PROBABILISTIC SEISMIC DEMAND ANALYSIS CONSIDERING RANDOM SYSTEM PROPERTIES BY AN IMPROVED CLOUD METHOD

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ABSTRACT

Probabilistic seismic demand analysis (PSDA) is an approach for computing the mean annual frequency of exceeding a specified seismic demand for a given structure at a designated site, which combines the conventional probabilistic seismic hazard analysis (PSHA) of the designated site with nonlinear dynamic analysis (NDA) of the given structure. Now PSDA has already become an important part of the new-generation Performance Based Earthquake Engineering (PBEE) proposed by PEER. The commonly used nonlinear demand estimation method in PSDA is the so called "cloud analysis" procedure in which a cloud of demand values are generated when a structure is subjected to a suite of earthquake ground motion records. However, this method can only take into account the record-to-record (RTR) variation in ground motions without considering the random system properties of structures. In this paper, an improved cloud method based on Latin Hypercube Sampling (LHS) method is proposed to build probabilistic seismic demand models (PSDM) to consider inherent randomness both in structures and in ground motions. Furthermore, probabilistic seismic demand fragility analysis (PSDFA) and probabilistic seismic demand hazard analysis (PSDHA) are made based on the improved probabilistic seismic demand models. The methodology is applied in a three-bay and five-storey R.C. frame as an example. The probabilistic seismic demand models with the conventional cloud method and the improved cloud method are compared, and the probabilistic seismic demand fragility curves and hazard curves are derived respectively.

KEYWORDS: Seismic Demand, Improved Cloud Method, Probabilistic Seismic Demand Model, Seismic Demand Fragility, Seismic Demand Hazard

1. INTODUCTION

Developing from the conventional probabilistic seismic hazard analysis (PSHA) of a designated site, probabilistic seismic demand analysis (PSDA) is an approach for computing the mean annual frequency (MAF) of exceeding a specified seismic demand for a given structure at a designated site, which couples a ground motion hazard curve for the designated site from PSHA with demand results from nonlinear dynamic analysis (NDA) of the given structure under a suite of earthquake ground motion records (Shome & Cornell, 1998; Shome, 1999). As one of central cores of probabilistic seismic performance assessment, seismic reliability analysis, seismic fragility analysis and seismic risk analysis, PSDA has already become an important part of the probabilistic decision-making framework of the new-generation performance-based earthquake engineering (PBEE) proposed by PEER (Cornell & Krawinkler, 2000; Moehle & Deierlein, 2004).
The commonly used nonlinear demand estimation method in PSDA is the so called "cloud analysis" procedure in which a cloud of demand values are generated when a structure is subjected to a set of earthquake ground motion records. However, this method can only take into account the record-to-record (RTR) variation in ground motions, or in other words, it generally takes the analyzed structure as a deterministic one without considering the inherent random properties of the system.

In order to overcome the shortcomings of the traditional cloud method in PSDA, an improved approach combining cloud method (Mackie & Stojadinovic, 2001) with Latin Hypercube Sample (LHS) method (Hwang, et al., 2001) is firstly proposed in this paper. The PSDA is run on three levels: probabilistic seismic demand model (PSDM) building, probabilistic seismic demand fragility analysis (PSDFA), and probabilistic seismic demand hazard analysis (PSDHA). PSDM is the core of PSDA, and also the basis for PSDFA and PSDHA, which represents the functional relationships between ground motion intensity measures (IMs) and engineering demand parameters (EDPs) of structures. PSDFA is used to provide the conditional failure probabilities of exceeding demand limit states of structures given a spectrum of IM levels, and generates a series of seismic demand fragility curves of a structure corresponding to the demand limit states. On the other hand, PSDHA is done to estimate the mean annual frequency of exceeding a given demand level, and results in a seismic demand hazard curve of the structure in a conjectured seismic hazard environment. As a case study, the developed methodology is applied in a three-bay and five-storey R.C. frame structure designed according to Chinese seismic design code of buildings (GB50011-2001). The spectral acceleration corresponding to the fundamental period of structures, i.e. $S_a(5\%,T_1)$, is selected as IM. The maximum inter-storey displacement angle (ISDA) is chosen as the global EDP of structures. The classical cloud method and the improved cloud method are both employed to run the PSDA, and the PSDMs from the two methods are compared. The PSDFA and PSDHA are carried on using the different PSDMs. It is demonstrated that the PSDMs without considering random system properties in structures will under-estimate the potential hazard probability of structures.

2. CLOUD METHOD AND ITS IMPROVEMENT

2.1. Cloud Method

The commonly used nonlinear demand estimation method in PSDA is the so called “cloud analysis” approach in which a cloud of demand values are generated when a structure is subjected to a suite of earthquake ground motion records. The main ingredients of cloud method to formulate the PSDMs of interest are as follows:

(1) Assemble a suite of N ground motions which are applicable to the area of interest. This suite should represent a broad range of values of the chosen intensity measure. Actually, the “cloud” describes the selection of earthquakes with variable IMs. The artificial or synthetic site-specific ground motions can be used just like in Monte Carlo simulation. However, the most commonly used method is the bin approach which subdivides ground motions into hypothetical bins based on magnitude (M_w), epicentral distance (R), and local soil type (Shome et al., 1998), as shown in Figure 1(a). As a result, the record-to-record random properties in ground motions can be considered thoroughly. In this paper, the ground motion records are selected according to the four bins shown in Figure 1(a), and the spectral acceleration $S_a(T_1)$ corresponding to the fundamental period of the structure is selected as an intensity measure.

(2) Select the class of structures to be investigated. Associated with this class are a set of engineering demand parameters that can be measured during analysis to assess structural performance under the considered motions. In this paper, the maximum inter-storey displacement angle (ISDA) $\theta_{\max}$ of structures is chosen as an engineering demand parameter.

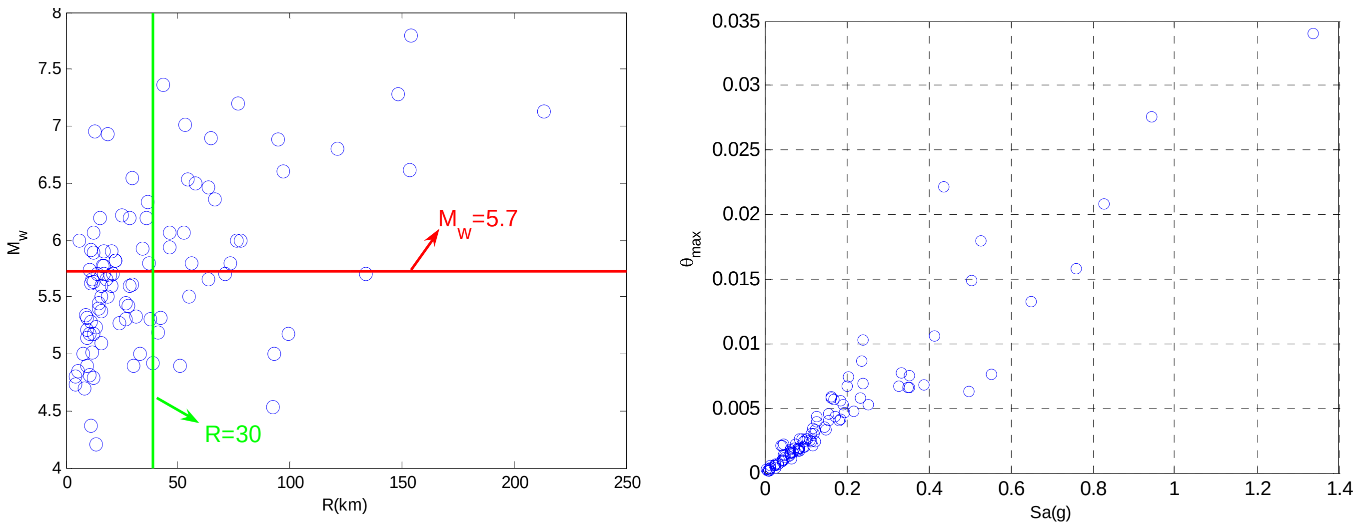
(3) Build the nonlinear finite element model to character the class of structures selected, with provisions to vary designs of the class through the use of design parameters. Thus, a portfolio of structures is generated by different realizations of the design parameters. The finite element model should capture the material and geometric nonlinear characteristics in the structure.

(4) Perform a full nonlinear time history analysis of the structure subjected to the selected ground motion record until all ground motions and structural model combinations have been exhausted. Key responses or demand parameters should be monitored throughout the analysis.

(5) Record and plot the peak values of the selected engineering demand parameter (EDP) versus the value of intensity measure (IM) for that ground motion, as shown in Figure 1(b). A regression of this cloud of data is then used to derive the PSDMs between ground motion intensity measures and structural engineering demand parameters.

2.2 Improved Cloud Method

Earthquake is a low-probability, high-consequences, and large-uncertainty hazard event. The above-mentioned cloud method can only consider the inherent randomness in ground motions in the form of record-to-record variation. It cannot take the inherent randomness in structural systems and the epistemic uncertainty in modeling into account, although it does consider a class of structures through varying the design parameters.



(a) Ground motion bins (b) PSDM

Figure 1 Sketch diagram of cloud method

In order to consider the random system properties of structures, the samples of structures should be generated by Monte Carlo simulation (MCS) to group into motion-structure pairs with the selected ground motions. However, the crude MCS usually requires a large number of seismic response simulations in order to obtain accurate statistics of the structural EDPs of interest. To reduce the number of crude MCS, many variance reduction techniques have been invented, among which, Latin hypercube sampling (LHS) is a comparatively efficient method (Hoshino & Takemura, 2000). As a stratified sampling approach, LHS can provide reliable estimates of the response statistics with a significant smaller number of samples that required by crude MCS. It has been proved by numerous researchers that for a given number of random samples of structures, the variance in the response statistics obtained from LHS is much smaller than that obtained from crude MCS.

To combine Latin hypercube sampling with the cloud method in PSDA in order to consider the random system properties of structures, in this paper, the range of each random system property is divided into the same number of segments with equal probability as the number of input ground motions at the specified seismic hazard level. From each segment, one representative sample, taken as the median value of the segment, is selected. Then, in the nonlinear time history simulations, the samples of all variables are randomly matched without replacement in sets containing one sample from each input random variable; these sets are finally matched randomly to the input ground motions. As a result, Latin hypercube sampling causes the entire range of each random variable to be represented in the set of variables used in the simulation. The basic procedure of the improved cloud method based on LHS is as follows:

(1) Assemble a suite of N ground motions according to the bin approach shown in Figure 1(a);
(2) Select the class of structures of interest and the corresponding engineering demand parameters;
(3) Build the nonlinear finite element model of the subject structure;
(4) Quantify the aleatory randomness and epistemic uncertainty in structural systems;
(5) Generate N statistical samples of the subject structure by LHS, these samples should be generated by sampling on various modeling parameters which may be deemed significant (e.g. material strength, geometric size, damping ratio, etc.);
(6) Establish a set of N ground motion-structure pairs according to the strategy in LHS;
(7) Perform a full nonlinear time history analysis for each ground motion-structure sample to simulate a cloud of structural response data;
(8) Regress this cloud of data to derive the PSDMs between IMs and EDPs.

3. PROBABILISTIC SEISMIC DEMAND ANALYSIS BY IMPROVED CLOUD METHOD

3.1. Probabilistic Seismic Demand Model (*PSDM*)

The probabilistic seismic demand models (PSDMs) are results of the improved cloud analysis in PSDA, which

provide the relationships between IMs and EDPs. The randomness in seismic demand of structures comes both from the random input earthquake ground motions and from the random system properties.
Let us denote the ground motion intensity measure by S_a, the random engineering demand parameter by D. The demand D can be written in terms of the product of it median value $\hat{D}$ and a random error ε:

$$D = \hat{D}\varepsilon \tag{3.1}$$

It is generally assumed that ε can be represented by a lognormal distribution with median and standard deviation of lnε:

$$\hat{\varepsilon} = 1$$
$$\sigma_{\ln\varepsilon} = \beta_{D|S_a} \tag{3.2}$$

where, $\beta_{D|S_a}$ is called dispersion, which is a model error in nature depending to some degree on the level of S_a, we assume that it is constant.
The best estimate of the resulting demand model $\hat{D}$, is defined as the median, or the mean of the natural log of the N IM-EDP data points:

$$\ln(\hat{D}) = \frac{1}{N}\left(\sum_{i=1}^{N}\ln(D_i)\right) \tag{3.3}$$

where, D_i is the ith EDP data.
The dispersion is dependent on the number of parameters (df) being estimated in a linear regression on the EDP data:

$$\beta_{D|S_a} = \sqrt{\frac{\sum_{i=1}^{N}\left(\ln(D_i) - \ln(\hat{D})\right)^2}{N - df}} \tag{3.4}$$

where, df =2, since we only have two parameters D and S_a.
It has been observed by numerous researchers that the median demand and the spectral acceleration follow the typical power law relationship:

$$\hat{D} = a(S_a)^b \tag{3.5}$$

where, a and b are parameters, which can be determined from linear least-squares regression technique.
In evidence, the relationship between the median demand and the spectral acceleration in the log space is linear:

$$\ln(\hat{D}) = \ln(a) + b\ln(S_a) \tag{3.6}$$

3.2. Probabilistic Seismic Demand Fragility Analysis (*PSDFA*)

PSDM can be used to provide probabilities of exceeding demand limit states, given measure of intensity. Such a relationship is termed a demand fragility curve. Therefore, the demand fragility can be defined as a conditional failure probability that drift demand D exceeds a certain drift limit d, given a specific spectral acceleration S_a:

$$F_R(x) = P[D > d \mid S_a = x] \tag{3.7}$$

The fragility is modeled commonly by a lognormal cumulative distribution function (CDF), a choice that has been supported by numerous research programs during the past decade in disparate fields. Then it is described by

$$F_R(x) = \Phi[\ln(x/m_R)/\beta_R] \tag{3.8}$$

where, m_R is the median spectral capacity, β_R is the logarithmic standard deviation, and $\Phi[]$ is the standard normal integral. The parameters m_R and β_R measure the inherent randomness in seismic capacity.
With the assumption that the seismic demand can also be described by a lognormal distribution, with median as defined in Eq. (3.5) and dispersion as defined in Eq. (3.4), we can determine the parameters in Eq. (3.8). Note that Eq. (3.7) can be further rewritten as:

$$F_R(x) = 1 - \Phi\left[\frac{\ln(d) - \ln(\hat{D})}{\beta_{D|S_a}}\right] \tag{3.9}$$

Substituting Eq. (3.5) in Eq. (3.9), one has

$$F_R(x) = 1 - \Phi\left[\frac{\ln(d) - \ln\left[a(x)^b\right]}{\beta_{D|S_a}}\right] = \Phi\left[\frac{\ln(x) - \dfrac{\ln(d) - \ln(a)}{b}}{\dfrac{\beta_{D|S_a}}{b}}\right] \tag{3.10}$$

Comparing Eq. (3.8) with Eq. (3.10), we can obtain the parameters m_R and β_R as follows:

$$m_R = \exp\left(\frac{\ln d - \ln a}{b}\right) \tag{3.11}$$

$$\beta_R = \frac{\beta_{D|IM}}{b} \tag{3.12}$$

The demand limit states are multiple levels corresponding to the different seismic damage states, or seismic performance levels of structures. For example, we generally classify the seismic damage states of structures into five levels: intact, minor damage, moderate damage, severe damage, and collapse. Therefore, the corresponding four levels of demand limit states are: equal to or larger than minor damage demand, equal to or larger than moderate damage demand, equal to or larger than severe damage demand, and equal to or larger than collapse demand. In order to characterize the four performance levels, we denote the four deterministic limit values of drift demand as $d^i_{LS}\ (i = 1,2,3,4)$. Then, we can derive the four levels of seismic demand fragilities:

$$F_{Ri}(x) = P[D > d^i_{LS} \mid S_a = x] = \Phi[\ln(x / m_{Ri}) / \beta_R] \tag{3.13}$$

$$m_{Ri} = \exp\left(\frac{\ln d^i_{LS} - \ln a}{b}\right) \tag{3.14}$$

3.3. Probabilistic Seismic Demand Hazard Analysis (*PSDHA*)

PSDM can be also used to estimate the mean annual frequency of exceeding a given demand, and results in a structural demand hazard curve in a conjectured hazard environment. Actually, demand hazard curve is the final result of PSDHA, just like the seismic hazard curve of the site in the conventional PSHA.

The demand hazard is defined as the probability of failure of exceeding a given demand under a spectrum of possible earthquakes, which is determined by convolving the demand fragility curve of the structure and the seismic hazard curve of the designated site:

$$H_D(d) \triangleq P[D > d] = \int P[D > d \mid S_a = x] \mid dH_{S_a}(x) \mid = \int F_R(x) \mid dH_{S_a}(x) \mid \tag{3.15}$$

where, $H_{S_a}(x)$ is the seismic hazard function derived from PSHA of the site.

For specific building sites, it has been shown by many researchers that at moderate to large values of ground acceleration, there is a logarithmic linear relation between first modal spectral acceleration S_a and the exceedance probability $H_{Sa}(x)$ (Cornell et al., 2002). This relationship implies that S_a can be described by a Type II distribution of largest values

$$H_{S_a}(x) = p[S_a > x] = 1 - \exp[-(x / k_0)]^{-k} \approx k_0 x^{-k} \tag{3.16}$$

where k_0 = characteristic extreme, k = shape parameter.

The hazard curve can be approximately linear on a log-log plot in the region of interest (Cornell et al., 2002):

$$H_{S_a}(x) \approx k_0 x^{-k} \tag{3.17}$$

After integration of lognormal demand fragility function defined in Eq. (3.10) and derivative of seismic hazard function presented in Eq. (3.17), the demand hazard in Eq. (3.15) can be obtained as an explicit formulation (Cornell et al., 2002):

$$H_D(d) = H_{S_a}(S^d_a) \exp\left[\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a}\right] \tag{3.18}$$

where, S^d_a represents the spectral acceleration corresponding to a specific drift limit d. Their relationship is defined in Eq. (3.5), which can be transformed to the following expression

$$S^d_a = (d / a)^{1/b} \tag{3.19}$$

Then, $H(S_a^d)$ is the seismic hazard with spectral acceleration of S_a^d, representing the probability of ground motion intensity that cause drift d or more.
Eq. (3.18) is the most important result of PSDHA, from careful inspection of it, we can see that the seismic hazard curve for the drift demand $H_D(d)$ is equal to the seismic hazard function of the site evaluated at the spectral acceleration corresponding to this drift demand, times a correction factor related to dispersion in the drift demand estimation for a given spectral acceleration.

4. CASE STUDY: R.C. FRAME STRUCTURES

4.1. Basic Data

A three-bay and five-storey R.C. frame structure, as shown in Figure 2, is taken as the example in this case study. It is designed according to Chinese seismic design code of buildings (GB50011-2001).

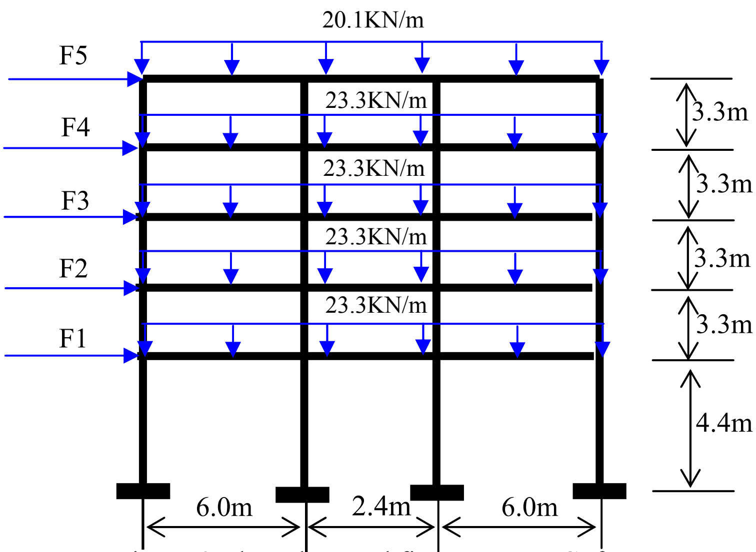


Figure 2 Three-bay and five-storey R.C. frame

4.2. PSDM Based on Cloud Method

The $S_a(5\%,T_1)$ is chosen as the IM, and the maximum ISDA $\theta_{\max}$ is chosen as the EDP. 100 ground motion records are selected from the database of PEER. We take Ms = 5.7 and D = 30(km) as the limit values to carve these ground motion records into four bins, as shown in Figure 1(a).
The nonlinear finite element model of the structure is developed by OpenSees platform. Without considering the random system properties, the nonlinear time history analysis is done for the model subjected to the 100 sets of ground motion records. The PSDM based on cloud method is obtained via log-linear regression method as

$$\ln(\hat{\theta}_{\max}) = 1.0071\ln(S_a) - 3.6852\,,\quad \beta_{D|S_a} = 0.27 \tag{4.1}$$

The PSDM in this case is shown in Figure 3(a).

4.3. PSDM Based on Improved Cloud Method

In this case, we consider four random material properties of the structure: the yield strength f_y and initial elastic modulus E of steel; the compressive strength f_c and crushing strength f_{cr} of concrete. The statistics of the four random variables are shown in Table 4.1. All variables are assumed as normal distribution, and independent on each other.

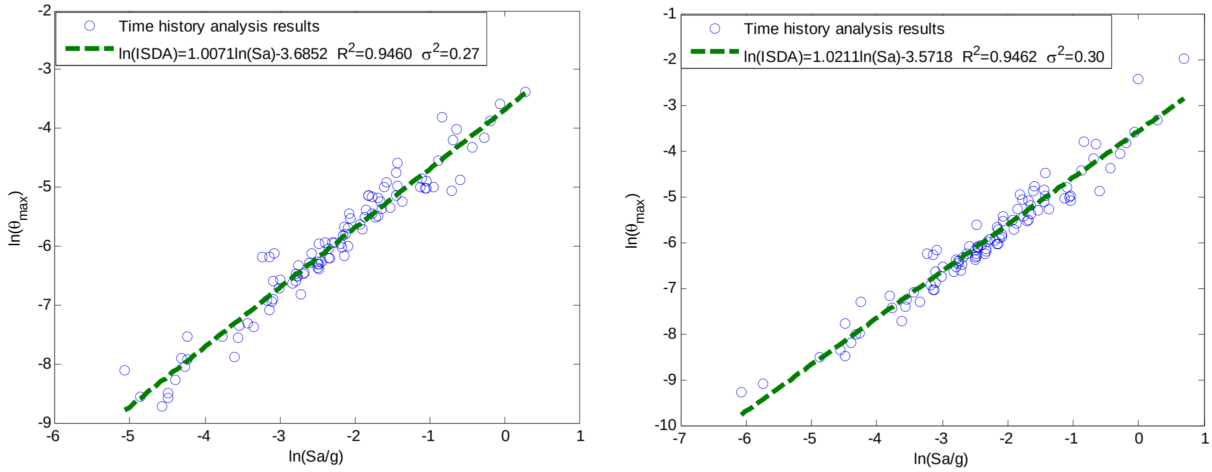
Table 4.1 Statistics of random variables

Variables	Distribution parameters		Variables	Distribution parameters	
	Mean value	Std		Mean value	Std
f_y(N/mm^2)	384.80	28.59	f_{cr}(N/mm^2)	27.32	4.44
E(N/mm^2)	204000	2040	f_c(N/mm^2)	26.10	4.44

According to the strategy in the improved cloud method, the 100×4 matrix of random samples is firstly generated by using LHS method, and 100 samples of the finite element model are obtained. Then 100 couples of ground motion records and structural samples are combined. The nonlinear time history analysis is exhausted for all ground motion-structure pairs. From the recorded spectral acceleration-ISDA results, the PSDM considering the random system properties is derived by using the improved cloud method:

$$\ln(\hat{\theta}_{\max}) = 1.0211\ln(S_a) - 3.5718\,,\quad \beta_{D|S_a} = 0.30 \tag{4.2}$$

The PSDM in this case is shown in Figure 3(b).



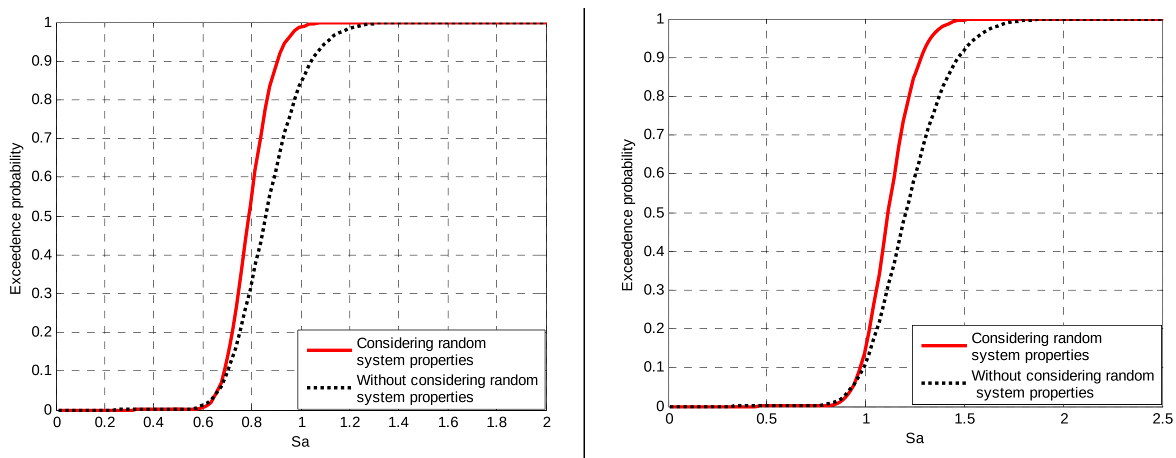
(a) Cloud method (b) Improved cloud method

Figure 3 PSDM in two cases

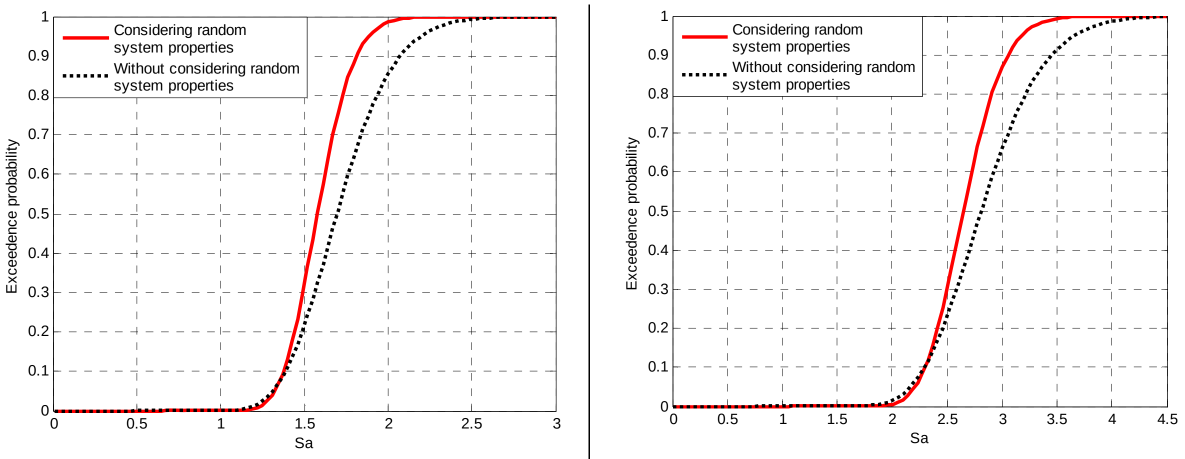
4.4. *Drift Demand Fragility Curves*

In Chinese seismic design code of buildings (GB50011-2001), the limit value of the maximum ISDA for R.C. frame structure in elastic serviceability limit state is 1/550, while in elastoplastic collapse limit state 1/50. In this paper, the four levels of drift demand limit state are chosen as: $d_{\mathrm{LS}}^1 = 1/550$, $d_{\mathrm{LS}}^2 = 2/550$, $d_{\mathrm{LS}}^3 = 4/550$, and $d_{\mathrm{LS}}^4 = 1/50$.

The drift demand fragility curves based on the above PSDMs are shown in Figure 4. From Figure 4, we can see that the drift demand fragility curves based on the PSDMs without considering random system properties tend to be less than that based on the PSDMs considering random system properties. It means that drift demand fragility curves without considering random system properties under-estimates the conditional failure probability given the occurrence of the earthquake ground motion.



(a) $d_{\mathrm{LS}}^1 = 1/550$ (b) $d_{\mathrm{LS}}^2 = 2/550$



(c) $d_{LS}^{3} = 4/550$ (d) $d_{LS}^{4} = 1/50$

Figure 4 Drift demand fragility curves based on different PSDMs

4.5. Drift Demand Hazard Curves

As shown in Figure 5, the seismic hazard curve of PSHA of the site in this example is approximated as

$$H_{Sa}(x) = 0.038 \cdot (x)^{-2.381} \tag{4.3}$$

In Figure 5, S_a(63,2%, 50yr), S_a(10%, 50yr) and S_a(2%, 50yr) are the values of S_a(5%, T_1) corresponding to different exceedance probabilities in 50 years.

The drift hazard curves coupling the seismic hazard curve and the drift fragility curves based on different PSDMs are derived, as shown in Figure 6.

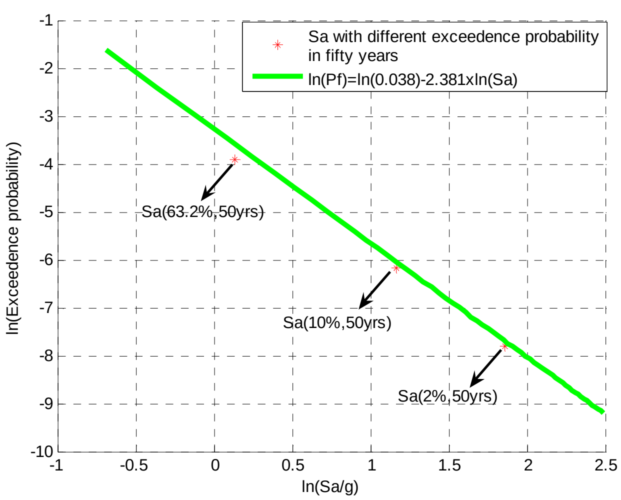


Figure 5 Seismic hazard curve for the designated site

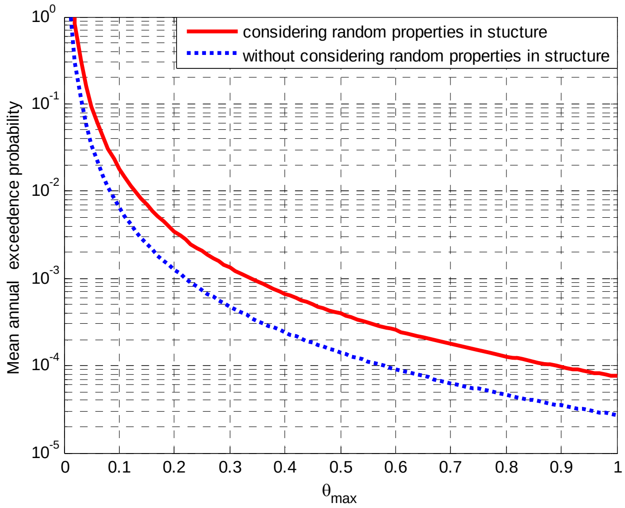


Figure 6 Drift demand hazard curves based on different PSDMs

From Figure 6 we can see that, the drift demand hazard curve based on the PSDM without considering random system properties is lower than that based on the PSDM considering random system properties. That is to say, PSDFA without considering random system properties can under-estimate the potential hazard probability of structures. It is similar to the conclusion obtained from Figure 4. Therefore, it is necessary to consider random system properties of structures when carrying on PSDA.

5. CONCLUSIONS

This paper proposes an improved cloud method in PSDA to consider random system properties of structures, which is a coupling of the conventional cloud method and Latin hypercube sampling (LHS) method. The probabilistic seismic demand analysis of structures is carried on three levels: PSDM building, PSDFA and PSDHA. A three-bay and five-storey R.C. frame structure is taken as a case-study example. The PSDMs based on cloud method and improved cloud method are derived and compared. The demand fragility and hazard curves are obtained by PSDFA and PSDHA, respectively. The conclusions are summarized as follows.

(1) The improved cloud method proposed in this paper can efficiently consider random system properties in PSDA.

(2) Both PSDFA and PSDHA without considering random system properties tend to under-estimate the structural potential seismic hazard. Therefore, it's necessary to make PSDA considering random system properties.

6. ACKNOWLEDGEMENTS

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