find a relative one anyway. Substituting Eqn.2.1 into Eqn.1.1, the governing equation of motion in absolute displacement form can be written as,

$$\mathbf{m}\ddot{\mathbf{u}}_1^t + \mathbf{c}\dot{\mathbf{u}}_1^t + \mathbf{k}\mathbf{u}_1^t = \mathbf{c}\dot{\mathbf{u}}_g + \mathbf{k}\mathbf{u}_g \tag{3.1}$$

Likewise, Eqn.1.2 can be expressed in absolute displacement form with the introduction of Eqn.2.2 as follows,

$$\mathbf{m}\ddot{\mathbf{u}}_2^t + \mathbf{c}\dot{\mathbf{u}}_2^t + \mathbf{k}\mathbf{u}_2^t = -\mathbf{c}_g\dot{\mathbf{u}}_g - \mathbf{k}_g\mathbf{u}_g \tag{3.2}$$

Inconsistency between Model 1 and Model 2 is seemingly obvious since the $\mathbf{c}_g$ and $\mathbf{k}_g$ in Eqn.3.2 have different dimensions from the $\mathbf{c}$ and $\mathbf{k}$ in Eqn.3.1. Can we then draw a conclusion that Model 2 contradicts Model 1? To dwell the problem in further details, three typical structures are examined to explore the intricacies. Case 1 is a 2-DOF structure with one support shown in Figure 4, Figure 5 illustrates a 2-DOF structure with two supports, and a more general 5-DOF structure with 3 supports shown in Figure 6 are further considered. The three cases represent three typical structures. Both Model 1 and Model 2 are to be applied to the three cases and to be questioned about their consistency.

3.2. Agreement of both models in Case 1

In case 1, the effective force in Model 1, i.e. the r.h.s. of Eqn.3.1 is,

$$\mathbf{p}_{eff1} = \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{u}_g \\ \dot{u}_g \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_g \\ u_g \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{u}_g \\ \dot{u}_g \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_g \\ u_g \end{bmatrix} = \begin{bmatrix} c_1 \\ 0 \end{bmatrix}\dot{u}_g + \begin{bmatrix} k_1 \\ 0 \end{bmatrix}u_g \tag{3.3}$$

On the other hand, the partitioned damping and stiffness matrices in Model 2 is,

$$\mathbf{c} = \begin{bmatrix} c_1+c_2 & -c_2 & -c_1 \\ -c_2 & c_2 & 0 \\ -c_1 & 0 & c_1 \end{bmatrix} = \begin{bmatrix} \mathbf{c} & \mathbf{c}_g \\ \mathbf{c}_g^T & \mathbf{c}_{gg} \end{bmatrix}, \mathbf{k} = \begin{bmatrix} k_1+k_2 & -k_2 & -k_1 \\ -k_2 & k_2 & 0 \\ -k_1 & 0 & k_1 \end{bmatrix} = \begin{bmatrix} \mathbf{k} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix}$$

Consequently, the effective force in Model 2, i.e. the r.h.s. of Eqn.3.2 is,

$$\mathbf{p}_{eff2}(t) = -\begin{bmatrix} -c_1 \\ 0 \end{bmatrix}\dot{u}_g - \begin{bmatrix} -k_1 \\ 0 \end{bmatrix}u_g = \mathbf{p}_{eff1}(t) \tag{3.4}$$

Therefore, the effective force in Model 1 equals exactly to the effective force in Model 2 when the structure in both models are subjected to uniform ground motion. As a result, Model 2 agrees with Model 1 in Case 1.

Furthermore, the physical significance of both Eqn.3.3 and Eqn.3.4 are interesting. Both equations show that only the first mass is experiencing the earthquake velocity and displacement. The mass not directly connected to the ground bears no direct earthquake forces, but reacted to the ground motion though the coupling of stiffness and damping devices between the masses, which abbeys common senses.

3.3. Agreement of both models in Case 2

In case 2, the effective force in Model 1, i.e. the r.h.s. of Eqn.3.1 can be simplified as follows,

$$\mathbf{p}_{eff1}(t)=\begin{bmatrix}c_1+c_3 & -c_3\\ -c_3 & c_2+c_3\end{bmatrix}\begin{bmatrix}\dot{u}_g\\ \dot{u}_g\end{bmatrix}+\begin{bmatrix}k_1+k_3 & -k_3\\ -k_3 & k_2+k_3\end{bmatrix}\begin{bmatrix}u_g\\ u_g\end{bmatrix}$$
$$=\begin{bmatrix}c_1 & 0\\ 0 & c_2\end{bmatrix}\begin{bmatrix}\dot{u}_g\\ \dot{u}_g\end{bmatrix}+\begin{bmatrix}k_1 & 0\\ 0 & k_2\end{bmatrix}\begin{bmatrix}u_g\\ u_g\end{bmatrix}=\begin{bmatrix}c_1\\ c_2\end{bmatrix}\dot{u}_g+\begin{bmatrix}k_1\\ k_2\end{bmatrix}u_g \quad (3.5)$$

On the other hand, the partitioned damping and stiffness matrices in Model 2 is,

$$\mathbf{k}=\begin{bmatrix}k_1+k_3 & -k_3 & -k_1 & 0\\ & k_2+k_3 & 0 & -k_2\\ & & k_1 & 0\\ Sym. & & & k_2\end{bmatrix}=\begin{bmatrix}\mathbf{k} & \mathbf{k}_g\\ \mathbf{k}_g^T & \mathbf{k}_{gg}\end{bmatrix},\mathbf{c}=\begin{bmatrix}c_1+c_3 & -c_3 & -c_1 & 0\\ & c_2+c_3 & 0 & -c_2\\ & & c_1 & 0\\ Sym. & & & c_2\end{bmatrix}=\begin{bmatrix}\mathbf{c} & \mathbf{c}_g\\ \mathbf{c}_g^T & \mathbf{c}_{gg}\end{bmatrix}$$

Consequently, the effective force in Model 2, i.e. the r.h.s. of Eqn.3.2 is,

$$\mathbf{p}_{eff2}(t)=-\begin{bmatrix}-c_1 & 0\\ 0 & -c_2\end{bmatrix}\begin{bmatrix}\dot{u}_g\\ \dot{u}_g\end{bmatrix}-\begin{bmatrix}-k_1 & 0\\ 0 & -k_2\end{bmatrix}\begin{bmatrix}u_g\\ u_g\end{bmatrix}=\begin{bmatrix}c_1\\ c_2\end{bmatrix}\dot{u}_g+\begin{bmatrix}k_1\\ k_2\end{bmatrix}u_g=\mathbf{p}_{eff1}(t) \quad (3.6)$$

Therefore, Model 2 agrees with Model 1 in Case 2. In this case, both masses are directly connected to the ground and bear the same earthquake velocity and displacement simultaneously.

3.4. Agreement of both models in Case 3

For the saving of space, only the stiffness terms in both models are treated explicitly in this subsection and the following Section 4. Similar relationship of damping terms exists like that of stiffness terms, and the derivations to the stiffness terms can be applied to damping terms with minor modifications. In this sense, the damping components and stiffness components in the governing equation of motion in both Eqn.3.1 and 3.2 can be regarded as equivalent.

For the structure shown in Figure 6, the effective force in Model 1, i.e. the r.h.s. of Eqn.3.1 is,

$$\mathbf{p}_{eff1}(t)=\begin{bmatrix}k_1+k_4+k_6 & -k_6 & 0 & -k_4 & 0\\ -k_6 & k_2+k_5+k_6+k_7 & -k_7 & 0 & -k_5\\ 0 & -k_7 & k_3+k_7 & 0 & 0\\ -k_4 & 0 & 0 & k_4+k_8 & -k_8\\ 0 & -k_5 & 0 & -k_8 & k_5+k_8\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}u_g+\mathbf{c}u_g=\begin{bmatrix}k_1\\k_2\\k_3\\0\\0\end{bmatrix}u_g+\begin{bmatrix}c_1\\c_2\\c_3\\0\\0\end{bmatrix}\dot{u}_g \quad (3.7)$$

On the other hand, the partitioned stiffness matrices in Model 2 is,

$$\mathbf{k}_{5x5}=\begin{bmatrix}k_1+k_4+k_6 & -k_6 & 0 & -k_4 & 0\\ -k_6 & k_2+k_5+k_6+k_7 & -k_7 & 0 & -k_5\\ 0 & -k_7 & k_3+k_7 & 0 & 0\\ -k_4 & 0 & 0 & k_4+k_8 & -k_8\\ 0 & -k_5 & 0 & -k_8 & k_5+k_8\end{bmatrix}$$

$$\mathbf{k} = \begin{bmatrix} & -k_1 & 0 & 0 \\ \mathbf{k}_{5\times5} & 0 & -k_2 & 0 \\ & 0 & 0 & -k_3 \\ & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & k_1 & 0 & 0 \\ & & k_2 & 0 \\ Sym. & & & k_3 \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{5x5} & \mathbf{k}_g \\ \mathbf{k}_g^T & \mathbf{k}_{gg} \end{bmatrix}$$

The effective force term in the r.h.s of Eqn(3.2) can then be expressed as,

$$\mathbf{p}_{eff\,2}(t) = -\begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u_g - \mathbf{c}u_g = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ 0 \\ 0 \end{bmatrix} u_g + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ 0 \\ 0 \end{bmatrix} \dot{u}_g = \mathbf{p}_{eff\,1}(t) \tag{3.8}$$

Therefore, Model 2 agrees also with Model 1 in this case. In this case, three masses are directly connected to the ground. It is interesting to note that only the ground-directly-connected masses bear the earthquake motions in Eqn.3.7 and Eqn.3.8. Similar to case 1, the superstructure masses which are not directly connected to the ground bears no direct earthquake forces, but reacted to the ground motion though the coupling of stiffness and damping devices between the masses. The following observations are also of interest. When an earthquake comes, the land under the structure moves a bit distance at first and the elastic and damping mechanism directly connected to the foundation can sense immediately the change and reflect in the restoration forces in the r.h.s. of both Eqn.3.7 and Eqn.3.8. At this moment the ground-directly-connected masses are still at rest, therefore, there is no acceleration related terms in the r.h.s. of both equations. Only after the ground-directly-connected masses have moved, the other masses located on the superstructure react as what ground-directly-connected masses have experienced like a chain reaction.

4. AGREEMENT OF BOTH MODELS IN GENERAL MDOF STRUCTURES

With the primary conclusion that Model 1 agrees to Model 2 for the three typical cases drawn in Section 3, the same rationale can be extended to a general MDOF structure with multiple supports. Without loss of generality, it is assumed here that there are m masses connected to the foundation through s supports according to Eqn.1.3. When the stiffness matrix is partitioned as that of Eqn.1.3, the effective force in Model 1, i.e. the r.h.s. of Eqn.3.1 turns out to be,

$$\mathbf{p}_{eff\,1}(t) = \mathbf{k}\mathbf{u}_g = \mathbf{k}_{m\times m} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{m\times1} u_g = \begin{bmatrix} \sum_{j=1}^{m} k_{1j} \\ \sum_{j=1}^{m} k_{2j} \\ \vdots \\ \sum_{j=1}^{m} k_{mj} \end{bmatrix} u_g = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_s \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{m\times1} u_g \tag{4.1}$$

$$where, \sum_{j=1}^{m} k_{lj} = \begin{cases} k_l, \text{when } l \text{ is a directly gound connected stiffness} \\ 0, \text{when } l \text{ is not directly gound connected stiffness} \end{cases}$$

Note that the damping term $\mathbf{c}_g \dot{\mathbf{u}}_g$ has a similar relationship as that of the elastic force term in the r.h.s of Eqn.(3.1) and is dropped here for the convenience of clearance. The sum of the stiffness connected to a mass along a row equals to 0 if the mass is not directly fixed to the foundation, which is a basic principle in finite element analysis. This relationship roots back also to the Betti's principle, or even Newton's Third Law. For those masses directly connected to the foundation, the sum of the stiffness along a row equals to the connection stiffness. It is because that the connection point mass at the foundation is assumed to zero and has been removed from system matrices, which should be a complete one if all the zero masses are included and has zero natural frequencies.

On the other hand, the effective force term in the r.h.s of Eqn(3.2) can be rewritten as,

$$\mathbf{p}_{eff\,2}(t) = -\mathbf{k}_g \mathbf{u}_g = -\begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}_{m \times s} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{s \times 1} u_g = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_s \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{m \times 1} u_g = \mathbf{p}_{eff\,1}(t) \tag{4.2}$$

When the damping terms are included, similar relationships can be derived likewise. Therefore, the effective force in Model 1 equals exactly to the effective force in Model 2 for all structures, and Model 2 are completely consistent with Model 1. Furthermore, the earthquake force transferring mechanism from the foundation to the upper structure is clearly explained when the governing equation of motion is written in absolute displacement form. Namely, only the masses directly connected to the foundation bears the earthquake force and the other masses react to the earthquake force through the coupling stiffness and damping devices in between.

5. CONLUSIIONS

The paper resolved an apparent discrepancy between the uniform earthquake input model and another model for multiple support excitations by analyzing both models in absolute reference framework. Furthermore, the inherent mapping relationship between the stiffness terms related to the superstructure and the stiffness terms related to the foundation for the earthquake input model with multiple support excitations is revealed and used to prove that the effective earthquake forces in both models are equivalent, which results in the same structural responses if computed by both models. Therefore, both models are consistent when the ground motions at the different supports of a structure are uniform.

REFERENCES

Clough, R.W. and Penzien J. (1995), Dynamics of Structures, Computers & Structures, Inc. C.A., U.S.A.
Chopra, A.K. (2001), Dynamics of Structures: Theory and Applications to Earthquake Engineering (2nd Edition), Prentice-Hall, U.S.A.
D.S. Li, (2008), Lectures Notes on Structural Dynamics, unpublished manuscript.