MODAL PARAMETER IDENTIFICATION OF STRUCTURE MODEL USING SHAKING TABLE TEST DATA

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ABSTRACT :

The objective of the paper is to present a method for modal parameter identification of structure model by using simulated earthquake response data in the shaking table experiment. An adaptive on-line system identification method is introduced to investigate whether the time-varying response occurs or not during the process of vibration. The acceleration response time history only in time-invariant stage is adopted to identify modal parameters by off-line system identification method. To show the availability and accuracy, the method is applied to shaking table test data of a 12-story RC-frame building model (scale 1:10) to obtain its modal frequencies, damping ratios and mode shapes after every test, and the results are compared with the modal analysis results. It is shown that the dynamic characteristics can be evaluated from the shaking table test data. The results from the earthquake response time history can be as the supplement of the modal analysis results.

KEYWORDS: Shaking table test, Modal parameter, System identification, Transfer-function

1. INTRODUCTION

The dynamic parameters of the structure model in the shaking table experiment are usually obtained by the modal analysis, and white-noise time histories with low amplitude are required as the input for the purpose. Many methods have been developed for the modal analysis in the last several decades, and almost all of them are based on the time-invariant and linear stability signal processing. For structure shaking table test, the modal analysis of structure model is based on the white-noise input and output response during the shaking table test, because all the signals are linear and stationary, and it is very easy to obtain the modal parameters by using the existing method. In fact, the main target of modal analysis is to determine the dynamic properties or damage state of the building model after several strong motion data input, but in most cases modal analysis is carried out after several strong motion input not after every strong motion input, which means the property of the model can't be obtained after every strong motion input. However, it is necessary and valuable to know the dynamic properties and state of the model after each strong motion input. In recent years, some researchers attempt to identify the structural parameters by using the seismic response data for real structures. Loh and Lin (1996) analyzed the dynamic characteristics of a seven-story reinforced concrete building during four earthquakes, and the modal frequencies and damping ratios were determined in their paper, and the time-varying properties of the parameters were also detected by using the online identification method. Sanli and Çelebi (2001) studied the dynamic characteristics of a 13-story building during four earthquakes, and the natural frequency and normalized mode shape of the building were identified using the transfer functions between different channels of each earthquake record, then the damage was detected by the variation of the modal parameters. Lin and Betti (2004) identified the time-varying structural parameters for damage detection purposes, and the least-square (Kalman Filter) based identification algorithm was adopted in their study. The authors (Gong & Xie, 2005) had presented a method to identify the time-invariant and time-varying modal parameters including the frequencies and damping ratios based on earthquake response data of a 7-story reinforced concrete building which suffered moderate damage in Chi-Chi Earthquake.

In the paper, the simulated earthquake response data of structure model during shaking table test are adopted to identify the modal parameters such as frequency, damping ratio and mode shape. From the point of system identification, it is useful to detect the system is time-invariant or time-varying. Herein, an adaptive on-line system identification method (Andersson, 1985), the adaptive forgetting through multiple models, is introduced to investigate whether the time-varying response occurred or not during the process of vibration, and then the whole acceleration response time history is divided into three segments according to the adaptive system identification result if time-varying response occurs. Only the time-invariant third segment data is used to identify the modal parameters by using the off-line system identification method of ARX (Auto-Regression with eXogenous variables) model (Ljung, 1999) to determine the dynamic properties of structural model after one strong motion input. On the contrary, the whole response time history is adopted to identify the modal parameters if the response is time-invariant. The method is applied to the shaking table test data of a 12-story RC-frame structure model (Lu et al, 2003) to identify its modal frequencies, damping ratios and mode shapes, and it is shown that the dynamic properties can be easily estimated. The results identified from the simulated seismic response data can be as the useful supplements of modal analysis in structural model experiment to check the variation of modal parameters and the structural state after strong motion input during the experiment.

2. SYSTEM IDENTIFICATION METHOD

Determining the dynamic properties of a structural system from the response data is well known as system identification, and many kinds of methods have been developed for the purpose during the last several decades. In the study, to identify the dynamic properties of shaking table structure model based on output seismic response data, the ARX which is off-line system identification method in discrete-time domain is adopted. The ARX model can be simply described as below:

$$A(q)y(t) = B(q)u(t-nk) + e(t) \tag{2.1}$$

where $A(q) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_{na} q^{-na}$ and $B(q) = b_1 + b_2 q^{-1} + \cdots + b_{nb} q^{-nb+1}$, *na* and *nb* are the order of the system output *y*(*t*) and input *u*(*t*). In other words, the numbers *na* and *nb* are the orders of the respective polynomials, and *nk* is the number of delays from input to output and *e*(*t*) is the disturbance. The solution of Eqn. 2.1 can be estimated by the linear least square method (Ljung, 1999) as below:

$$\hat{\theta}_N = \left[\sum_{t=1}^{N} \varphi(t)\varphi^{\mathrm{T}}(t)\right]^{-1} \sum_{t=1}^{N} \varphi(t)y(t) \tag{2.2}$$

where $\hat{\theta}_N$ is the estimated value of θ, $\theta = [a_1 \ \cdots \ a_{na} \ b_1 \ \cdots \ b_{nb}]^{\mathrm{T}}$ is the vector of system parameter, and $\varphi(t) = [-y(t-1) \ \cdots \ -y(t-n) \ u(t-nk) \ \cdots \ u(t-nk-nb+1)]^{\mathrm{T}}$ is the vector including input and output data of the system. The transfer function can be easily obtained from the estimated system parameters $\hat{\theta}_N$ as followed:

$$G(q) = \frac{B(q)}{A(q)} \tag{2.3}$$

Eqn. 2.3 should be transferred into frequency domain in order to get the modal parameters. The *r*-order modal frequency and damping ratio can be obtained by the following Eqn. 2.4 and Eqn. 2.5.

$$\omega_r = \frac{1}{\Delta t}\sqrt{\ln Z_r \ln Z_r^*} \tag{2.4}$$

$$\xi_r = \frac{-\ln(Z_r Z_r^*)}{2\sqrt{\ln Z_r \ln Z_r^*}} \tag{2.5}$$

where (Z_r, Z_r^*) is the r-th discrete complex conjugate eigenvalue pair and Δt is the sampling period.

The ARX method is a kind of off-line system identification model, and it is suitable for the time-invariant system. However, it is necessary to track the time-varying phenomena of structural model under the excitation of simulated earthquake strong motion during the shaking table test. Herein, a recursive identification method presented by Anderson (1985) is adopted to track the time-varying phenomenon of the structural model. The method can effectively track, detect and capture any abrupt changes in the system with jumping or rapidly changing parameters because of a failure was combined for time-piecewise fitting to get the exact time and the parameter values. It can be considered as a particular way of implementing adaptive gains or adaptive forgetting factors for recursive identification. For a time-discrete system with jumping parameters, the state space model can be written as:

$$\left.\begin{aligned} \theta(t+1) &= \theta(t) + w(t) \\ y(t) &= \varphi^{\mathrm{T}}(t)\theta(t) + e(t) \end{aligned}\right\} \tag{2.6}$$

where $\theta(t)$ is an n-dimensional vector containing the true parameters describing the system at time t, $\varphi(t)$ is a vector containing old inputs and outputs as mentioned before. $w(t)$ and $e(t)$ are disturbances. The best estimate of $\theta(t)$ can be given by a special case of Kalman filter if the $w(t)$ is assumed to satisfy some special conditions (Ljung, 1999). In order to track the time-varying system clearly, Andersson (1985) presented an algorithm to estimate the system parameter $\theta(t)$ as shown in Eqn. 2.7:

$$\hat{\theta}(t) = \sum_{i=1}^{M} \alpha_i(t)\bar{\theta}_i(t) \tag{2.7}$$

The more detailed description about the method can be found in the paper (Anderson, 1985). In fact, the method consists of multiple recursive least-square algorithms running in parallel, each with a corresponding weighting factor. The method had been used by Loh et al (1996) to track the time-varying of some actual structure under the excitation of earthquake motion load.

3. STRUCTURE MODEL

A 12-storey reinforced concrete frame model (scale 1:10) was tested at large-scale earthquake simulator facility of the State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University. The properties of the model, sensor locations and orientations of test system are shown in Figure 1 from which we can know the height of each story is 0.3m, total height is 3.6m, and floor plan is 0.6m*0.6m. Strong ground motion for input of the test were El Centro wave, Kobe wave, Shanghai artificial wave and bed rock wave, and the input level (PGA) were increased step by step until the structure was totally damaged in the end. White-noise excitation test was also carried out in the progress of the test in order to obtain vibration characteristics of the structure after and before one group of strong motion input related to fortification intensity. Installed sensors were accelerometers, strain gauges, and the accelerometers were installed in order to record the response time-history of the excitation direction. More detailed information of the model, test progress and test results can be found in the experiment report (Lu et al, 2003). The simulated earthquake response data under the excitation of El Centro wave with different PGA are used for the analysis in the paper.

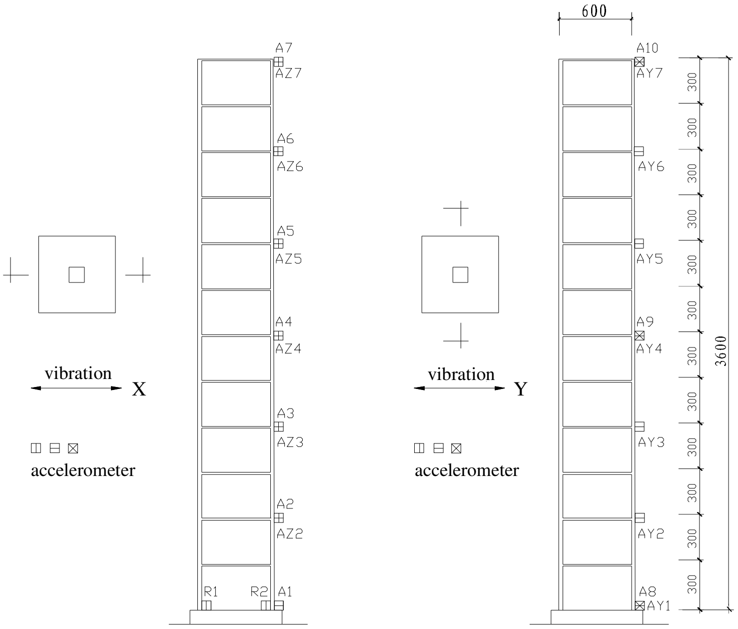


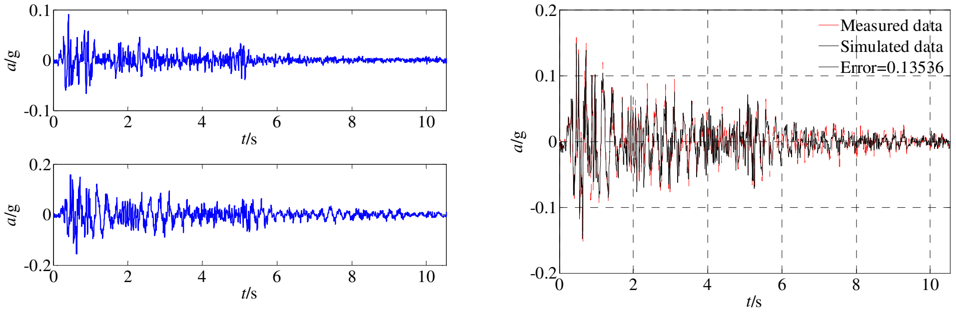
Figure1 Sensor locations and orientations of test system and dimensions of the model

4. MODAL PARAMETER IDENTIFICATION

The modal parameters were identified based on the output seismic response by using the method mentioned in part 2 of the paper. First, the recursive identification method (Anderson, 1985), adaptive forgetting through multiple models, was used to analyze if the response was time-varying or not. Then, the whole response time histories were used to identify the modal parameters if the system was time-invariant. On the contrary, the whole response time-histories were divided into three segments and only the third segment was used for the analysis during which the structure was with time-invariant response. The acceleration response of the roof (12F) and the response of the base were considered as output and input signal respectively because the ARX is single-input and single-output (SISO) system identification model, and the parameters of the X-direction was analyzed in the paper. The model was tested for 62 times totally including 8 white noise inputs. It was found from the test phenomenon that the structure was not damaged during the first 9 inputs, and the slight cracks were found on some beams of the fourth floor after the 10th input. Furthermore, the obvious damage was found after the 16th, 18th, and 21st input, and the structure was almost totally damaged after the 62nd input when the test was finished. The authors had introduced some results in other paper (Gong & Xie, 2007), and herein, only some parts of the results are included in the paper because of the paper length limitation.

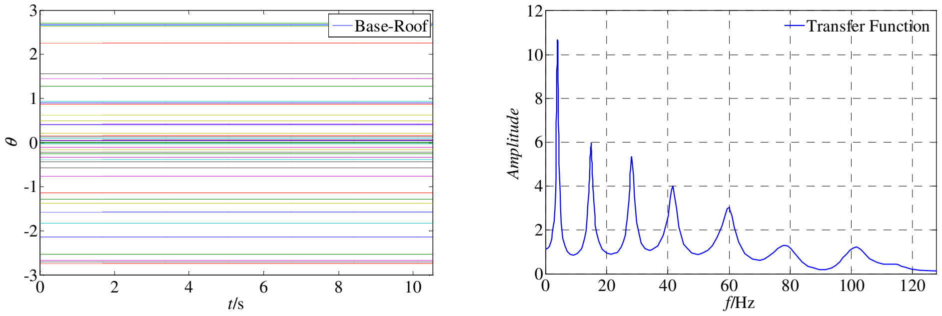
4.1. Time-invariant Response

The analysis shows that there are no time-varying responses during the first 9 inputs, and the results accord with the test phenomena. The results of the 2nd test which is first strong motion input (El Centro strong motion with PGA=0.09g) as an example are shown in Figure 2. It can be concluded that the system parameter θ is constant during the input as shown in Figure 2(b), and the system is time-invariant. The whole response time-history is used to identify the modal parameters, and the comparison of identified data and measured data is shown in Figure 2(c) from which we can see that the error is very small. The transfer function is shown in Figure 2(d) from which the modal frequencies can be identified. The first 3 modal frequencies of the structure model are 4.01Hz, 14.91Hz, and 28.19Hz respectively, and the corresponding damping ratios obtained from Eqn. 2.5 are 6.49%, 3.81%, and 2.59% respectively. The results are similar as the results of traditional modal analysis of white-noise input in the 1st test. The first 3 modal frequencies from first modal analysis are 3.99Hz, 14.82Hz, and 28.77Hz respectively. The mode shapes identified from the earthquake response data are shown in Figure 3 in which only the six mode shapes can be obtained because only the response of the 2nd, 4th, 6th, 8th, 10th and 12th story are measured during the test. The comparison between transfer functions is shown in Figure 4. It shows that there is little difference between the two transfer functions.



(a) input (base) and output response (roof)

(c) comparison of simulated and measured data



(b) estimated system parameters

(d) transfer function

Figure 2 Identification results of El Centro wave (PGA=0.09g) input

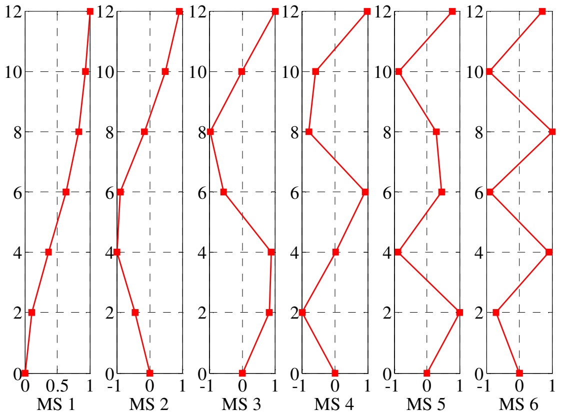


Figure 3 Identified mode shapes

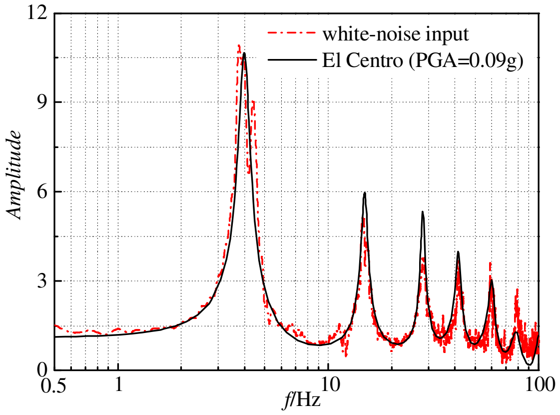
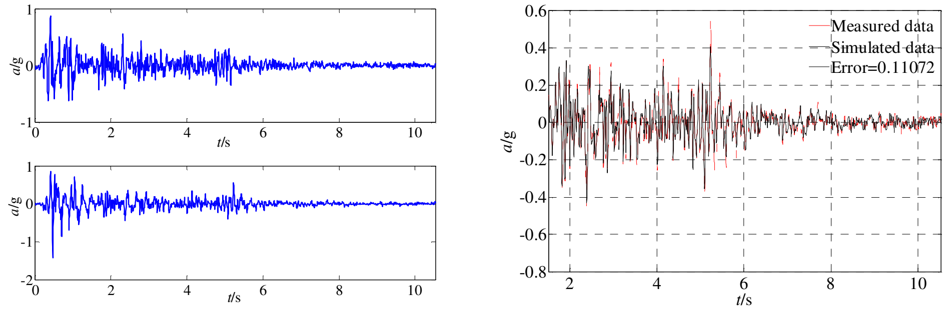


Figure 4 Comparison of transfer function

4.2. Time-varying Response

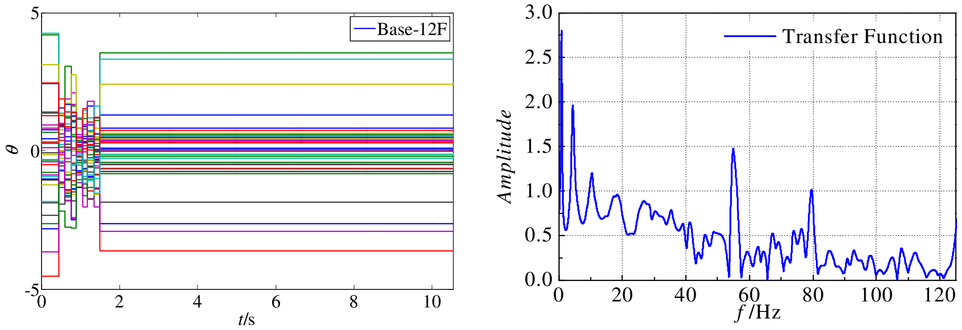
The time-varying analysis results of the 54th test as an example are shown in Figure 5 which is the last El Centro wave input. The white-noise was input for the modal analysis (53rd test) just before this input. It can be concluded that the parameter θ is changed as shown in Figure 5(b), so the system is time-varying, and the parameter is changed from 0.46s to 1.51s during the 54th test. It means that the structure model was damaged during this time range according to the result. However, the parameter was not changed after 1.51s, and the modal parameters were identified by using the data behind 1.51s which was defined as the third segment. The comparison of identified data and measured data is shown in Figure 5(c) from which we can see that the identified result is with high accuracy. The transfer function is shown in Figure 5(d) from which the modal frequencies can be identified. The first 3 modal frequencies of the structure are 0.76Hz, 4.36Hz, and 10.34Hz respectively, and the damping ratios are 18.67%, 11.01%, and 7.90% respectively. Compared with the results of the first strong motion data input, the frequencies decrease very much, on the contrary damping ratios increase very much. The mode shapes determined from third segment of the response data are shown in Figure 6 and the comparison with the results identified from the first strong motion input will be discussed in the next part.

The modal analysis was performed before this strong motion input. The comparison of the transfer functions are shown in Figure 7 from which we can know the frequencies obtained from the third segment data are a little bit smaller than the modal analysis results from the white-noise input. It means the frequencies decrease during the strong motion input and the parameters of the building changed under the excitation of the earthquake load, and the building model was damaged during the input of El Centro wave with PGA 0.904g. The time-varying detection of the system shown in Figure 5(b) is confirmed. As a result, the results identified from the third segment can be considered as the structural parameters just after the earthquake wave input.



(a) input (base) and output response (roof)

(c) comparison of identified and measured data



(b) estimated of system parameters

(d) transfer function

Figure 5 Identification results of El Centro wave (PGA=0.904g) input

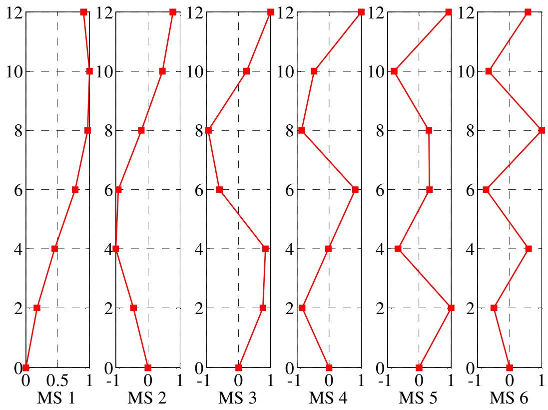


Figure 6 Identified mode shapes

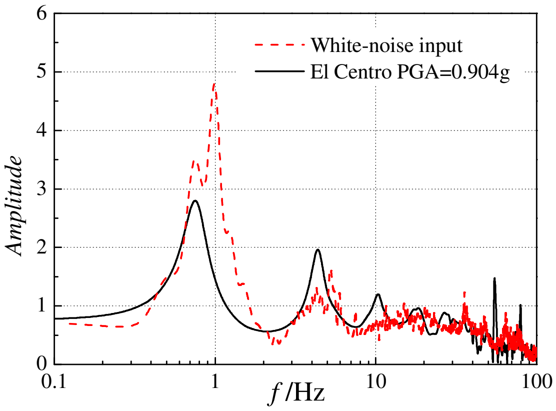


Figure 7 Comparison of transfer function

4.3. Comparison

The results identified from the response data of El Centro wave input with different PGA are compared in Figure 8 from which it can be observed that the modal frequencies decrease with the increasing of PGA. The natural frequency decreases from 4.01Hz to 0.76Hz in Figure 8, and the amplitudes of the transfer functions also decrease with the increasing of PGA. It can be concluded that the structure suffered very severe damage in the process of experiment and the damage was cumulated gradually with the input of simulated earthquake motion. It is also found that the damping ratios increase in the process of experiment according to the identification results. In fact, the experiment phenomenon shows that the structure model is almost totally damaged and becomes an unstable system when the test is finished.

The comparison of the mode shapes identified from the first and last El Centro input is shown in Figure 9 from which we can know that the mode shapes are also changed during the experiment especially the first mode shape. From the comparison of frequencies, damping ratios and the mode shapes determined by the earthquake response data, it can be concluded that the model properties are totally changed after the experiment.

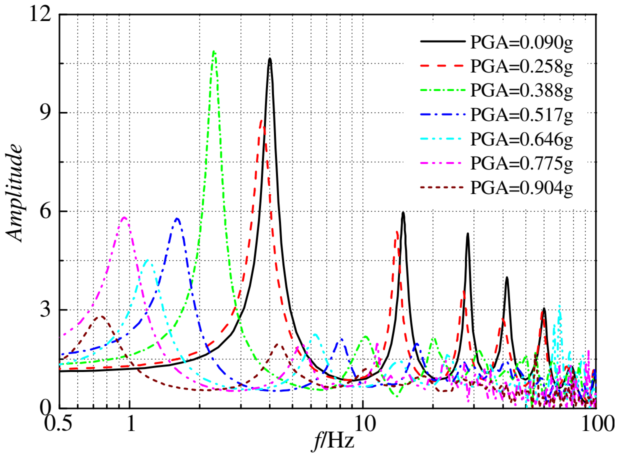


Figure 8 Comparison of transfer functions

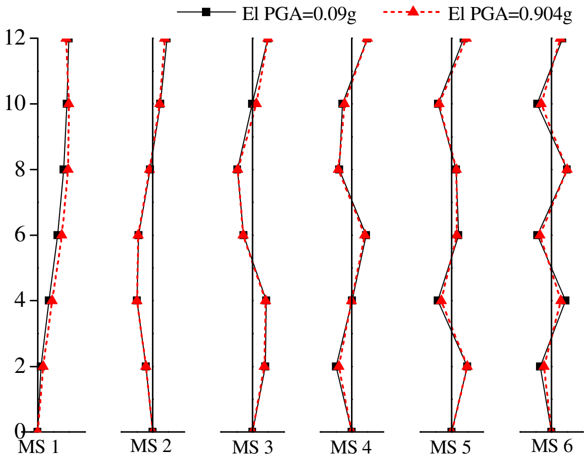


Figure 9 Comparison of mode shapes

5. CONCLUSION

The authors attempt to identify the modal parameters from the simulated earthquake response data of structure model in the shaking table test. The shaking table test data of one 12-story RC frame model are used for the analysis to obtain its modal parameters such as frequencies, damping ratios and mode shapes. Some conclusions are summarized as follows:

(1) The output simulated seismic response data can be analyzed to get the modal parameters, and the results can be the useful complements of modal analysis in structure model experiment to investigate the variation of modal parameters and structure state after earthquake input.
(2) The adaptive forgetting through multiple models identification method could be used to track if the response of structure is time-varying or not, and the third segment not whole response time-history should be used to analyze the modal parameters if the response is time-varying.
(3) The whole response time-history can be adopted to identify the modal parameters if the system is time-invariant.
(4) It is shown that the modal frequencies decrease while the damping ratios increase with the increasing of PGA of strong earthquake motion input.
(5) The damage of structural model is cumulated gradually during the experiment process.

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