IDENTIFICATION OF INSTANTANEOUS MODAL PARAMETERS OF A TIME VARYING STRUCTURE VIA A NEURAL NETWORK

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ABSTRACT :

The present work develops a novel procedure of establishing a neural network for a time varying system and estimating the instantaneous modal parameters of the system from the established neural network. The connective weights and thresholds in a neural network are assumed as functions of time and are expanded by shape functions constructing by a moving least-squares technique with polynomial basis functions. The instantaneous modal parameters of the system are directly estimated from the connective weights. The feasibility of the proposed procedure is demonstrated by processing numerically simulated dynamic responses of a time-varying linear system. The proposed procedure is also applied to process the dynamic responses of a five-story steel frame, subjected to 10% and 60% of the strength of the Kobe earthquake, in shaking table tests. The steel frame responded nonlinearly when it subjected to 60% Kobe earthquake, while it responded linearly under 10% Kobe earthquake input. This work further assesses the possible damage in the steel frame based on the instantaneous modal parameters identified from the responses corresponding to different strengths of the Kobe earthquake.

KEYWORDS: Time varying, neural network, instantaneous modal parameters, damage assessment

1. INTRODUCTION

A structure may sustain damage either when subjected to severe loading like a strong earthquake or when its material deteriorates. Damage is traditionally assessed by visual inspection, which method is costly but inefficient. Various innovative sensor technologies have recently been developed and applied to monitor buildings and infrastructure. It is desirable to use the measured data to determine whether a structure is damaged and, further, the nature of any such damage.

The damage of a structure is often assessed from observed dynamic responses by detecting changes in the modal parameters of the structure (Hearn and Testa, 1991; Alampalli and Fu, 1993; Koh *et al*., 1995). The concept underlying such an approach is that damage to a structure reduces its natural frequencies, increases the modal damping, and changes the modal shapes. When a structure is damaged under a severe dynamic loading, the structure often shows some nonlinear behaviors, which means that the dynamic characteristics of the structure are time dependent under such dynamic loading and the modal parameters at any moment can be different. However, in the mentioned convention approaches for diagnosing the health of a structure based on modal parameters, the modal parameters are found from an equivalent linear system fitting the nonlinear dynamic responses. They cannot really describe the time dependent behaviors of modal parameters. Consequently, it is very desirable to develop an approach to accurately determine the modal parameters varying with time.

Over the last two decades, artificial neural networks (ANN) have gradually been established as a powerful tool in pattern recognition, signal processing, control, and complex mapping problems, because of their excellent learning capacity and their high tolerance to partially inaccurate data. Various types of neural networks have been applied to establish input-output relationship of a nonlinear system from its dynamic responses and input forces. For example, Huang *et al*. (2003) applied a conventional back-propagation neural network to establish input-output relationship of a time-invariant nonlinear system from its seismic response data while Hung *et al.* (2003) and Adeli and Jiang (2006) utilized wavelet neural networks. Gu *et al.* (2003) proposed different methodologies for constructing an ANN for a time-variant nonlinear system. However, accurately establishing an ANN model itself is not enough for performing damage assessment for a structure. Huang *et al.* (2003) are the pioneers to develop an proper procedure to estimate the modal parameters from an ANN. Then, health diagnosis for a structure was carried out through examining the estimated modal parameters.

This paper extends the work of Huang *et al.* (2003) to a time-variant system. The concept of establishing an ANN model from dynamic responses and input forces of a structure is similar to that proposed by Gu *et al.* (2003). Instead of using Slepian base to represent the time-variant weights in an ANN, this paper proposes a moving least-squares algorithm to construct shape functions for expanding the time-variant weights and thresholds. After establishing a proper ANN for a time-variant system from its dynamic responses and input forces, an approach is proposed to determine the time-variant modal parameters (also called as instantaneous modal parameters) of the system from the weights of the established ANN. Numerical simulations are performed to validate the proposed procedure in accurately estimating the instantaneous modal parameters. Then, the proposed procedure is applied to process the dynamic responses of a five-story steel frame, subjected to 10% and 60% of the strength of the Kobe earthquake, in shaking table tests. The steel frame responded nonlinearly when it was subjected to 60% Kobe earthquake, while it responded linearly under 10% Kobe earthquake input. This work further assesses the possible damage in the steel frame based on the identified instantaneous modal parameters.

2. THEORECTICAL FORMULATION

Consider a three-layered ANN as depicted in Fig. 1, including an input layer, one hidden layer, and an output layer. The nodes in each layer are connected to each node in the adjacent layer. The computed output of the *i*th node in the output layer is defined as follows.

$$y_i = g\left(\sum_{j=1}^{N_h}\left(w_{ij} h\left(\sum_{k=1}^{N_i} v_{jk} x_k + \theta_{vj}\right) + \theta_{wi}\right)\right), \quad i = 1, 2, \cdots N_o \tag{2.1}$$

where w_{ij} are the connective weights between nodes in the hidden layer and those in the output layer, v_{jk} are the connective weights between nodes in the input layer and those in the hidden layer, θ_{wi} (or θ_{vj}) are bias terms representing the threshold of the transfer function g (or h), and x_k is the input of the k^{th} node in the input layer. Terms N_i, N_h, and N_o are the number of nodes in input, hidden, and output layers, respectively. The transfer functions can be linear or nonlinear.

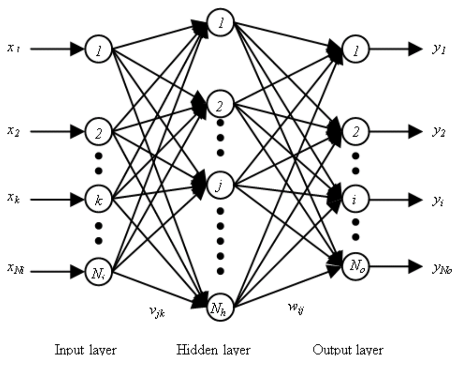

Figure 1 A typical three-layer neural network

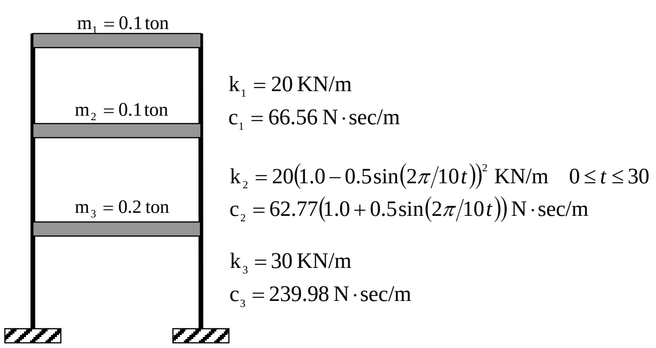

Figure 2 Numerical simulation model

To consider a time varying system, the connective weights and thresholds are assumed to be time dependent. Accordingly, expand them by shape functions as follows:

$$v_{jk}=\sum_{l=1}^{L_v}a_{jkl}\phi_l(t),\ \ w_{ij}=\sum_{l=1}^{L_w}b_{ijl}\phi_l(t),\ \ \theta_{vj}=\sum_{l=1}^{K_v}c_{jl}\phi_l(t),\ \ \theta_{wj}=\sum_{l=1}^{K_w}d_{jl}\phi_l(t), \tag{2.2}$$

where a_{jkl}, b_{ijl}, c_{jl}, and d_{jl} are constants, and $\phi_l(t)$ are shape functions. As a matter of fact, the weights and thresholds can be expanded by different sets of shape functions. However, for simplicity, the work uses the same set of shape functions for all weights and thresholds and sets $L_v=L_w=K_v=K_w=L$.

Various types of basis functions such as Legendre polynomials, Walsh functions and wavelets can be directly chosen as the needed shape functions. When polynomial basis functions are used, numerical difficulties are often enfaced when a large number of basis functions are used. To overcome this problem, a moving least-squares algorithm is applied to construct $\phi_l(t)$ from polynomial basis functions. Use v_{ij} as an example to show the procedure of constructing $\phi_l(t)$. Let

$$v_{ij}(t)=\sum_{n=0}^{\bar{N}_v}\bar{a}_{ijn}t^n=\mathbf{p}^T\mathbf{a}\ , \tag{2.3}$$

where $\mathbf{p}^T=(1,t,t^2,\ldots,t^{\bar{N}_v})$ and $\mathbf{a}^T=(\bar{a}_{ij0},\bar{a}_{ij1},\bar{a}_{ij2},\ldots,\bar{a}_{ij\bar{N}_v})$. Let $\tilde{v}_{ijk}$ denote the true value of $v_{ij}(t_k)$. Vector $\mathbf{a}$ is determined by minimizing the error function defined as

$$E(t)=\sum_{l=1}^{L_v}W(t,t_l)(\mathbf{p}^T(t_l)\mathbf{a}\ -\tilde{v}_{ijl})^2, \tag{2.4}$$

where $W(t,t_l)$ is a weighting function that is positive definite, and L_v is the number of nodal points for $v_{ij}(t)$. Notably, one can use different values of L_v or different weighting functions for different $v_{ij}(t)$. Herein, $W(t,t_l)$ defined in Eqn (2.5) is used,

$$W(t,t_l)=\begin{cases}\dfrac{\exp(-((t-t_l)/c)^2)-\exp(-(d_m/c)^2)}{1-\exp(-(d_m/c)^2)} & \mathit{for}\ |t-t_l|\le d_m\\[2ex] 0 & \mathit{elsewhere}\end{cases} \tag{2.5}$$

where d_m is the support of *W*, and *c* defines the decrease rate of *W* from t=t_l to $|t-t_l|=d_m$. Minimizing *E* and following the procedure given in Liu (2003) for developing a meshfree approach to solve solid mechanics problems yield

$$v_{ij}(t)=\tilde{\boldsymbol{\varphi}}\ (t)\tilde{\mathbf{v}}_{ij}, \tag{2.6}$$

where $\tilde{\boldsymbol{\varphi}}(t)=(\phi_1(t),\phi_2(t),\ldots\phi_{L_v}(t))^T=\mathbf{p}^T(t)\mathbf{A}^{-1}(t)\mathbf{Q}\ (t)$, $\tilde{\mathbf{v}}_{ij}=(\tilde{v}_{ij1},\tilde{v}_{ij2},\ldots,\tilde{v}_{ijL})^T$,

$$\mathbf{A}(t)=\sum_{l=1}^{L_v}W(t,t_l)\mathbf{p}(t_l)\mathbf{p}_i^T(t_l)\,,\quad \mathbf{Q}\ (t)=[\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_{L_v}]\,,\quad \mathbf{q}_l=W(t,t_l)\mathbf{p}\ (t_l)\,.$$

It should be noted that the established $\phi_l(t)$ has a finite support if the chosen weighting function $W(t,t_l)$ in Eqn. 2.4 has a finite support, so that $\phi_l(t)$ has better ability of describing local behaviors of $v_{ij}(t)$ than the traditional polynomial basis functions have.

Substituting Eqn. 2.2 into Eqn. 2.1 leads to

$$y_i=g(\sum_{j=1}^{N_h}\sum_{l=1}^{L}a_{ijl}\phi_l(t)h(\sum_{k=1}^{N_i}\sum_{l=1}^{L}b_{jkl}\phi_l x_k+\sum_{l=1}^{L}c_{jl}\phi_l(t))+\sum_{l=1}^{L}d_{il}\phi(t)))\,. \tag{2.7}$$

Then, to construct an ANN, one needs to determine the coefficients a_{jkl}, b_{ijl}, c_{jl}, and d_{jl} that can be found through the traditional least-squares approach. A system error function is defined as

$$\tilde{E}=\frac{1}{2P}\sum_{p=1}^{P}(\tilde{\mathbf{Y}}_p-\mathbf{Y}_p)(\tilde{\mathbf{Y}}_p-\mathbf{Y}_p)^T \tag{2.8}$$

where $\tilde{\mathbf{Y}}=(\tilde{y}_1\ \tilde{y}_2\cdots\tilde{y}_i\cdots\tilde{y}_{N_o})$; $\mathbf{Y}=(y_1\ y_2\cdots y_i\cdots y_{N_o})$; $\tilde{y}_i$ is the desired (or measured) value of output node *i*. Notably, $\tilde{E}$ depends on a_{jkl}, b_{ijl}, c_{jl}, and d_{jl}. One determines a_{jkl}, b_{ijl}, c_{jl}, and d_{jl} by minimizing $\tilde{E}$.

Since a time-variant ANN at any instantaneous moment is equivalent to a time-invariant ANN, one can follow the approach developed by Huang *et al*. (2003) to estimate the modal parameters at that moment from the known connective weights $w_{ij}(t)$ and $v_{ij}(t)$. Accordingly, this work uses the following function for the transfer functions *g* and *h* in Eqn. 2.1,

$$g(y)=h(y)=\begin{cases}1 & when\ y>1\\ y & when\ -1\le y\le 1\\ -1 & when\ y<-1\end{cases} \tag{2.9}$$

3. NUMERICAL VERIFICATION

Processing numerical simulation responses was carried out to demonstrate the feasibility of the proposed procedure. The Runge-Kutta method with time increment (Δt) equal to 0.001 second was applied to determine the dynamic responses of a three-story shear building (see Fig.2) subjected to base excitation. The stiffness and damping at the second floor of the shear building is assumed to be time dependent. Figure 3 shows the time histories of displacement responses at each floor and input acceleration.

In constructing an ANN from the dynamic responses and input accelerations, the nodes in input layer are $y_1(t-1)$, $y_1(t-2)$, $y_2(t-1)$, $y_2(t-2)$, $y_3(t-1)$, $y_3(t-2)$, $f(t)$, $f(t-1)$, and $f(t-2)$, where $f(t-j)$ and $y_k(t-j)$ are the input acceleration and the displacement response of the k^{th} degree of freedom relative to the base at the (*t*-*j*) time step, respectively. The nodes in output layer are $y_1(t)$, $y_2(t)$, and $y_3(t)$. Notably, the values for nodes in the input layer are normalized to the range between 1 and -1.

Figure 4 demonstrates the effects of the weighting function (Eqn. 2.5) and the number of shape functions (L) for representing the connective weights and thresholds on identifying instantaneous modal parameters. The results shown in Fig. 4 were obtained by using d_m =30 seconds for the weighting function given in Eqn 2.5. It is observed that mean values (μ) of the relative errors in identifying instantaneous natural frequencies (f_n) and modal damping ratios (ξ) generally decrease with the increase of L. When the number of shape functions is sufficiently large, the identified instantaneous modal parameters are not considerably affected by the values of c in the chosen weighting function. The same trend is found for the mean absolute error (*MAE*) between the output predicted by the trained ANN and the measured displacement responses. Hence, one can use *MAE* as an index for constructing a proper ANN from the measured data.

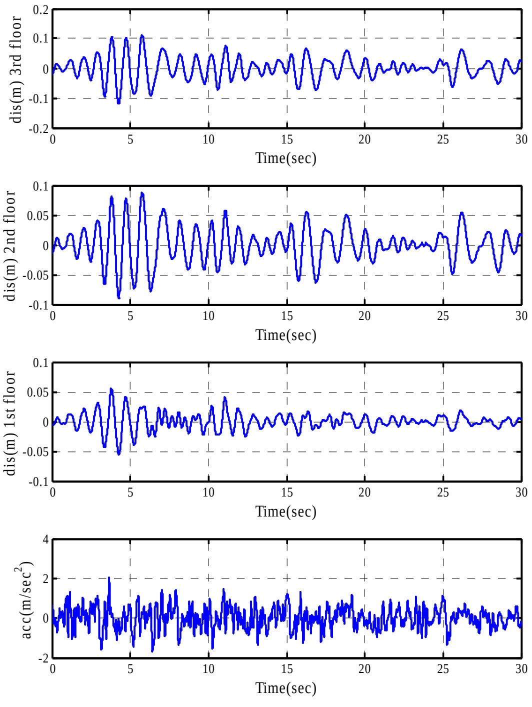

Figure 3 Time histories of responses and input

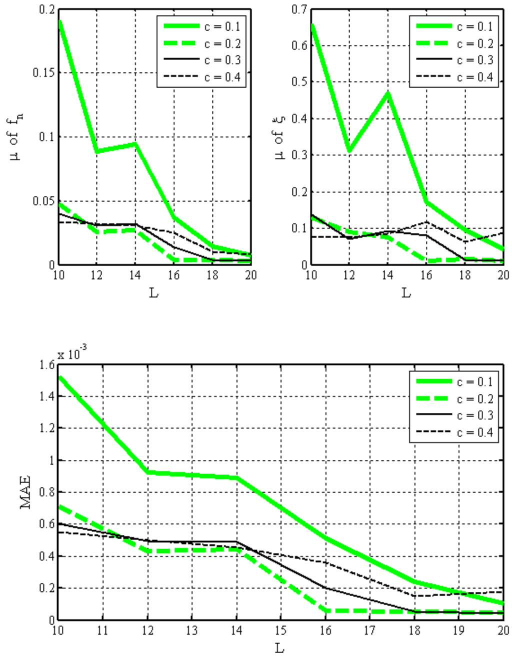

Figure 4 Identified results varying with L and c

Figure 5 depicts the compassion of the identified instantaneous modal parameters with the true values. The present results were obtained by using L=15 and c=0.2. The following index e (Trifunac, 1971) was applied to show the agreement between identified instantaneous modal shapes and true ones,

$$e = \sqrt{((\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^T (\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^* / \boldsymbol{\varphi}_{iR}^T \boldsymbol{\varphi}_{iR}^*)} ,$$

where $\boldsymbol{\varphi}_{iR}$ and $\boldsymbol{\varphi}_{iC}$ represent the i^{th} complex mode shapes for the reference state and the current state to which it is to be compared, respectively; the complex constant, a, is obtained by minimizing $(\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^T (\boldsymbol{\varphi}_{iR} - a\boldsymbol{\varphi}_{iC})^*$; $*$ denotes the complex conjugate. When the two modal shapes are highly correlated, e is close to zero. The feasibility of the proposed approach is validated by the excellent agreement between the identified instantaneous modal parameters and true ones shown in Fig. 5.

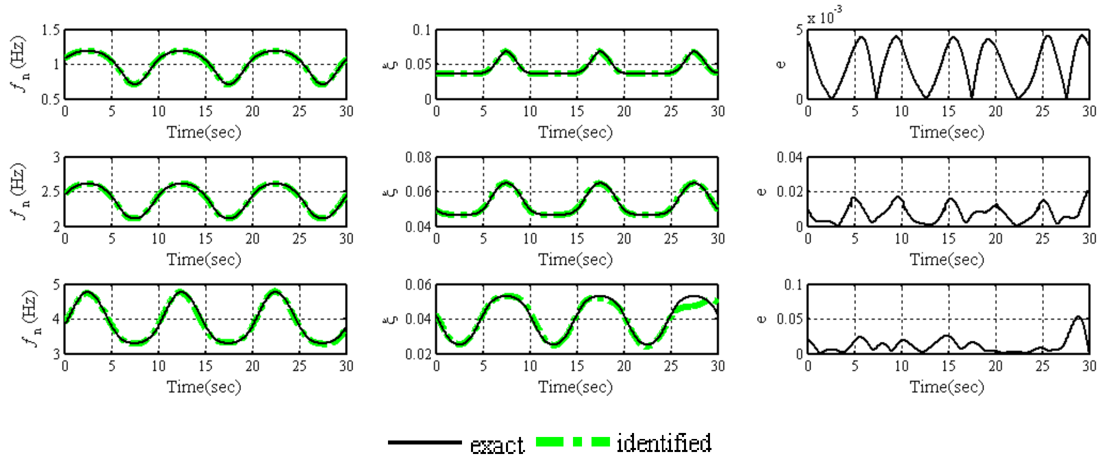

Figure 5 Comparison of identified results with true values

4. APPLICATIONS TO EXPERIMENTAL RESPONSES

Shaking table tests are often carried out in a laboratory to examine the behaviors of structures in earthquakes. The Center for Research on Earthquake Engineering (NCREE) in Taiwan undertook a series of shaking table tests on a 3 m long, 2 m wide, and 6.5 m high steel frame to generate a set of earthquake response data of the five-story steel frame (Yeh *et al*., 1991). The mass of each floor with piled lead blocks was approximately 3664 kg. The displacement, velocity, and acceleration response histories of each floor were measured with a sampling rate of 1000 Hz during the shaking table tests. Additionally, some strain gauges were installed in one of the columns and near the first floor.

Yeh *et al*. (1991) reported that the frame responded linearly when it subjected to 10% of the strength of the Kobe earthquake, and the steel columns near the first floor yielded when the frame subjected to 60% of the strength of the Kobe earthquake. Measured strains shown in Fig. 6 evidence the observations. There have been residual strains since *t* was around 2 seconds when the frame was subjected to 60% of the Kobe earthquake. Because of the symmetry of the steel frame, only the displacement responses and inputs in the long span direction were processed to find the instantaneous modal parameters. Figure 6 also depicts the displacement responses of the third and fifth floors in the long-span direction, subjected to 60% of the Kobe earthquake.

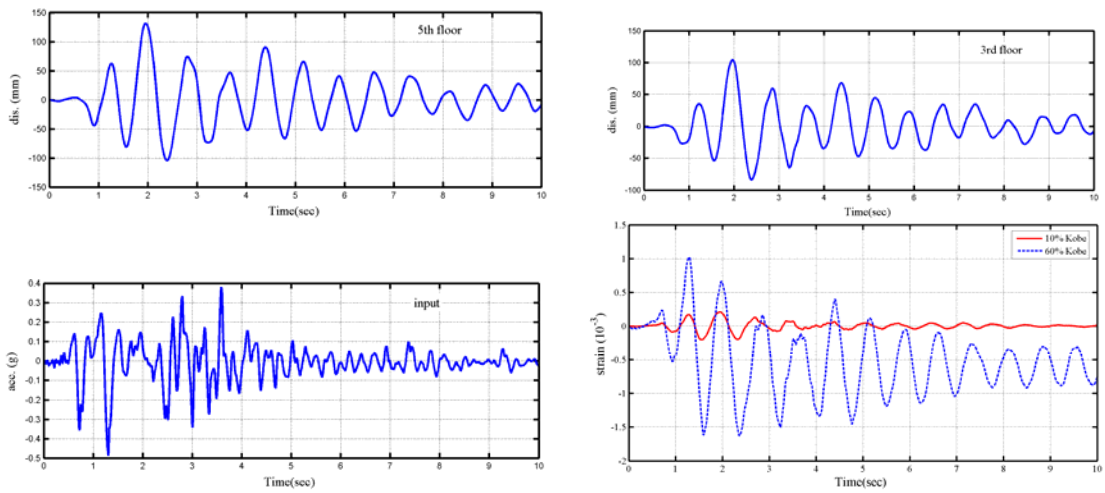

Figure 6 Input and response histories for 60% Kobe earthquake input.

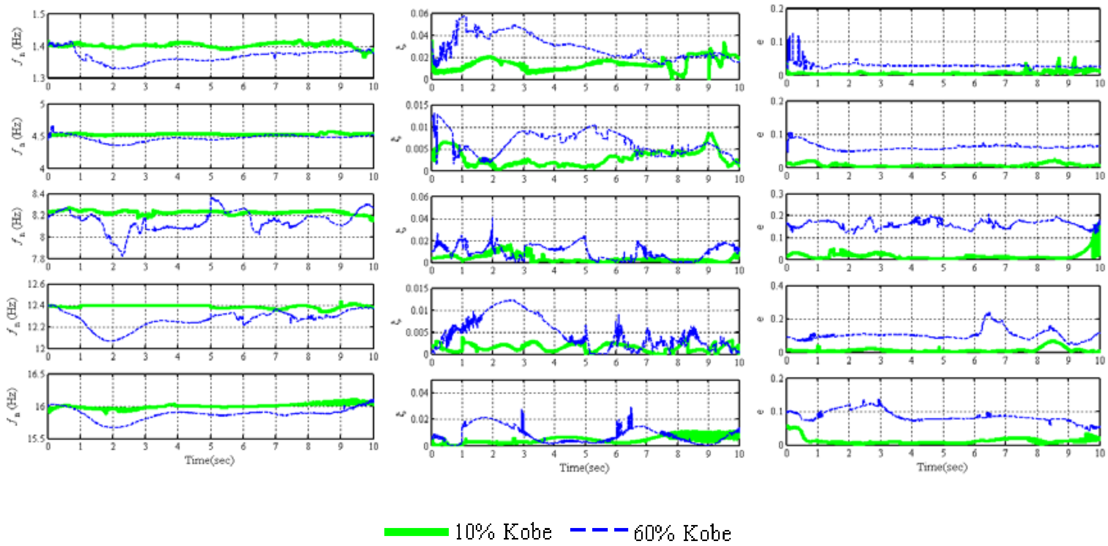

Figure 7 Instantaneous modal parameters obtained from experimental data

The measured base acceleration and displacement responses of each floor relative to the base were used to train an ANN. The instantaneous modal parameters shown in Fig. 7 were identified for the steel frame subjected to 10% and 60% of the Kobe earthquake. In computing *e*, the mode shapes for the reference state were the mode shapes determined by using a time invariant ANN to process the dynamic responses of the frame under 10% of the Kobe earthquake (Huang *et al*, 2003).

As expected, small variations of instantaneous natural frequencies with time are observed in Fig. 7 for the frame under 10% of the Kobe earthquake because no damage occurred to the frame. The identified instantaneous modal parameters for different modes, especially for natural frequencies and mode shapes, are much consistent with those obtained by Huang *et al*. (2003).

Comparison of the instantaneous modal parameters identified for 60% Kobe input with those for 10% Kobe input reveals that $f_n(\mathrm{t})$ identified for 60% Kobe input are generally smaller than those identified for 10% Kobe input, while $\xi(t)$ and *e* values show the opposite trend. These observations obey the well-known physical phenomenon that damage in a structure induces the decrease of natural frequency and increase of damping ratio for the structure. Furthermore, significant decrease in $f_n(\mathrm{t})$ is observed at *t*=2 seconds around, especially for the first mode showing more than 5% decrease in $f_n(\mathrm{t})$, which indicates possible damage initiated in the frame at that moment. This finding is consistent with what is observed from the measured strains in Fig.6.

5. CONCLUDING REMARKS

This work has presented a procedure to identify the instantaneous modal parameters of a time varying or nonlinear structure from its dynamic responses, using an ANN model. To catch the time varying feature of the structure, the connective weights and thresholds of an ANN are assumed to be functions of time and expanded by a set of shape functions established by a moving least-squares approach. The dynamic displacement responses and input forces of the structural system are used to train a proper ANN. Then, the instantaneous modal parameters of the structural system are directly estimated from the connective weights of the trained ANN. The feasibility of the proposed procedure has been demonstrated by processing numerically simulated dynamic responses of a three-story shear building whose stiffness and damping at the second floor are time dependent.

To demonstrate the applicability of the present approach to real data, the present approach has also been applied to process the displacement responses of a five-story steel frame, subjected to 10% and 60% of the Kobe earthquake, in shaking table tests. The frame was first shaken under 10% Kobe input, then subjected to a large earthquake (60% Kobe) input and yielded. The identified instantaneous modal parameters are consistent to the observed physical phenomena in the tests. The identified instantaneous modal parameters indeed indicate possible damage in the structure and when the damage initiated. To identify where the damage occurs is the next study subject in future.

ACKNOWLEDGEMENTS

The work reported herein was sponsored by the Taiwan Science Council, through research grant no. NSC94-2625-Z-009-006. This support is gratefully acknowledged. The appreciation is also extended to the Center for Research on Earthquake Engineering (NCREE) in Taiwan for providing shaking table test data.

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