Modified linear viscoelastic model for elimination of the tension force in the linear viscoelastic

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SUMMARY:

In recent times, earthquake-induced structural pounding has been intensively studied through the use of different impact force models. The numerical results obtained from the previous studies verified that the linear viscoelastic model is recommended in simulating the pounding force time histories during impact and also nominated this model for simulating pounding at high peak ground acceleration levels, as long as there is no tensile force involved. The aim of this paper is to overcome this disadvantage by introducing an improved version of the linear viscoelastic model by redefining the contact force of the model during two stages of contact, i.e. the approach period and the restitution period. This requires the reassessment of the relation between the impact damping ratio and the coefficient of restitution. The results for two different impact experiments are used in this study. In addition, a suit of thirty ground motion records from thirteen different earthquakes is applied to simulate pounding between two single degree of freedom systems of different period ratios. The final outcome of this study demonstrates that the results obtained through the modified linear viscoelastic model are comparably similar to those found by using the linear viscoelastic model without the tension force.

KEY WORDS: approach period; restitution period; pounding force; damping ratio; coefficient of restitution; earthquakes.

1. INTRODUCTION

Structural pounding during earthquakes has been recently intensively studied by using different models of collision applied to different types of structures. Anagnostopoulos (1998) modeled adjacent buildings as single degree of freedom lumped mass system with linear viscoelastic model of collision to simulate structural pounding. Jankowski et al. (1998) used the same model to study pounding of superstructure segments in bridges.

Other simplified models for pounding investigation have been introduced to represent pounding force during collision, for example, the linear elastic model (Maison and Kasai, 1990), the nonlinear elastic model (Pantelides and Ma, 1998), the nonlinear viscoelastic model (Jankowski, 2005), and the Hertzdamp model (Muthukumar and DesRoches, 2006).

Jankowski (2005) showed that the linear viscoelastic model and the nonlinear viscoelastic model give the smallest simulation errors in the pounding force time histories during impact but, in the case of the linear viscoelastic model, a negative force just before separation has been observed.

Muthukumar and DesRoches (2006) performed a comparison study using two single degree of freedom (SDOF) systems of different period ratios subjected to a suit of 27 ground motion records of different peak ground acceleration (PGA) levels from 13 different earthquakes to assess the performance of various pounding models for capturing pounding. Numerical results shown in (Muthukumar and DesRoches, 2006) indicate that, for high PGA levels the linear viscoelastic model with a coefficient of restitution equal to 0.6, provides smaller acceleration amplifications among the considered models, however, it has been shown that the linear viscoelastic model is known to result in a sticky tensile forces just before separation of colliding bodies (Hunt and Crossley, 1975).

Valles and Reinhorn (1996) proposed a variation of the Kelvin element with force reformulation, where the viscous part of the element is only active for positive velocities and allow the masses to release at time $t_r = t_{\max} + \frac{\pi}{2\omega_d}$ where $t_{\max} = \frac{1}{\omega_d}\tan^{-1}(\frac{\omega_d}{\xi_c \omega_c})$ and hence, the equivalent restitution coefficient is $e = \sin(\omega_d t_{\max})\exp(-\xi_c \omega_c t_{\max})$. The previous expression given to compute the coefficient of restitution e is not only linked to damping coefficient but also to the masses of the colliding bodies, the stiffness parameter, the damped and natural frequency, and the approach time for maximum deformation $t_{\max}$ which leads to a level of complexity during the computation process. However, no numerical experiments have been given to investigate the performance of the proposed impact Kelvin element (Muthukumar and DesRoches, 2006).

In the linear viscoelastic model, the damper is activated during the whole period of collision, i.e. the approach period and the restitution period. The pounding force F(t) follows the relation (Anagnostopoulos, 1998)

$$\begin{aligned} &F(t) = k_k \delta(t) + c_k \dot{\delta}(t); \qquad && \delta(t) > 0 \\ &F(t) = 0; && \delta(t) \le 0 \end{aligned} \tag{1.1}$$

where $\delta(t), \dot{\delta}(t), k_k$ and c_k denote the deformation of the colliding bodies, the relative velocity, linear spring stiffness and the damping coefficient, which in turn can be related to the linear spring stiffness, the colliding bodies masses m_1 and m_2, and the damping ratio ξ through (Anagnostopoulos, 1998, 2004)

$$c_k = 2\xi\sqrt{k_k \frac{m_1 m_2}{m_1 + m_2}}\ , \quad \xi = -\frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}} \tag{1.2}$$

This paper aims to eliminate the major shortcoming of the linear viscoelastic model, i.e. the sticky tensile force just before separation of the colliding bodies. With this in mind, two steps are used. The first is neglecting the minor energy loss during the restitution period, i.e. the damper is activated only during the approach period of collision. The second one is reassessing the relation between the damping ratio and the coefficient of restitution. To verify the performance of the modified linear viscoelastic model, both the linear viscoelastic model and its modified version are used to simulate the pounding-involved structural response for the following numerical experiment. In the first numerical experiments, we use the results of two impact experiments. The second numerical experiments applies a suite of thirty ground motion records from thirteen different earthquakes with different peak ground acceleration (PGA) levels applied to pounding between two single degree of freedom (SDOF) building systems with varying period ratios.

2. METHODOLOGY

2.1. Force Reformulation

The damper of the modified linear viscoelastic model is activated only during the approach period of collision in order to simulate the process of energy dissipation which takes place mainly during that period (see (Goldsmith, 1960)). The pounding force can be expressed as

$$\begin{aligned} &F(t) = k_k \delta(t) + c_k \dot{\delta}(t); \qquad && \delta(t) > 0, \qquad && \dot{\delta}(t) > 0 \qquad && \text{(approach period)} \\ &F(t) = k_k \delta(t) && \delta(t) > 0, && \dot{\delta}(t) \le 0 && \text{(restitution period)} \\ &F(t) = 0; && \delta(t) \le 0 \end{aligned} \tag{2.1}$$

2.2. Derivation of the Relations Relating ξ and e

The relation between the damping ratio and the coefficient of restitution in Eqn. 1.2 is no longer valid due to the activation of the damper in the approach stage only. A reassessment of the relation between the damping ratio and the coefficient of restitution based on this reformulation of the pounding force can be derived as follows:
Let v_i be the approach velocities of the two bodies 1, 2 respectively. The loss in kinetic energies before and after impact can be expressed in terms of the coefficient of restitution e and the relative approach velocity $\dot{\delta}_o$ as (Goldsmith, 1960)

$$\Delta E = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1-e^2)(\dot{\delta}_o)^2 \tag{2.2}$$

where $\dot{\delta}_o = v_1 - v_2$.
In the modified linear viscoelastic model the damper is activated only during the approach period of collision, the dissipated energy by the damper follows the relation

$$\Delta E = \int_0^{\delta_{\max}} c_k \dot{\delta} \quad d\delta = 2\xi\sqrt{k_k\left(\frac{m_1 m_2}{m_1 + m_2}\right)}\int_0^{\delta_{\max}} \dot{\delta} \quad d\delta \tag{2.3}$$

where $\dot{\delta}$ and $\delta_{\max}$ denote the relative velocity and the maximum relative displacement between the colliding bodies during the approach period ($\dot{\delta} > 0$), respectively. An expression for the relative velocity $\dot{\delta}$, during the approach period, in terms of the relative displacement δ has to be obtained to evaluate the integral in Eqn. 2.3. For simplicity, we first obtain a formula for the relative velocity $\dot{\delta}$ during the restitution period, which is considered as elastic period, and later, based on the assumed approximating functions in (Jankowski, 2006), the relative velocity $\dot{\delta}$, during the approach period, can be obtained in terms of the relative displacement δ. Equating the accumulated elastic strain energy at the beginning of the restitution period (i.e. at the point of maximum deformation, $\delta_{\max}$) with the kinetic energy at the time of separation

$$\int_0^{\delta_{\max}} F d\delta = \int_0^{\delta_{\max}} k_k \delta \quad d\delta = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\dot{\delta}_f)^2 \tag{2.4}$$

where $\dot{\delta}_f$ is the final velocity. The solution of the Eqn. 2.4, for $\delta_{\max}$ yields

$$\delta_{\max} = \left(\frac{m_1 m_2 (\dot{\delta}_f)^2}{(m_1 + m_2)k_k}\right)^{1/2} \tag{2.5}$$

The relative velocity $\dot{\delta}$ can be related to the relative displacement $\delta \in (0, \delta_{\max})$ in the restitution period as follows

$$\int_0^{\delta} k_k \delta \quad d\delta + \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\dot{\delta}^2 = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(\dot{\delta}_f)^2 \tag{2.6}$$

Solving Eqn. 2.6, yields

$$\dot{\delta} = \sqrt{(\dot{\delta}_f)^2 - k_k\left(\frac{m_1 + m_2}{m_1 m_2}\right)\delta^2} \tag{2.7}$$

Taking into consideration Eqn. 2.7, and assuming the validation of the formulation $e = \frac{|\dot{\delta}_f|}{\dot{\delta}_o}$ for defining the relation between the approaching and rebounding relative velocities for $\delta = 0$, as well as all other values of deformation during contact, $\delta \in (0, \delta_{\max})$, the formula for the relative velocity, $\dot{\delta}$, during the approach period ($\dot{\delta} > 0$) can be expressed as

$$\dot{\delta} = \frac{1}{e}\sqrt{(\dot{\delta}_f)^2 - k_k\left(\frac{m_1 + m_2}{m_1 m_2}\right)\delta^2} \tag{2.8}$$

After substituting Eqn. 2.8, into Eqn. 2.3, we obtain

$$\Delta E = 2\frac{\xi}{e}\sqrt{k_k\left(\frac{m_1 m_2}{m_1 + m_2}\right)}\int_0^{\delta_{\max}}\sqrt{(\dot{\delta}_f)^2 - k_k\left(\frac{m_1 + m_2}{m_1 m_2}\right)\delta^2}\,d\delta \tag{2.9}$$

Deriving the formula for $\left(\dot{\delta}_f\right)^2$ from Eqn. 2.5, and substituting it into Eqn. 2.9, and simplifying the resulting expression leads to

$$\Delta E = 2\frac{\xi k_k}{e}\int_0^{\delta_{\max}}\sqrt{\delta_{\max}^2 - \delta^2}\,d\delta \tag{2.10}$$

By integrating Eqn. 2.10, the loss in kinetic energy can be written as.

$$\Delta E = \frac{\pi}{2}\frac{\xi k_k \delta_{\max}^2}{e} \tag{2.11}$$

Equating Eqn. 2.11, and Eqn. 2.2 yields

$$\frac{\pi}{2}\frac{\xi k_k \delta_{\max}^2}{e} = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - e^2)(\dot{\delta}_o)^2 \tag{2.12}$$

Substituting Eqn. 2.5, into Eqn. 2.12, and solving for ξ gives

$$\xi = \frac{e(1 - e^2)}{\pi}\frac{(\dot{\delta}_o)^2}{(\dot{\delta}_f)^2} \tag{2.13}$$

Substituting the formula $e = \frac{|\dot{\delta}_f|}{\dot{\delta}_o}$ into the Eqn. 2.13, allows us to describe the relation between the coefficient of restitution and the damping coefficient for the modified linear viscoelastic model according to the formula

$$\xi = \frac{1}{\pi}\frac{1 - e^2}{e} \tag{2.14}$$

3. NUMERICAL EXPERIMENTS

We assess the performance of the modified linear viscoelastic model in capturing pounding compared with the linear viscoelastic model, through two different procedures of comparison. The first procedure is based on two impact experiments conducted for different types of structural members with various materials and contact surface geometries. The accuracy of each model is assessed by calculating the normalized error (*N.E.*) to indicate the difference between the experimental and numerical results (Jankowski, 2005)

$$N.E = \frac{\|F - \overline{F}\|}{\|F\|}.100\% \tag{3.1}$$

where F is the response time history obtained experimentally, $\overline{F}$ is the response time history obtained numerically, and $\|.\|$ is the Euclidean norm. The second procedure is based on simulation using thirty ground motion records from thirteen different earthquakes of peak ground acceleration (PGA) levels ranging from 0.1 to 1 (Muthukumar, 2003; Muthukumar and DesRoches, 2006). The two single degree of freedom (SDOF) systems of structures shown in Figure 1 with equal masses and three different period ratios are subjected to the ground motion records. Two cases are considered. In the first case, the initial separation distance is chosen such that pounding occurs. The performance of the models is evaluated by comparing the numerically obtained displacement and acceleration amplifications to each other.

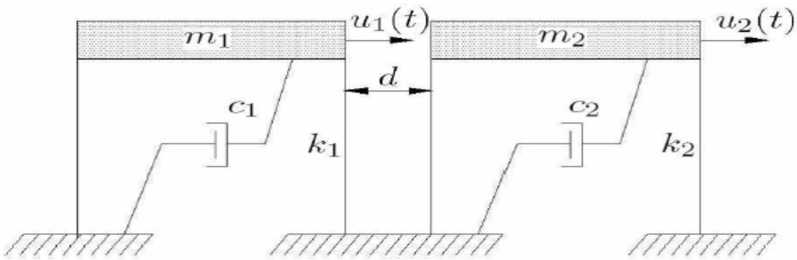

Figure 1 Model idealization of adjacent structures

3.1. COMPARISON BASED ON CONDUCTED IMPACT EXPERIMENTS

3.1.1. Steel-to-steel impact

Goland et al. (1955) carried out an experiment to measure load time histories and strain propagation in a square beam of different dimensions subjected to sharp lateral impacts by letting a steel ball with diameter ranging from $\frac{1}{8}$ inch to $\frac{9}{32}$ inch, drop onto the top of the beam from a specific height. A force gauge was used to measure the force-time history exerted by the ball on the beam. The strain gauges were placed at different locations on

the beam to record the strain time histories at those locations. The dynamic equation of motion for pounding between a ball of mass m_1 dropping onto a beam can be written by drawing the free body diagram (Jankowski, 2005) as shown in Figure 2

$$m_1\ddot{u}_1(t) + F(t) = m_1 g \tag{3.3}$$

where $\ddot{u}_1(t)$, g, and $F(t)$ denote the acceleration, gravitational acceleration, and the pounding force respectively. The pounding force $F(t)$ follows the relation (1.1), for the linear viscoelastic model and relation (2.1), for the modified linear viscoelastic model.. In the experiment the maximum pounding force was 80.7 N when a ball of diameter $\frac{5}{32}$ inch fell from a height of 2 inches (Goland et al. 1955). As for the stiffness parameter in the linear viscoelastic model, we set k_k = 2.08 x 10^7 N/m$^{3/2}$ which was determined through an iterative procedure in order to keep the maximum pounding force in the numerical analysis and experiment to the same (see (Jankowski, 2005)). The same procedure is used to obtain the stiffness parameter of the modified linear viscoelastic model, which is found to be k_k = 2.10 x 10^7 N/m$^{3/2}$. For the two models, the coefficient of restitution e = 0.6 is used.

The results from the numerical analysis and the experiment are shown in Figure 3. Using eqn. 3.1, the normalized errors for pounding force histories are found to be equal to 14.95% for the linear viscoelastic model and 21.65% for the modified version

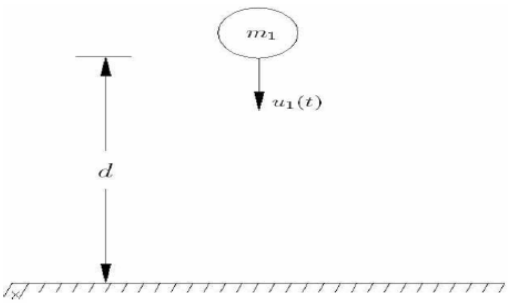

Figure 2 Free body diagram of a ball dropping onto a beam

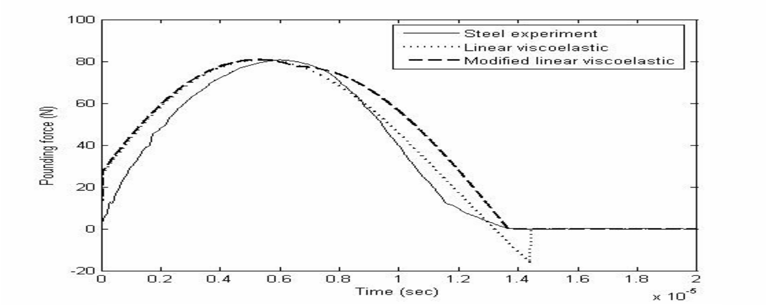

Figure 3 Pounding force time histories during impact between falling ball and a beam

3.1.2. Concrete-to-concrete impact

Van Mier et al. (1991) carried out an experiment on collisions between a prestressed concrete pile and a concrete striker. The dynamic equation of motion for pounding between a striker of mass m_1 and a prestressed fixed pile can be written by drawing the free body diagram (Jankowski, 2005) as shown in Figure 4

$$m_1\ddot{u}_1(t) + \frac{m_1 g}{l} u_1(t) + F(t) = 0 \qquad (3.4)$$

The pounding force *F*(t) follows the relation (1.1), for the linear viscoelastic model and relation (2.1), for the modified linear viscoelastic model. The stiffness parameter used for the linear viscoelastic model is k_k = 9.29 x 10^7 N/m$^{3/2}$. For the modified linear viscoelastic model, k_k = 9.35 x 10^7 N/m$^{3/2}$ was found through an iterative procedure in order to keep the maximum pounding force in the numerical analysis equals the maximum pounding force of the experiment as 102.5 N. For the two models, the coefficient of restitution is e = 0.6. The results from the numerical computations and the experiment are shown in Figure 5. It is found that the normalized error equals 23.89% for the linear viscoelastic model and 32.61% for the modified linear viscoelastic model.

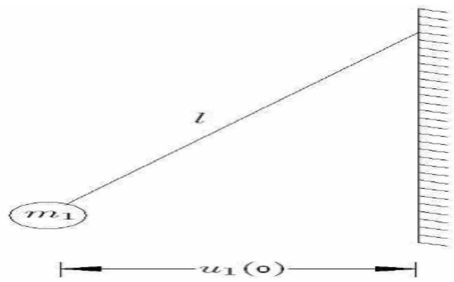

Figure 4 Free body diagram for pounding between a concrete pendulum and prestressed concrete pile

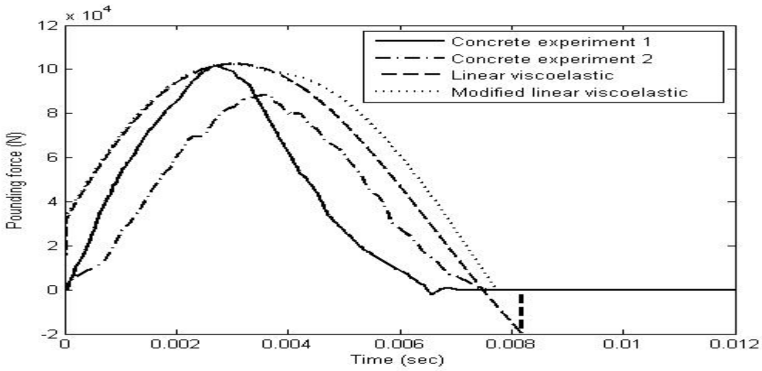

Figure 5 Pounding force time histories during impact between a concrete pendulum and prestressed concrete pile

3.2. COMPARISON BASED ON GROUND MOTION RECORDS

Muthukumar (2003) and Muthukumar and DesRoches (2006) assessed the performance of the Hertzdamp model by comparing it to the linear spring, Kelvin, Hertz, and stereomechanical models using two single degree of freedom (SDOF) building systems shown in Figure 1. For i = 1, 2, let m_i be the masses, c_i be the viscous damping coefficients, and k_i be the stiffness for SDOF 1 and SDOF 2, accordingly. The coupling equation of motion for two adjacent buildings subjected to horizontal ground motion $\ddot{u}_g(t)$ has the following form

$$\begin{aligned} m_1\ddot{u}_1(t) + c_1\dot{u}_1(t) + k_1u_1(t) + F(t) &= -m_1\ddot{u}_g(t) \\ m_2\ddot{u}_2(t) + c_2\dot{u}_2(t) + k_2u_2(t) - F(t) &= -m_2\ddot{u}_g(t) \end{aligned} \tag{3.5}$$

where $u_i(t), \dot{u}_i(t),$ and $\ddot{u}_i(t)$ represent the displacement, velocity and acceleration of the system respectively. The pounding force $F(t)$ follows the relation (1.1), for linear viscoelastic model and relation (2.1). for modified linear viscoelastic model. The values of structural stiffness and damping coefficients: k_i, c_i can be calculated from the formulas (Harris and Piersol, 2002)

$$k_i = \frac{4\pi^2 m_i}{T_i^2}; \ c_i = 2\xi_i\sqrt{k_i m_i} \tag{3.6}$$

where $T_i, \xi_i (i = 1,2)$ denote the natural structural vibration period and structural damping, respectively. The systems were subjected to a suite of thirty ground motion records (Muthukumar, 2003) from thirteen different earthquakes with peak ground accelerations (PGA) levels varying from 0.1 to 1. Three ground motion records were used at each PGA level with the following parameters $m_1 = m_2 =$ 7.8 kip-s^2 /in, $\xi_1 = \xi_2 =$ 5% three different period ratios of 0.3, 0.5, and 0.7, and the models impact stiffness parameter k_k = 25000 kip-in$^{-3/2}$ with coefficient of restitution e=0.6 (see (Muthukumar, 2003)). The performance of the two models is investigated in the process of studying buildings pounding. As suggested in (Muthukumar and DesRoches, 2006), two cases are considered. First is a no pounding case where the gap distance d is set to be large enough to avoid pounding. In the second case, the gap distance d is chosen to be small enough to have pounding. The ratio of the maximum responses obtained in second case and the first case, called the amplification response, is used in the analysis. We solve the equation of motion (3.5), for the thirty ground motion records and the three different period ratios. The obtained amplification factor for displacement and acceleration of both SDOF 1 and SDOF 2 indicates that the two models provide similar displacement, velocity and acceleration amplifications for the whole PGA levels and the different period ratios considered herein, due to the space limitations, only numerical results for period ratio of 0.3 presented in Figure 4.

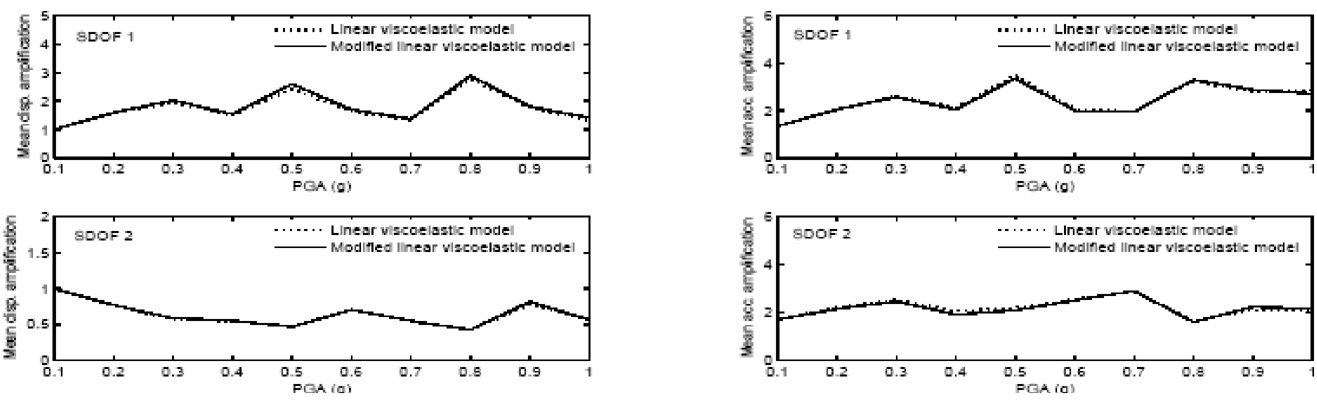

Figure 4 Mean displacement and acceleration amplification for elastic system $T_1/T_2 = 0.3$

CONCLUSIONS

The elimination of the sticky tensile force of the linear viscoelastic model has been conducted in this paper. An improved version of the linear viscoelastic model which overcomes the disadvantage of the tension force appearing just before separation in the linear viscoelastic model is introduced. The used technique is based on neglecting the minor energy loss during the restitution period by activating the damper only during the approach period of collision in order to simulate the energy dissipation which takes place mainly during this period. In addition, a relation relating the damping ratio and the coefficient of restitution has been derived due to the disregard of the minor energy loss. The validity of the proposed modified model with respect to the linear

viscoelastic model has been assessed through conducted impact experiments as well as pounding between two single degree of freedom systems with different period ratios subjected to thirty ground motion records of different PGA levels.
The normalized errors obtained using both the linear viscoelastic model and the modified linear viscoelastic model, in the impact force time histories, show a small difference between the two models.
The results of further analysis indicate that both the linear viscoelastic and the modified linear viscoelastic models provide similar displacement, velocity, and acceleration amplifications of the response of colliding SDOF building systems for three different period ratios and different PGA levels of ground motions.

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