PROPOSING INPUT-DEPENDENT MODE CONTRIBUTION FACTORS FOR SIMPLIFIED SEISMIC RESPONSE ANALYSIS OF BUILDING SYSTEMS

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ABSTRACT :

When the excitation frequency at the base of a building is close to one of its modal frequencies other than its first mode, the contribution of that mode in comparison with the first mode is much more comparing with other cases of excitation, however, this fact is not taken into accoutn by the common definitions of "Mode Participation / Contribution Factors" in which always the first mode is dominant. If the effect of input frequency is incorporated by some way in the definition of these factors, it will be possible to calculate the approximate structural responses to various dynamic loads, including seismic forces, with no need to time history analysis. For this purpose, in this paper at first various definitions, proposed by different scholars, for modal participation factors have been introduced and discussed briefly. Then a simple MDOF system has been considered subjected to sinusoidal base excitations with various frequencies. For each input, the ratio of the corresponding maximum response of every mode in each degree of freedom of the system to the total response of that degree of freedom has been calculated and compared to those of other modes. Then the responses of the considered system to some seismic inputs with various frequency contents have been calculated and the same ratios have been obtained again to find out the possibility of defining 'input-frequency dependent mode participation factors'. Numerical results show that this definition is possible and these factors can be used for 'Simplified Seismic Response Analysis' of building systems.

KEYWORDS: Input-frequency Dependent Mode Participation Factors, Simplified Dynamic Analysis

1. INTRODUCTION

In almost all resources of 'structural dynamics' the mode participation factors are independent of the input characteristics, and are only functions of specifications of the structure. This basically results in dominancy of the first natural mode of the system in its dynamic response to any base excitation, regardless of the type and frequency content of the excitation. However, it is clear that in case of a building, excited at its base by an excitation having a frequency close to one of its modal frequencies other than its first mode, the contribution of that mode in its dynamic response in comparison with the contribution of the first mode is much more than other cases of excitation. Therefore, if the effect of the input frequency can be incorporated in the definition of mode contribution factors by some way, it will be possible to calculate the approximate values of maximum responses of structures to various dynamic loads, including earthquake induced forces, by some easy calculations with no need to any 'time history analysis'. This can be called a 'Simplified Dynamic Analysis Approach'. For this purpose, in this paper at first various definition proposed by different scholar for modal participation factors have been introduced and discussed briefly. Then a simple MDOF system has been considered subjected to sinusoidal base excitations with various frequencies. For each input the ratio of the corresponding maximum response of every mode in each degree of freedom of the system to the total response of that degree of freedom has been calculated and compared to those of other modes. Then the responses of the considered system to a set of seismic inputs with various frequency contents, from low to high, have been calculated and the same ratios have been obtained again to find out the possibility of defining 'input-frequency dependent mode participation factors'. The details of the study are explained in the next sections of the paper.

2. MODE PARTICIPATION CONCEPTS AND DEFINITIONS

In calculation of dynamic response of structures to seismic excitations by mode combination technique the modal equations of motion of the form given by Eqn (1) are used:

$$M_j\ddot{y}_j + C_j\dot{y}_j + K_j y_j = P_j(t) \tag{1}$$

where j is the mode number, M_j, C_j and K_j are respectively the modal mass, modal damping, and modal stiffness, and y_j and its time derivatives are modal displacement, velocity, and acceleration respectively, and $P_j(t)$ is the modal load. In cases of ordinary multi-story buildings subjected to just a horizontal seismic excitation the modal load is given by:

$$P_j(t)=\left\{\varphi_j\right\}^t\left\{P_{eff}(t)\right\}=L_j\ddot{x}_g(t) \tag{2}$$

in which $\{\varphi_j\}^t$ is the transpose of j^{th} modal vector or mode shape of the structure, and $\{P_{eff}(t)\}$ is the earthquake effective load vector given by:

$$\{P_{eff}(t)\}=-[M]\left\{1\right\}\ddot{x}_g(t) \tag{3}$$

where [M] is the mass matrix, {1} is the nx1 earthquake influence vector having n elements of unity, and $x_g(t)$ is the time history of ground displacement. On this basis the modal response to base excitations are obtained by:

$$y_j(t)=\frac{\left\{\varphi_j\right\}^T[M]\{1\}}{\left\{\varphi_j\right\}^T[M]\left\{\varphi_j\right\}}V_j(t)=\frac{L_j}{M_j\omega_{Dj}}V_j(t) \tag{4}$$

In Eqn (4) L_j is called the modal influence factor and $V_j(t)$ is the modal integral of seismic response, which is a function of ground acceleration as well as the values of modal frequency, ω_j, and modal damping, ξ_j. Up to this point there is no difference in the formulation proposed by various scholars for calculation of seismic response of building systems, however, considering the algebraic term beside $V_j(t)$ in Eqn (4) as a factor which is only dependent on the modal characteristics of the system, while $V_j(t)$ is a function of both earthquake and system characteristics, it has been tried to introduce some kind of modal participation or contribution factor for simplification of seismic response calculations, and several definitions have been proposed for these factors so far. Clough and Penzien (1975) have suggested the following formulas, calling γ_js as the modal effective masses.

$$\gamma_j=\frac{\dfrac{L_j^{\,2}}{M_j}}{M_T} \qquad \sum_{j=1}^{n}\gamma_j=1 \qquad M_T=\{1\}^T[M]\{1\} \tag{5}$$

They have called M_T the total mass of the building. Amini (1983) have introduced the following formula:

$$\alpha_j=\frac{\left\{\varphi_j\right\}^T[M]\{r\}}{\left\{\varphi_j\right\}^T[M]\left\{\varphi_j\right\}} \qquad \sum_{j=1}^{n}\alpha_j=1 \tag{6}$$

calling α_js the mode participation factors and {r} has the same definition of {1} in Eqn (3). Adeli (1986) has proposed the following formula, calling L_js the modal participation factors, which their summation is unity, implying that the whole modes together makes the whole response of the structure:

$$L_j = \frac{\{\phi_j\}^T [M]\{1\}}{\{\phi_j\}^T [M]\{\phi_j\}} \qquad \sum L_j = 1 \tag{7}$$

Gaylord and Gaylord (1989) and also Paz (1994) have used another form of Eqn (7) by using γ_j instead of L_j, which can be used when the mass matrix is diagonal, such as the case of multistory buildings with m_i as mass of the i^{th} story.

$$\gamma_j = \frac{\sum_{i=1}^{n} m_i \phi_{ij}}{\sum_{i=1}^{n} m_i \phi_{ij}^2} \qquad \sum \gamma_j = 1 \tag{8}$$

Naeim (1993) have suggested the following formulas.

$$M_{ej} = \frac{L_j^2}{M_j} \qquad \sum_{j=1}^{N} M_{ej} = \text{Total Mass} \tag{9}$$

where M_{ej} is called the effective modal mass. It can be seen that Eqns (6), (7) and (8) are all based on the same concept. Eqns (5) and (9) are in fact very similar in the concept, and they have just used different notations. By paying attention to Eqns (6) to (8) it can be easily seen that if the modal vectors are calculated by different approaches, surprisingly, different values will be obtained for each of the so-called "modal participation factors". Chopra (1995) has mentioned implicitly this shortcoming and has introduced some other form of modal contribution factors which are different kinds of responses such as displacements, story shear forces and story moments. Eqns (5) and (9), on the other hand, do not have this shortcoming, but their use is limited to only the case of shear-beam-type structures subjected to just horizontal ground excitation, and they can not be used for other types of structures, and even for the case of shear-beam-type buildings they can not be used for rotational or torsional excitations. Hosseini and Yaghoobi Veyeghan (1998, 1999) have proposed, by using the definition of Clough and Penzien, a somehow new definition for modal participation factors of a structure subjected to multicomponent ground motion by introducing Eqn (10).

$$\gamma_{jk} = \frac{\dfrac{L_{jk}^2}{M_j}}{M_{ek}} \qquad \sum_{j=1}^{n} \gamma_{jk} = 1 \tag{10}$$

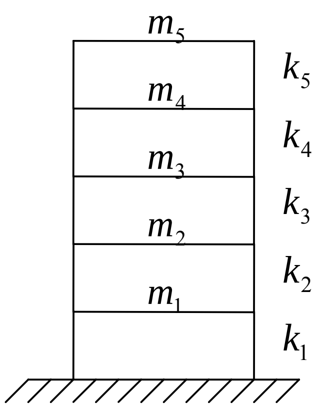
in which γ_{jk} and M_{ek} are respectively the modal participation factor of the structure and its effective mass for the k^{th} component of ground motion. This effective mass is defined as:

$$M_{ek} = \{r_k\}^T [M]\{r_k\} \tag{11}$$

where $\{r_k\}$ is the earthquake influence vector for the k^{th} component of ground motion, of which the element r_{jk} is defined as the generalized displacement created in the i^{th} degree of freedom of the structure due to the unite positive generalized displacement of the k^{th} component of ground motion. By Eqns (10) and (11), in fact, instead of one participation factor for each mode as a single valued parameter which is not dependent on how the structure is excited by the ground motion, up to six different componential factors can be defined for each mode of the structure, depending on the number of considered component of ground motion. Although by this approach the modal participation factors are defined much deliberately and are dependent to the excitation component(s), still they are quite independent from the excitation frequency. This is while, obviously, exciting a structure by an excitation having a dominant frequency very close to one of the modal frequencies of the structure will cause the participation of that mode increase remarkably. This fact is not taken into consideration in the definition of modal participation factors, even by using modal-componential factors. In the next section it is tried to define a new concept of "input-frequency dependent mode participation factor" by which it is believed that the seismic response analysis of building systems can be done with very little calculations.

3. INPUT-FREQUENCY DEPENDENT MODE PARTICIPATION FACTOR

The modal participation or contribution factors by the definitions given in previous section all results in the dominancy of the first mode of any structure in calculation of its seismic response, regardless of the dominant frequency of the excitation. To find out if it is possible to define mode participation factors which depend in some way on the input frequency a 5-story building shown in Figure 1 is considered.



$$m_1 = m_2 = \ldots.. = m_5 = 150\ ton$$
$$K_1 = 5\times10^6 \quad KN/m$$
$$K_2 = 4.7\times10^6 \quad KN/m$$
$$K_3 = 4\times10^6 \quad KN/m$$
$$K_4 = 3\times10^6 \quad KN/m$$
$$K_5 = 1.7\times10^6 \quad KN/m$$

Figure 1. The 5-story building model used in the study

In this model the height of all stories has been assumed to be 3.30 m, and stories' mass and stiffness values are as given in Figure 1. For seismic design of this building model the soil type has been assumed to be of type II, and the seismicity of the region has been considered to be moderate to high. The building has been designed based on drift control. The modal frequencies and modal shapes are obtained as:

$$\{\omega\} = \begin{Bmatrix} 4.72 \\ 11.61 \\ 18.32 \\ 24.97 \\ 31.68 \end{Bmatrix} \text{ rad/Sec} \qquad [\phi] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1.993 & 1.634 & 0.992 & 0.074 & -1.140 \\ 2.992 & 1.553 & -0.265 & -1.193 & 0.636 \\ 3.991 & 0.399 & -1.497 & 0.837 & -0.188 \\ 4.969 & -2.112 & 0.763 & -0.186 & 0.024 \end{bmatrix}$$

Now, by introducing the following formulas:

$$\begin{aligned} \eta_{ij} &= \frac{v_{ij}(t)}{v_i(t)} \\ v_{ij}(t) &= \sum_{i=i}^{n} \overline{k\varphi}_{ij}\, y_j(t) \\ \overline{k\varphi}_{ij} &= \sum_{l=1}^{n} k_{il}\varphi_{lj} \\ v_i(t) &= \sum_{j=1}^{n}\left(\sum_{i=i}^{n} \overline{k\varphi}_{ij}\, y_j(t)\right) \end{aligned} \qquad (12)$$

where η_{ij} is the modal story shear force ratio, v_{ij} is the contribution of mode j in the shear force of story i, v_i, it is possible to relate the modal contribution factors to the input frequency as described hereinafter. Two sets of base excitations have been considered, a sinusoidal and a seismic. In case of sinusoidal excitations two frequency values, corresponding to the second and the forth modes of the building, respectively 11.61 r/s and 24.97 r/s have been used. In case of seismic excitations the accelerograms of three earthquakes of Emeryville, Chichi, and Corralitos, have been used, respectively as low frequency, mid frequency, and high frequency excitation. The results of dynamic response calculations for sinusoidal excitations are shown in Figures 2 to 6, where the horizontal axes shows the time in sec and vertical axes show the modal shear values in kN.

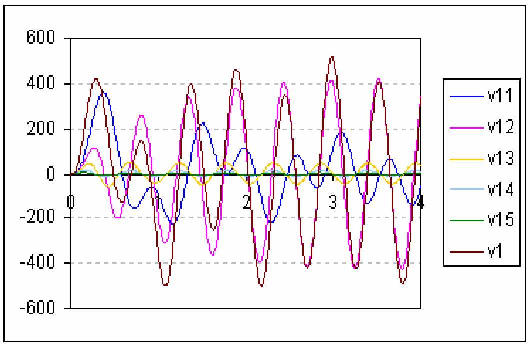


Figure 2. The modal and total shear force values at the 1st story of the building for the case of sinusoidal base excitation with the second modal frequency

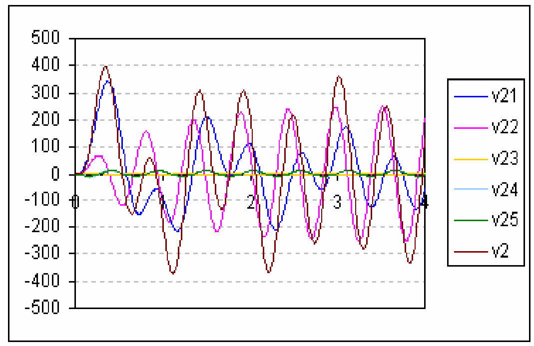


Figure 3. The modal and total shear force values at the 2nd story of the building for the case of sinusoidal base excitation with the second modal frequency

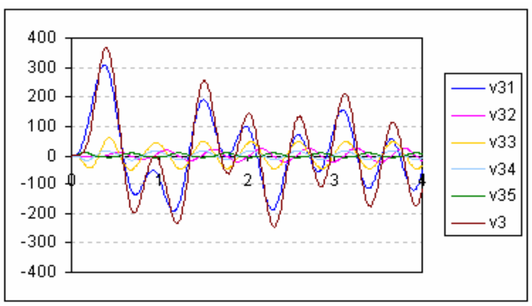


Figure 4. The modal and total shear force values at the 3rd story of the building for the case of sinusoidal base excitation with the second modal frequency

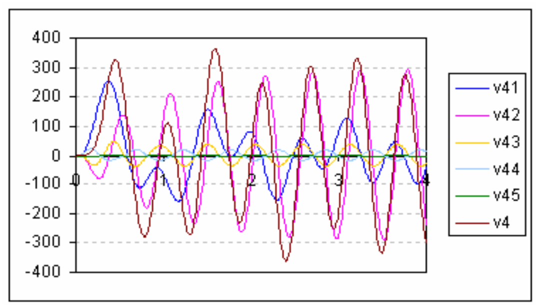


Figure 5. The modal and total shear force values at the 4th story of the building for the case of sinusoidal base excitation with the second modal frequency

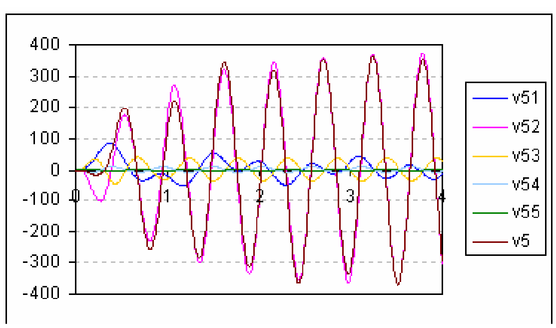


Figure 6. The modal and total shear force values at the 5th story of the building for the case of sinusoidal base excitation with the second modal frequency

It is seen in Figures 2 to 6 that the first mode does not have much contribution, particularly in higher stories. The same is true for the case of excitation of the building by the frequency of its forth mode of vibration, of which the results can not be shown here because of lack of space (the complete results can be found in the main report of the study (Abbasi 2008)). The results of seismic excitations are shown in Figures 7 to 16.

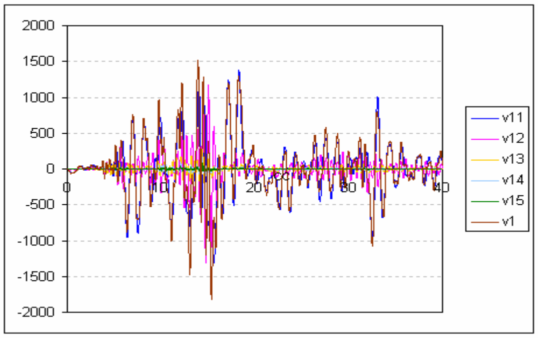


Figure 7. The modal and total shear force values at the 1st story of the building for the case of seismic excitation by Chichi (mid frequency) earthquake accelerogram

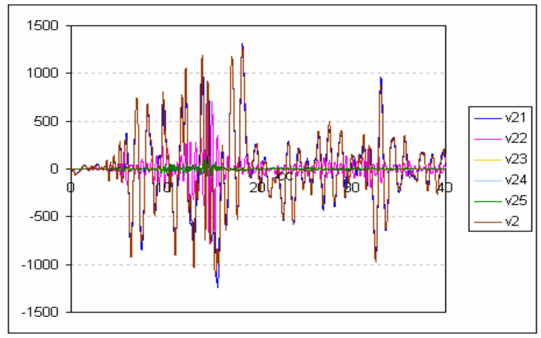


Figure 8. The modal and total shear force values at the 2nd story of the building for the case of seismic excitation by Chichi (mid frequency) earthquake accelerogram

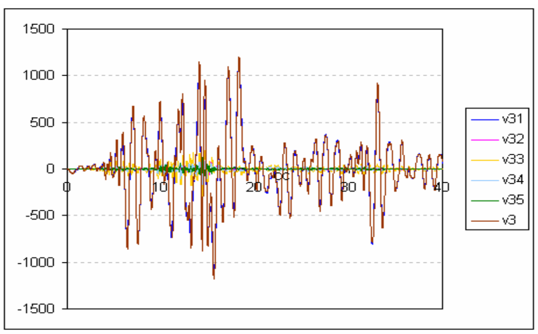


Figure 9. The modal and total shear force values at the 3rd story of the building for the case of seismic excitation by Chichi (mid frequency) earthquake accelerogram

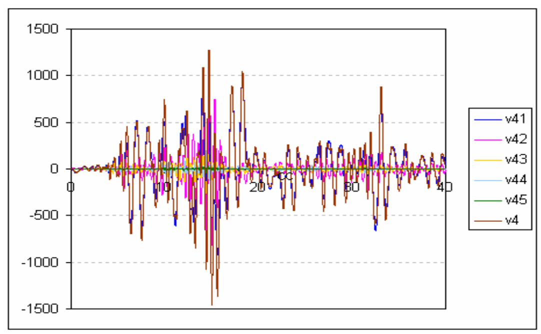


Figure 10. The modal and total shear force values at the 4th story of the building for the case of seismic excitation by Chichi (mid frequency) earthquake accelerogram

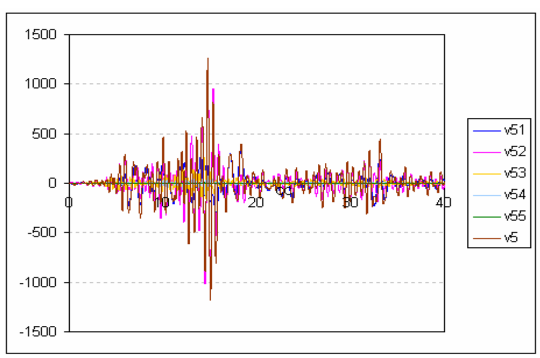


Figure 11. The modal and total shear force values at the 5th story of the building for the case of seismic excitation by Chichi (mid frequency) earthquake accelerogram

It can be seen in Figures 7 to 11 that contribution of the second mode is remarkable in the response of the system, and even in some cases, more than the first mode, particularly in the higher stories.

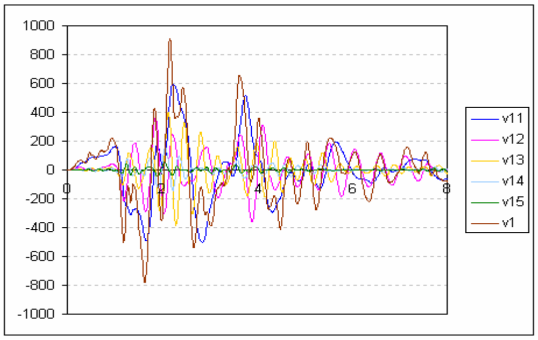


Figure 12. The modal and total shear force values at the 1st story of the building for the case of seismic excitation by Corralitos (high frequency) earthquake accelerogram

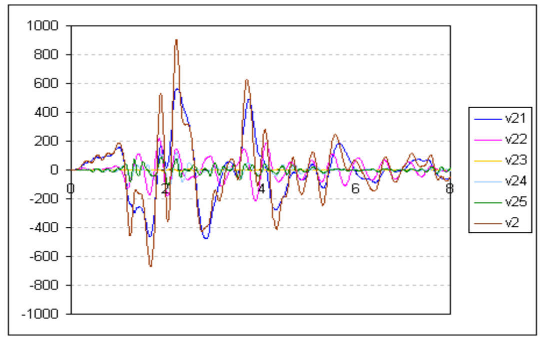


Figure 13. The modal and total shear force values at the 2nd story of the building for the case of seismic excitation by Corralitos (high frequency) earthquake accelerogram

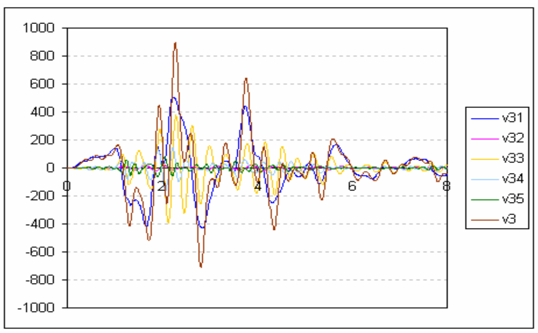


Figure 14. The modal and total shear force values at the 3rd story of the building for the case of seismic excitation by Corralitos (high frequency) earthquake accelerogram

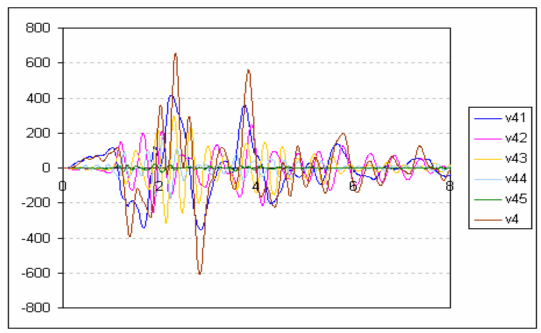


Figure 15. The modal and total shear force values at the 4th story of the building for the case of seismic excitation by Corralitos (high frequency) earthquake accelerogram

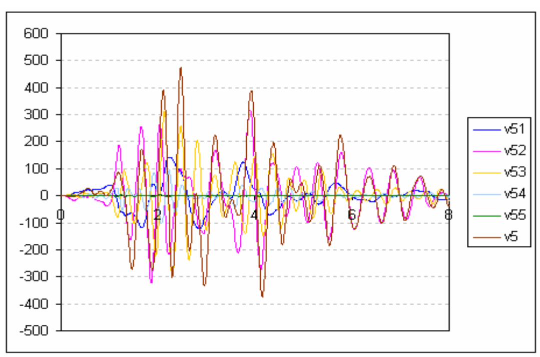


Figure 16. The modal and total shear force values at the 5th story of the building for the case of seismic excitation by Corralitos (high frequency) earthquake accelerogram

It can be seen in Figures 12 to 16 that in the case of a high frequency earthquake the contribution of one of the higher modes (in this case the third mode) is quite dominant comparing with the contribution of the first mode. Now, going back to Eqns (12) it is seen that defining the story shear modal contribution factors is reasonable, but, as it can be seen in the shown results that exciting the building with an excitement whose frequency is very close to one specific mode of the building does not necessarily leads to the dominancy of that mode in the shear values in all stories. A very good sample of such case can be seen in Figure 4, which shows that the contribution of the second mode in the shear force of the 3rd story of the building is very little in spite of that the excitation frequency is exactly equal to the second modal frequency of the building. The reason behind this fact is described here. The story shear force vector can be written as:

$$\{V(t)\}=[I][K][\varphi]\{y(t)\} \qquad [I]=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{(in the case of 5-story building)} \tag{13}$$

By defining $[\lambda]=[I][K][\varphi]$, any element of this matrix, λ_{ij}, can be called the coefficient of the j^{th} modal response in the shear force of the i^{th} story. For the considered 5-soty building $[\lambda]$ is obtained as:

$$[\lambda]=\begin{bmatrix} 5 & 5 & 5 & 5 & 5 \\ 4.76 & 2.98 & 0.10 & -4.29 & -10.05 \\ 4.27 & -0.31 & -4.90 & -5.10 & 7.09 \\ 3.52 & -3.45 & -3.90 & 5.93 & -2.44 \\ 1.21 & -4.40 & 3.92 & -1.64 & 0.34 \end{bmatrix}$$

In fact, $[\lambda]$ shows the structure dependent part of "story shear modal contribution factors". It is seen in the above matrix that all mode has the same contribution in the shear force of the 1^{st} story, while for other stories the modal contributions are quite different, and sometimes they have opposite effects. The reason of little contribution of the second mode in the shear force of third story, which was mentioned before, can also be found in this matrix by comparing -0.31 with other values in the third row of the matrix.

4. CONCLUSIONS

Based on the numerical results it can be said that the "input-frequency dependent mode participation factors" are meaningful factors that can be defined for any structure, and their number is equal to n^2 (for an n-degree of freedom system) instead of n. Furthermore, by using these factors the calculation of seismic response of structures can be done by little effort, with no need to time history analysis. Therefore, they are very useful tools for 'Simplified Seismic Response Analysis' of building systems.

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