GLOBAL PROBABILISTIC SEISMIC CAPACITY ANALYSIS OF STRUCTURES THROUGH STOCHASTIC PUSHOVER ANALYSIS

Xiao-Hui Yu[1], Da-Gang Lu[2], Peng-Yan Song[3], Guang-Yuan Wang[4]

[1] *Graduate Student, School of Civil Engineering, Harbin Institute of Technology, Harbin. China*
[2] *Professor, School of Civil Engineering, Harbin Institute of Technology, Harbin. China*
[3] *Graduate Student, School of Civil Engineering, Harbin Institute of Technology, Harbin. China*
[4] *Professor, School of Civil Engineering, Harbin Institute of Technology, Harbin. China*
Email: yuxiaohui2830582@163.com, ludagang@hit.edu.cn, songpengyan@sina.com, wanggy@hit.edu.cn

ABSTRACT

Probabilistic seismic capacity analysis (PSCA) plays an important role in the field of seismic performance evaluation, seismic reliability analysis, seismic fragility analysis, and seismic risk analysis of civil engineering structures. In general, PSCA has two levels of research context: global PSCA and local PSCA. The local PSCA has been studied by numerous research programs; however, the global PSCA has just recently been paid attention to in the field of earthquake engineering and structural engineering. In this paper, in order to build global probabilistic seismic capacity model (GPSCM) of structures considering random system properties, a stochastic pushover analysis (SPA) method is developed, which combines four kinds of approximating methods for estimating statistical moments of complex random function with deterministic pushover analysis. The four random analysis methods include mean value first order second moment (MVFOSM) approach, Monte Carlo simulation (MCS) approach, improved point estimation approach, and Zhou-Nowak numerical integration approach. The methodology proposed is applied in R.C. frame structure. A three-bay and five-storey plane R.C. frame is taken as an example in case study, the global seismic capacity curves of the structure corresponding to four levels of limit states are derived. It is demonstrated by this example that the approach proposed in this paper is an efficient and accurate tool for global probabilistic seismic capacity analysis of structures.

KEYWORDS: Stochastic Pushover Analysis, Global Seismic Capacity, MVFOSM Method, MCS Method, Point Estimation Method, Zhou-Nowak Numerical Integration Method

1. INTODUCTION

Probabilistic seismic capacity analysis (PSCA) plays an important role in the field of seismic performance evaluation, seismic reliability analysis, seismic fragility analysis, and seismic risk analysis of civil engineering structures. Recently, it has been introduced into the new-generation Performance Based Earthquake Engineering (PBEE) proposed by PEER as one of its four building blocks (Moehle & Deierlein, 2004).

PSCA has two levels of research context: global PSCA and local PSCA. The local PSCA has been studied by numerous research programs; however, the global PSCA has just recently been paid attention to in the field of earthquake engineering and structural engineering. Actually, global PSCA is more difficult than local PSCA. First, the global seismic capacity of structures has many influencing factors, such as: structural configuration, structural dynamic properties (stiffness and damping), geometric sizes of structural elements, constitutive relationships of structural materials, modeling uncertainty of structures, and so on. Second, all the influencing factors mentioned above have multi-scale characteristics. In fact, they can be classified into five scales, i.e., material scale, section scale, element scale, substructure scale and system scale. The propagation of the uncertainties across the different levels of scales makes global PSCA very difficult to implement. Third, these influencing factors may be dependent on each other. Forth, the global performance of structures is generally discretized into multiple levels. For example, in the common practice in earthquake engineering, the damage states are usually grouped into five discrete levels: intact, minor damage, moderate damage, severe damage, and collapse. It is difficult to clarify the limit values of different performance levels. Fifth, the global seismic capacity of structures behaves nonlinearity, especially during the stages of severe damage and collapse. Last, it usually needs numerical analysis method in global PSCA. However, the global seismic capacity of structures is a highly implicit function of the basic random variables. Therefore, it is difficult to derive the statistics of the random global seismic capacity of structures.

To overcome the difficulties mentioned above, this paper develops a stochastic pushover analysis method combining four kinds of statistical moment estimating methods, i.e., MVFOSM method, MCS method, point estimation method and Zhou-Nowak numerical integration method, with deterministic pushover analysis approach driven by a new force controlling method. The proposed methodology is then applied in R.C. frame structures. A three-bay and five-storey plane R.C. frame is taken as an example in case study, the probabilistic models and their corresponding fragility curves of the global seismic capacities of the structure corresponding to four levels of limit states are derived.

2. APPROXIMATE METHODS FOR ESTIMATING STATISTICAL MOMENTS OF COMPLEX RANDOM FUNCTIONS

2.1. Overview

Let us denote by C_θ the global seismic capacity of structures, e.g., the maximum inter-storey displacement angle (ISDA). It is a nonlinear function of the basic random variables **X**:

$$C_\theta = g(\mathbf{X}) = g(X_1, X_2, \cdots, X_n) \tag{2.1}$$

Generally, $g(\mathbf{X})$ is a complex and implicit function of **X**. The kth central moment of C_θ is obtained by

$$\mu_{kC_\theta} = E\left[g^k(\mathbf{X})\right] = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} g^k(x_1, \cdots, x_n) f_{\mathbf{X}}(x_1, \cdots, x_n) dx_1 \cdots dx_n \tag{2.2}$$

where， $E[\,]$ is the expectation operation, $f_{\mathbf{X}}(x_1, \cdots, x_n)$ is the joint probability density function (JPDF) of **X**. The mean value μ_{C_θ} and variance $\sigma_{C_\theta}^2$ of C_θ are

$$\mu_{C_\theta} = E\left[g(\mathbf{X})\right] \tag{2.3}$$

$$\sigma_{C_\theta}^2 = E\left[\left(g(\mathbf{X}) - E\left[g(\mathbf{X})\right]\right)^2\right] = E\left[g^2(\mathbf{X})\right] - E^2\left[g(\mathbf{X})\right] \tag{2.4}$$

Unfortunately, due to the implicit and complex characteristics of $g(\mathbf{X})$, it is difficult to directly solve the quadrature in Eqs. (2.2) to (2.3). Alternatively, some approximate methods for estimating statistical moments of complex functions are introduced. The approximation strategies can be divided into three categories:

(1) Approximate analytical methods. The Taylor series expansion of the function is the most frequently used. The well-known mean value first order second moment (MVFOSM) method belongs to this category, in which the function is expanded around the mean value point of the basic random variables with first order series function, and it only needs the first second moments information of the basic random variables to propagate the uncertainty.

(2) Numerical simulation methods. Monte Carlo simulation (MCS) method and its various variance-reduction techniques perhaps are the most rigorously employed.

(3) Numerical integration methods. The point estimation method originally proposed by Rosenblueth (1975) and the numerical integration approach developed by Zhou and Nowak (1988) enter into this category.

2.2. MVFOSM Method

In MVFOSM method, the random function is linearly expanded at the mean-value point of the basic variables:

$$g(\mathbf{X}) \approx g(\mathbf{M}) + \nabla_{\mathbf{M}} g^T \cdot (\mathbf{X} - \mathbf{M}) \tag{2.5}$$

where, $\mathbf{M} = [\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_n}]^T$ is the mean value vector of $\mathbf{X}$, $\mathbf{\Sigma} = \left[\rho_{ij}\sigma_i\sigma_j\right]_{n\times n}$ is the covariance matrix of $\mathbf{X}$, T denotes the transpose of a matrix.

Then, we can get the first two moments approximation of $g(\mathbf{X})$:

$$\mu_{C_\theta} \approx g(\mathbf{M}) \tag{2.6a}$$

$$\sigma_{C_\theta}^2 \approx \nabla_{\mathbf{M}} g^T \cdot \mathbf{\Sigma} \cdot \nabla_{\mathbf{M}} g \tag{2.6b}$$

2.3. MCS Method

Monte Carlo simulation can be used to estimate the mean value and standard deviation of a random function. Assume that a sample set of n input vectors has been generated, say $\{\mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(n)}\}$. The usual estimators of the first two moments of C_θ respectively read:

$$\hat{\mu}_{C_\theta} \approx \frac{1}{n} \sum_{i=1}^{n} g\left(\mathbf{x}^{(i)}\right) \tag{2.7a}$$

$$\hat{\sigma}_{C_\theta}^2 \approx \frac{1}{n-1} \sum_{i=1}^{n} \left[g\left(\mathbf{x}^{(i)}\right) - \hat{\mu}_{C_\theta} \right]^2 \tag{2.7b}$$

2.4. Point Estimation Method (PEM)

Point estimation method (PEM) was proposed by Rosenblueth (1975) to approximate the lower-order moments of functions of random variables. It is a special case of numerical quadrature based on orthogonal polynomials. For normal variables, it corresponds to Gauss-Hermite quadrature. Zhao and Ono (2000) introduced a new point estimation method based on Rosenblatt transformation (Hohenbichler & Rackwitz, 1981) in which the numerical quadrature is completed in standard normal space. Unfortunately, Rosenblatt transformation cannot deal with the case of random variables with given marginal distributions and correlation information. We herein introduce Nataf transformation (Liu & Der Kiureghian, 1986) into Zhao-Ono point estimation method to replace Rosenblatt transformation.

For a single-variable function $C_\theta = g(X)$, the kth central moment of the function is estimated by

$$M_{kC_\theta} = \sum_{j=1}^{m} \frac{w_j}{\sqrt{\pi}} g^k \left[T^{-1}(\sqrt{2} x_j) \right] \tag{2.8}$$

where, x_j is the Gauss-Hermit integration point, i.e., estimating point; w_j is the corresponding weight.

For a multi-variable function $C_\theta = g(\mathbf{X})$, two function approximation approaches are used:

$$C_\theta \approx g'(\boldsymbol{X}) = Z_{\boldsymbol{\mu}} \prod_{i=1}^{n} \left(\frac{Z_i}{Z_{\boldsymbol{\mu}}} \right) \tag{2.9}$$

$$C_\theta \approx g''(\boldsymbol{X}) = \sum_{i=1}^{n} (Z_i - Z_{\boldsymbol{\mu}}) + Z_{\boldsymbol{\mu}} \tag{2.10}$$

in which,

$$Z_{\boldsymbol{\mu}} = g(\boldsymbol{\mu}) = g(\mu_1, \cdots, \mu_i, \cdots, \mu_n) \tag{2.11}$$

$$Z_i = g\left[T_N^{-1}(\boldsymbol{u}_i) \right] = G(\boldsymbol{u}_i) = G(u_{\mu 1}, u_{\mu 2}, \cdots, u_{\mu i-1}, u_i, u_{\mu i+1}, \cdots, u_{\mu n}) \tag{2.12}$$

where, $T_N^{-1}()$ denotes inverse Nataf transformation; $\boldsymbol{\mu}$ represents the vector in which all the random variables take their mean values; $\mathbf{u}_i$ represents the vector in which only u_i is a random variable, while other variables take the corresponding transformed values of their mean values in standard normal space; $u_{\mu j} (j \neq i)$ is the jth element of the transformed vector $\mathbf{u}_\mu$ who corresponds the vector $\boldsymbol{\mu}$ in standard normal space $\mathbf{u}$; $G(\mathbf{u}) = g[T_N^{-1}(\mathbf{u})]$ is the formulation of random function $g(\mathbf{x})$ in standard normal space based on Nataf transformation.

Based on the product-rule as shown in Eq. (2.9), the mean value and the kth central moment of the function can be estimated:

$$\mu_{C_\theta} \approx Z_{\boldsymbol{\mu}} \prod_{i=1}^{n} \left(\frac{\mu_{Z_i}}{Z_{\boldsymbol{\mu}}} \right) \tag{2.13a}$$

$$E\left[g^K(\boldsymbol{X})\right] \approx Z_{\boldsymbol{\mu}}^K \prod_{i=1}^{n} \frac{E\left[Z_i^K\right]}{Z_{\boldsymbol{\mu}}^K} \tag{2.13b}$$

Based on the non-product rule as shown in Eq. (2.10), the mean value and variance of the function can be estimated:

$$\mu_{C_\theta} \approx \sum_{i=1}^{n} (\mu_{Z_i} - Z_{\boldsymbol{\mu}}) + Z_{\boldsymbol{\mu}} \tag{2.14a}$$

$$\sigma_{C_\theta}^2 \approx \sum_{i=1}^{n} \sigma_{z_i}^2 \tag{2.14b}$$

In Eqs. (2.13) and (2.14), μ_i and σ_i are mean value and standard deviation of G_i by using point-estimation of single-variable function.

2.5. Zhou-Nowak Numerical Integration Method (NIM)

For the independent standard normal random vector $\mathbf{Z} = (Z_1, Z_2, \cdots, Z_n)$, Zhou and Nowak (1988) proposed a special numerical integration method for directly estimating the statistical moments of random function $g(\mathbf{Z})$. The integration points $(z_{1j}, z_{2j}, \cdots, z_{nj})$ and their corresponding weights w_j are listed in Table 2.1.

Table 2.1 Estimating points and weights for Zhou-Nowak numerical integration approach

Point number N	Integration points $\mathbf{Z}_i = (z_{1i}, ...z_{ni})$	Weight factors w_j
$n+1$	$\mathbf{Z}_1 = (\sqrt{n}, 0, 0, 0, \cdots 0)$	$w_1 = \frac{1}{n+1}$
	$\mathbf{Z}_2 = (-\sqrt{\frac{1}{n}}, \sqrt{\frac{(n+1)(n-1)}{n}}, 0, 0, \cdots 0)$	$w_2 = \frac{1}{n+1}$
	$\mathbf{Z}_3 = (-\sqrt{\frac{1}{n}}, \sqrt{\frac{(n+1)}{n(n-1)}}, \sqrt{\frac{(n+1)(n-2)}{(n-1)}}, 0, \cdots 0)$	$w_3 = \frac{1}{n+1}$
	…	…
	$\mathbf{Z}_n = (-\sqrt{\frac{1}{n}}, -\sqrt{\frac{(n+1)}{n(n-1)}}, -\sqrt{\frac{(n+1)}{(n-1)(n-2)}}, \cdots \sqrt{\frac{n+1}{2}})$	$w_n = \frac{1}{n+1}$
	$\mathbf{Z}_{n+1} = (-\sqrt{\frac{1}{n}}, -\sqrt{\frac{(n+1)}{n(n-1)}}, -\sqrt{\frac{(n+1)}{(n-1)(n-2)}}, \cdots -\sqrt{\frac{n+1}{2}})$	$w_{n+1} = \frac{1}{n+1}$
$2n$	$\mathbf{Z}_1 = -\mathbf{Z}_{n+1} = (\sqrt{n}, 0, 0, 0, \cdots 0)$	$w_1 = w_{n+1} = \frac{1}{2n}$
	$\mathbf{Z}_2 = -\mathbf{Z}_{n+2} = (0, \sqrt{n}, 0, 0, \cdots 0)$	$w_2 = w_{n+2} = \frac{1}{2n}$
	…	…
	$\mathbf{Z}_n = -\mathbf{Z}_{n+n} = (0, 0, 0, 0, \cdots \sqrt{n})$	$w_n = w_{n+n} = \frac{1}{2n}$
$2n^2+1$	$\mathbf{Z} = (0, 0, ..., 0)$	$w_j = \frac{2}{n+2}$
	$\mathbf{Z} = (\pm\sqrt{n+2}, 0, ..., 0)^*$	$w_j = \frac{4-n}{2(n+2)^2}$
	$\mathbf{Z} = (\pm\sqrt{\frac{n+2}{2}}, \pm\sqrt{\frac{n+2}{2}}, ..., 0)^*$	$w_j = \frac{1}{(n+2)^2}$

* Points include all possible permutations in coordinates

Using the abscissas and weights in Table 2.1, the kth central moment of random function $C_\theta = g(\mathbf{Z})$ is estimated by

$$\mu_{kC_\theta} = E\left[g^k(Z_1, Z_2, \cdots, Z_n)\right] \approx \sum_{j=1}^{m} w_j g^k\left(z_{1j}, z_{2j}, \cdots, z_{ij}\right) \tag{2.15}$$

For random function $C_\theta = g(\mathbf{X})$ in general random space, the inverse Nataf transformation $\mathbf{X} = T_N^{-1}(\mathbf{Z})$ is applied to result in,

$$\mu_{kC_\theta} = E\left[g^k(X_1, X_2, \cdots, X_n)\right] \approx \sum_{j=1}^{m} w_j g^k\left(T_N^{-1}(z_{1j}), T_N^{-1}(z_{2j}), \cdots, T_N^{-1}(z_{ij})\right) \tag{2.16}$$

Based on Eq. (2.16), the mean value and variance of C_θ can be obtained by Eq. (2.3) and Eq. (2.4).

3. STOCHASTIC PUSHOVER ANALYSIS CONSIDERING RANDOM SYSTEM PROPERTIES

In earthquake engineering practice, the global performance of structures is usually discretized into multiple levels. For example, five levels of damage states are usually adopted, i.e., intact, minor damage, moderate damage, severe damage, and collapse. However, how to characterize the limit values of consecutive damage states has been remaining a difficult problem due to the large uncertainty in modeling and statistical analysis.
In this paper, the conventional pushover analysis is improved to determine the global capacity limiting values C_θ^i $(i = 1, 2, 3, 4)$. A new force control technique is introduced to set up the relationships between C_θ^i $(i = 1, 2, 3, 4)$ and the total base shear V, by adding lateral load to the structure increasingly, until V is equal to the corresponding levels of the base shear V_i $(i = 1, 2, 3, 4)$, as shown in Figure 1.

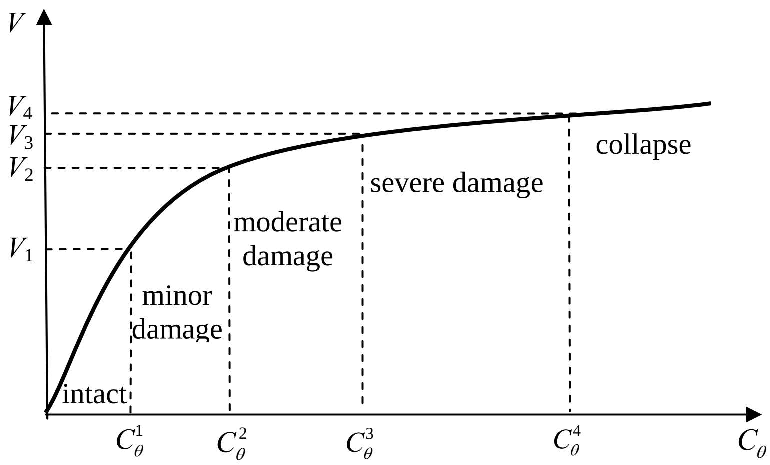

Figure 1 Sketch diagram of force-controlled pushover analysis

In Chinese seismic design code of buildings (GB50011-2001), three levels of seismic fortification for building structures are specified, which can be summarized as: do not be damaged under minor earthquake, can be repaired under moderate earthquake, and do not collapse under major earthquake. The moderate earthquake is the basic intensity I_0 of the designated site, whose exceedance probability in 50yrs is 10%. The exceedance probability in 50yrs of the minor earthquake is 63.2%, whose earthquake intensity is I_d - 1.55, where I_d is the fortification intensity. The exceedance probability in 50yrs of the major earthquake is 2%, whose earthquake intensity is I_d + 1.
The intensities of minor, moderate and major earthquakes can be directly used to control the lateral force added to the structure corresponding to C_θ^1, C_θ^3 and C_θ^4. To present the lateral force corresponding to C_θ^2, another level of earthquake fortification intensity, named as sub-moderate earthquake, or frequent earthquake, is

introduced in this paper, whose intensity takes the mean value of minor earthquake intensity and moderate earthquake intensity:

$$I=\frac{[(I_d-1.55)+I_d]}{2}=I_d-0.8 \tag{3.1}$$

Based on Chinese seismic design code of buildings (GB50011-2001), the relationships between the lateral seismic factors α_i $(i=1,2,3,4)$ and C_θ^i $(i=1,2,3,4)$ are listed in Table 3.1.

Table 3.1 Relationships between I_i, α_i and C_θ^i

Earthquake levels	C_θ^i	I_i	α_i		
			$I_d=7$	$I_d=8$	$I_d=9$
minor earthquake	C_θ^1	$I_d-1.55$	0.08	0.16	0.32
frequent earthquake	C_θ^2	$I_d-0.8$	0.12	0.23	0.47
moderate earthquake	C_θ^3	I_d	0.23	0.45	0.90
major earthquake	C_θ^4	I_d+1	0.50	0.90	1.40

Using α_i $(i=1,2,3,4)$ in Table 2, the lateral seismic force on the *j*th floor for each seismic level can be determined by

$$F_{ij}=\frac{\alpha_i F_j}{\alpha_3},\ (i=1,2,3,4)\ \ (j=1,\ldots,n) \tag{3.2}$$

where, F_j is the lateral seismic force on the *j*th floor under fortification intensity I_d, n is the number of structural storeys.

The controlling lateral base shear V_i $(i=1,2,3,4)$ is obtained by

$$V_i=\sum_{j=1}^{n}F_{ij},\ (i=1,2,3,4) \tag{3.3}$$

The pushover analysis under the lateral load F_{ij} is carried on until $V=V_i$ $(i=1,2,3,4)$, and then C_θ^i $(i=1,2,3,4)$ can be obtained from the pushover curve.

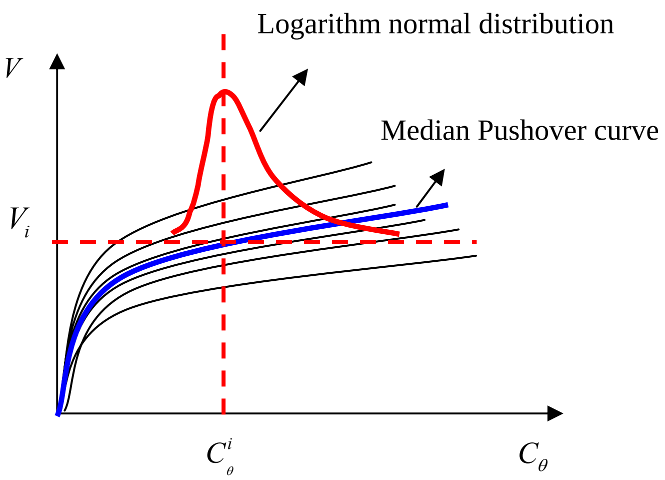

Figure 2 Sketch diagram of stochastic pushover analysis

The above-mentioned procedure can only derive the deterministic limiting values of the global seismic capacity of structures. Due to the random system properties, the limiting values of the global seismic capacity of structures behaves random dispersion in nature, as shown in Figure 2.
To consider random structural properties in pushover analysis, we herein propose a stochastic pushover analysis approach, which combines four approximate random analysis methods introduced in section 2, namely, MVFOSM method, MCS method, point estimation method and Zhou-Nowak numerical integration method, with the force-controlled pushover analysis method stated above. In this new approach, the global capacity measure C_θ of structures is assumed to be a function of basic random variables. Each time that the function is evaluated during the estimating the statistical moments of C_θ, the force-controlled pushover analysis is run for one time.
For stochastic pushover analysis based on MVFOSM method, because of Eq. (2.6b), it needs finite element response sensitivity analysis. The finite element reliability analysis module of OpenSees (Haukaas & Kiureghian, 2007) is utilized to compute the statistical moments of C_θ by direct differential method of response sensitivity analysis.
For stochastic pushover analysis based on MSC method, the random samples of the basic variables are firstly generated, and then, the random samples of finite element models of structures are obtained. The force controlled pushover analysis is then applied for each structural sample, and the statistical moments of C_θ^i $(i=1,2,3,4)$ are estimated by Eq. (2.8).
For stochastic pushover analysis based on point estimation method，we should choose Gauss-Hermit estimating points and weights at first, and then, the structural samples are obtained according to the strategy of PEM. The force controlled pushover analysis is carried on for each structural sample. The statistical moments of C_θ^i $(i=1,2,3,4)$ are then approximated by product rule in Eq. (2.13) and non-product rule in Eq. (2.14), respectively.
For stochastic pushover analysis based on Zhou-Nowak numerical integration approach, we should also select the estimating points and weights according to Table 2.1 at first, and then, the structural samples are obtained according to the strategy of this approach. The force controlled pushover analysis is again taken for each structural sample, and the statistical moments of C_θ^i $(i=1,2,3,4)$ are then estimated by Eq. (2.16).

4. PROBABILISTIC MODEL OF GLOBAL SEISMIC CAPACITY OF STRUCTURES

The probabilistic seismic capacity model of structures is defined as a conditional probability of by seismic demand D_θ exceeding the capacity C_θ, given the specific value of seismic demand:

$$F_{C_\theta}\left(d_\theta\right)=P\left[C_\theta \le D_\theta \middle| D_\theta = d_\theta\right] \tag{4.1}$$

where, D_θ is the global seismic demand parameter with the same unit as capacity parameter, e.g., the maximum ISDA; d_θ is the given value of demand parameter.
The model in Eq. (4.1) is also termed as seismic fragility of structures. It has been supported by numerous research programs that the fragility can be modeled by a lognormal cumulative distribution function:

$$F_{C_\theta}(d_\theta)=\Phi\left(\frac{\ln d_\theta-\lambda_{C_\theta}}{\zeta_{C_\theta}}\right) \tag{4.2}$$

where, λ_{C_θ} and ζ_{C_θ} are mean value and standard deviation of $\ln(C_\theta)$, respectively. They are related to mean value and standard deviation of C_θ as

$$\lambda_{C_\theta} = \ln\left(\frac{\mu_{C_\theta}}{\sqrt{1+\delta_{C_\theta}^2}}\right) \tag{4.3}$$

$$\zeta_{C_\theta} = \sqrt{\ln\left(1+\delta_{C_\theta}^2\right)} \tag{4.4}$$

where, μ_{C_θ} and δ_{C_θ} are mean value and coefficient of variation of C_θ, respectively.

The median value m_{C_θ} of C_θ is the exponent of λ_{C_θ}:

$$m_{C_\theta} = \exp(\lambda_{C_\theta}) \tag{4.5}$$

Therefore, the probabilistic model of seismic capacity can be re-written as a simpler formulation:

$$F_{C_\theta}(d_\theta) = \Phi\left(\frac{\ln(d_\theta / m_{C_\theta})}{\zeta_{C_\theta}}\right) \tag{4.6}$$

5 APPLICATION OF THE METHODOLOGY IN A R.C. FRAME STRUCTURE

A three-bay and five-storey R.C. frame structure, as shown in Figure 3, is taken as the example in this case study. It is designed according to Chinese seismic design code of buildings (GB50011-2001) with fortification intensity $I_d = 8$.

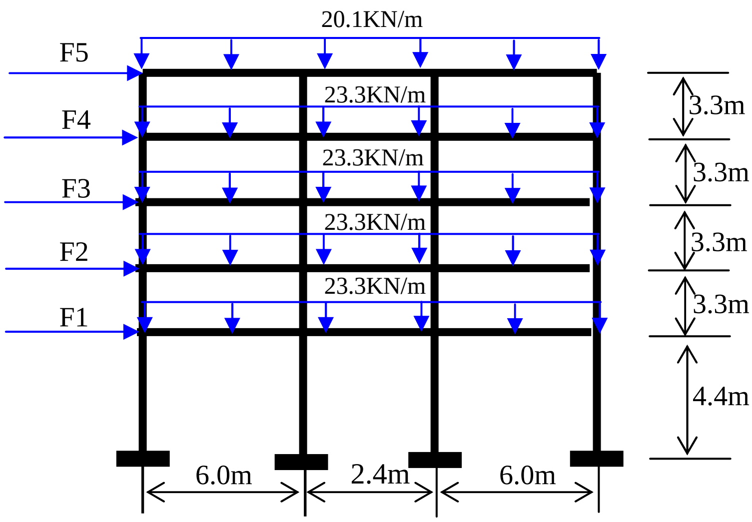

Figure 3 Three-bay and five-storey R.C. frame

For simplicity, only the random material properties are considered, which include six basic random variables: yield strength f_y and initial elastic modulus E of steel; compressive strength f_c, crushing strength f_{cr}, strain ε_c at compressive strength, and strain ε_{cu} at crushing strength of concrete. The statistics are shown in Table 5.1. All variables are assumed to follow normal distribution, and to be independent on each other.

Table 5.1 Statistics of basic random variables

Variables	Distribution parameters		Variables	Distribution parameters	
	Mean value	Std		Mean value	Std
f_y(N/mm^2)	384.80	28.59	f_{cr}(N/mm^2)	27.32	4.44
E(N/mm^2)	204000	2040	ε_c	0.0022	0.000308
f_c(N/mm^2)	26.10	4.44	ε_{cu}	0.021	0.00274

The values of $F_{ij}, (i=1,2,3,4; j=1,2,3,4,5)$ and V_i $(i=1,2,3,4)$ are listed in Table 5.2.

Table 5.2 Lateral controlling load F_{ij} and corresponding base shear

i	F_{ij}/KN					V_i/KN
	F_{i1}	F_{i2}	F_{i3}	F_{i4}	F_{i5}	
1	10.898	31.602	52.452	69.014	53.356	217.322
2	15.666	45.428	75.400	99.208	76.699	312.401
3	30.514	88.486	146.866	193.239	149.397	608.501
4	61.028	176.971	293.731	386.478	298.794	1217.002

Stochastic pushover analysis with four random analysis methods are applied to estimate the statistics of the global drift capacity of the structure, the computed results are shown in Table 5.3.

Table 5.3 Statistics of the global drift capacities

Methods	Performance levels	μ_{C_θ}	σ_{C_θ}	m_{C_θ}	ζ_{C_θ}
MCS	Minor damage	0.04371	0.00103	0.04371	0.02361
	Moderate damage	0.01489	0.00213	0.01472	0.14282
	Severe damage	0.00413	0.00015	0.00412	0.03631
	Collapse	0.00250	0.00005	0.00252	0.02002
PEM (product rule)	Minor damage	0.05469	0.01068	0.05371	0.19351
	Moderate damage	0.01487	0.00261	0.01462	0.17422
	Severe damage	0.00411	0.00018	0.00411	0.04381
	Collapse	0.00249	0.00005	0.00252	0.02002
PEM (Non-product rule)	Minor damage	0.05274	0.00129	0.26861	0.26821
	Moderate damage	0.01504	0.00236	0.01492	0.15472
	Severe damage	0.00413	0.00024	0.00412	0.05811
	Collapse	0.00250	0.00004	0.00251	0.00402
Zhou-Nowak numerical integration approach	Minor damage	0.05162	0.01380	0.11421	0.63221
	Moderate damage	0.01504	0.00236	0.01492	0.11932
	Severe damage	0.00413	0.00024	0.00411	0.05071
	Collapse	0.00250	0.00013	0.00252	0.05202
MVFOSM	Minor damage	0.03804	0.00060	0.03801	0.03601
	Moderate damage	0.01464	0.00030	0.01461	0.03152
	Severe damage	0.00412	0.00013	0.00411	0.02051
	Collapse	0.00250	0.00009	0.00251	0.01582

From Table 5.3, we can see that the results of stochastic pushover analysis method based on different approximating methods are approaching to each other; however, there exist some inconsistence in the results of collapse capacities, since the structure has gone into a highly nonlinear range.

Using the statistics in Table 5.3 and Eq. (4.2) or (4.6), the global seismic capacity fragility curves can be obtained, as shown in Figure 4. From this figure, we can assess the failure probability of different performance levels under a given seismic demand level.

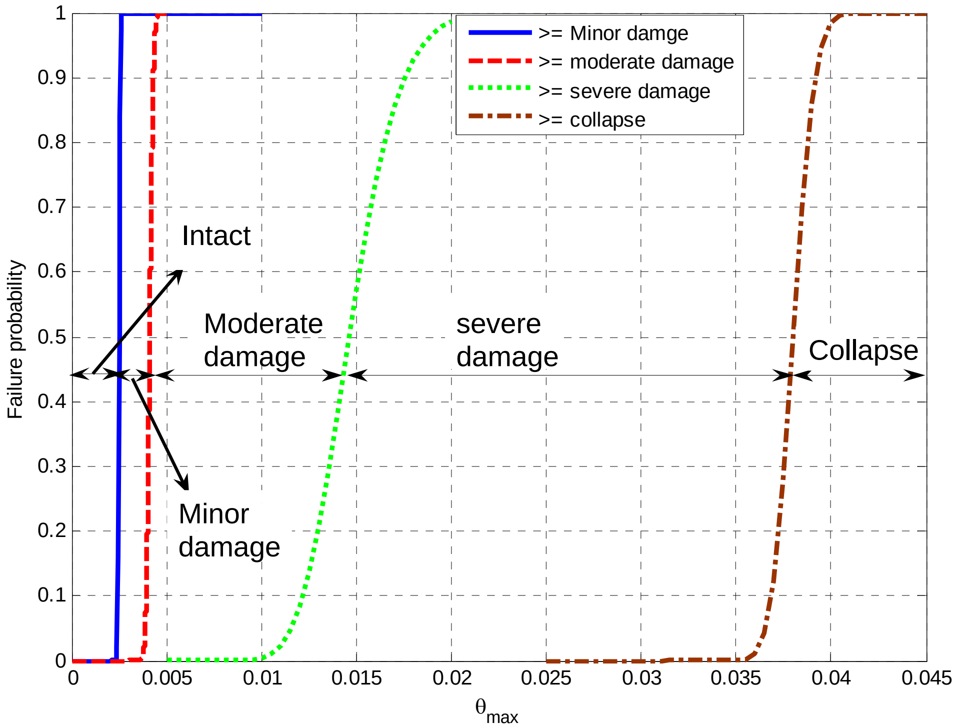

Figure 4 Fragility curves of global seismic capacities of the structure (MCS method)

6. CONCLUSION

This paper proposes a stochastic pushover analysis method for probabilistic seismic capacity analysis of structures, which combines four kinds of methods for estimating statistical moments of random function, namely MVFOSM method, MCS method, point estimation method (PEM) and Zhou-Nowak numerical integration method (NIM), with a force-controlled pushover analysis method, The developed methodology is applied in R.C. frame structures. A three-bay and five-storey plane R.C. frame is taken as an example in case study, the global seismic capacity curves of the structure corresponding to four levels of limit states are derived. It is demonstrated by this example that the approach proposed in this paper is an efficient and accurate tool for global probabilistic seismic capacity analysis of structures.

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REFERENCES

Haukaas, T., and Der Kiureghian, A. (2007). Methods and object-oriented software for FE reliability and sensitivity analysis with application to a bridge structure. *ASCE Journal of Computing in Civil Engineering* **21:3**, 151-163.

Hohenbichler, M., and Rackwitz, R. (1981). Non-normal dependent vectors in structural safety. *ASCE Journal of Engineering Mechanics* **107:6**, 1227-1238.

Liu, P.L., and Der Kiureghian, A. (1986). Multivariate distribution models with prescribed marginal and covariances. *Probabilistic Engineering Mechanics* **1:2**, 105-112.

Moehle, J., and Deierlein, G.G. (2004). A framework methodology for performance-based earthquake engineering. *Proceedings of the 13th World Conference on Earthquake Engineering*, Vancouver, Canada, 1–6 August, (CD-ROM).
National Standard of China P.R. (2001). Seismic Design Code of Buildings (GB50011-2001). Building Industry Press of China, Beijing.
Rosenblueth, E. (1975). Point estimates for probability moments. *Proceedings of the National Academy of Science* **72:10**, 3812-3814.
Zhao, Y.G., and Ono, T. (2000). New point estimates for probability moments. *ASCE Journal of Engineering Mechanics* **126:4**, 433-436.
Zhou, J.H., and Nowak, A.S. (1988). Integration formulas to evaluate functions of random variables. *Structural Safety* **5:4**, 267-284.