New Evaluation of Soil-Structure Interaction Effects on Optimal Control of Structures

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ABSTRACT :

A methodology is developed in this paper to include soil-foundation-structure interaction (SFSI) effects in optimal control algorithm LQG. Analysis of a SFSI system is usually formulated in the frequency domain, whereas the optimal control algorithm LQG in the time domain, hence using an equivalent fixed-base model of structure considering SFSI effects represents an effective approach. A system identification technique is applied to find an equivalent fixed-base system. This equivalent model is applied to combine for the determination of optimal control gain. In this study, two illustrative examples of a five-storey building resting on and in elastic half-space are used to investigate the effectiveness of the developed control algorithm LQG. It is shown that the LQG controller considering SFSI effects is more effective in suppressing the structural response in both embedded and surface footing, but requires more control force when foundation is shallow at surface ground. It is shown that the applied system identification in this study is useful for other analyses of structure considering SFSI effects.

KEYWORDS: SFSI,LQG controller, system identification

1. Introduction

Structural control for earthquake resistant structures may be classified into four categories: active , passive , semi-active and hybrid .Active control requires external power to generate the control forces; passive control forces are generated by mechanical devices and are induced by building motions; hybrid control is a system combining both the active and passive controllers ; semi-active control is a class of active system for which the external energy requirement is much smaller than for an active controller.
In most of the research work, structures are assumed to be fixed at base and the effect of soil-foundation-structure interaction on the performance of control systems is neglected. In practice, however, many structures are built on soft soil and the superstructure. On the other hand, Wolf (1985,1988) and Luco and Wong (1987) have shown that strong soil-foundation-structure interaction significantly modifies the dynamic characteristics of structures such as frequencies , damping , mode shapes , etc. Soil–foundation–structure interaction gives rise to kinematic and inertial effects, resulting in modifications of the dynamic properties of the structure and the characteristics of the ground motion around the foundation. Kinematic interaction due to the seismic wave effects reduces the translation response of the foundation and generates torsional and rocking responses. It represents the modification of the free-field ground motion by the presence of the foundation in absence of the structure.
Smith and Wu (1997) studied the effect of SFSI on the optimal structural control. They examined a 5 story shear building. Structure is supported on shallow rigid footing in soft soil. They followed by a methodology for using system identification techniques to find an equivalent fixed-base model of a MDOF SFSI system. They concluded that when soil is very soft, control algorithm considering SFSI effects is more effective than corresponding control algorithm assuming a fixed-base system model. Luco (1998) have been drawn similar conclusion on the equivalent 1-DOF oscillators which account for the effects of control and SFSI.
Recent study by Zhang et al (2006) led to same conclusion. In this study, authors are used inertia interaction effects only in SFSI modeling by impedance function items at the fundamental frequency of the SFSI system. Based on analyses of single-story and six-story structures, it is concluded that the intelligent strategy is effective for the hybrid control and SFSI needs to be include in the design of the intelligent hybrid system as well as other

types of control for building on soft soil.
The substructure method is commonly adopted in the SFSI analysis where a mixed boundary-value problem in elastodynamics is first solved to obtain the foundation impedances of soils and then utilized to analyze the structure-foundation model.
The major difficulty in including SFSI in optimal structural control comes from the fact that analysis of an SFSI system is usually formulated in the frequency domain due to the frequency-dependent foundation impedances and foundation input motions, whereas the conventional optimal control problems same as H_2 and LQG and LQR and etc. in the time domain.
The objective of this paper is evaluation of new method on equivalent fixed-base model of a MDOF SFSI system in time domain with a system identification and make use of LQG controller in structural control.

2. Modelling procedure for SFSI

There are two classical methods for modeling SFSI problem. The first is a direct approach, in which a computational method of the full structure, foundation and soil system is set up and excited by a complex and incoherent wave field. This approach is difficult to solve from a computational standpoint, specially when the system contains significant nonlinearities and hence the direct approach is rarely used in practice. In the second approach (referred to as the substructure approach) , the complex SFSI problem is divide into three steps. This three distinct parts are combined to formulate the complete solution. The superposition inherent to this approach requires an assumption of linear soil and structure behavior. The three steps in the analysis are as follows: evaluation of a foundation input motion (FIM) and determination of the impedance function and dynamic analysis of the structure supported on a compliant base represented by the impedance function and subjected to a base excitation consisting of the FIM.
The effects of both kinematic (transfer function that represent the ratio of foundation and free-field motions in the frequency domain) and inertial interaction are evaluated by using the SFSI system in Fig. 1.

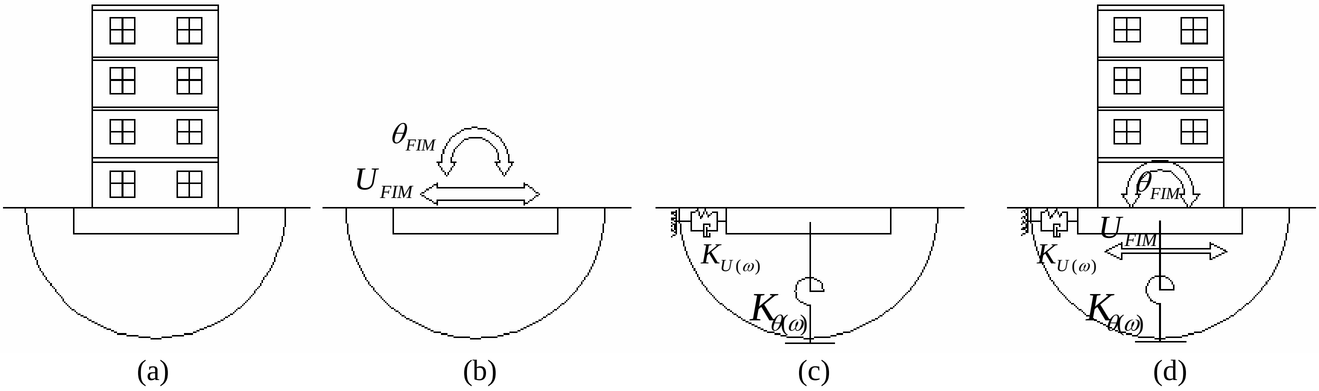


Figure 1 .a)complete system. b) Foundation input motion. c)Impedance functions (on rigid foundation). d)Dynamic analysis

The most complex and time-consuming task in analysis SFSI are the calculations of the impedance functions and the input motions for the foundation. In fact, the rigorous evaluation of these quantities normally demands the use of sophisticated methods of finite or boundary elements, which require substantial computation effort.
The impedance functions are complex-valued quantities strongly dependent on the excitation frequency. Its real part reflects the stiffness and inertia of the soil, whereas the imaginary part reflects the material and geometrical damping by hystertic behavior and wave radiation within the soil, respectively. The input motions representing the components of the base excitation are also complex-valued quantities dependent on the characteristics of the soil and foundation, as well as on the wave nature of excitation.

3. System Identification

As illustrated schematically in Fig. 2, the objective of system identification analyses is to evaluate the unknown properties of a system using a known input into and output from, that system.

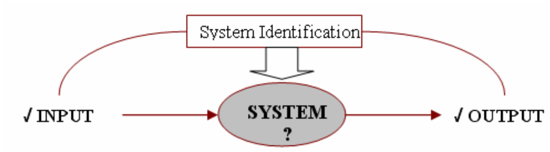


Figure 2 .Schematic of system identification problem

There are many advantages to using system identification techniques. Unlike conventional methods for determining system parameters, which often require a series of different measurement settings, system identification methods can determine all the parameters from a single measurement setting. This implies that all the estimated conditions. As another advantage, conventional methods are often difficult, time-consuming and costly, whereas system identification can be performed quickly, easily and inexpensively.
Analysis of a SFSI system is usually formulated in the frequency domain, whereas the optimal control algorithm LQG in the time domain, hence using an equivalent fixed-base model of structure considering SFSI effects represents an effective approach. In this paper, a structure considering SFSI effects is analyzed then displacement and velocity of stories (state vector of system) is obtained by earthquake excitation and soil structure interaction formulation. State vector and state matrix of system are named output from and input into system, respectively, hence from Eqn 3.1, applied lateral forces are determined.

$$\begin{cases} q_{(k+1)} = A.q_{(k)} + B'.F_{(k)} \\ y_k = C.q_k \end{cases} \tag{3.1}$$

$$B' = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}_{2n\times n}, F_{(k)} = -M.I.\ddot{x}_{g(k)} \tag{3.2}$$

$$\begin{cases} q_{(k+1)} = A.q_{(k)} + G_{(k)}.\ddot{x}_{g(k)} \\ y_k = C.q_k \end{cases} \tag{3.3}$$

In the above equations, C is measurement matrices associated with displacement and velocity, respectively and A , B' are state matrices of structure and y is output vector of plant or system. $q_{(k)} = [x \quad \dot{x}]^T$ is state vector of structure and $F_{(k)}$ is lateral force vector.
This approach has focused on lateral force coefficient ($G_{(k)}$) definition.

4. Control Design in Time domain and LQG Controller

The equation of motion of a MDOF structural system in a structural vibration control system design problem is often written in following general form:

$$M\ddot{x} + C\dot{x} + Kx = DU \tag{4.1}$$

In the above equation, M and C and K are mass and damping and stiffness matrices, respectively and $\ddot{x}, \dot{x}, x$ are acceleration and velocity and displacement vector of structure. $D \in \mathbb{R}^{n\times p}$ is a constant matrix specifying the locations of p applied loads, which could be external forces or control forces denoted by a vectors $f \in \mathbb{R}^p$. For controller design purpose, the above equation is further transformed to state space form as follows,

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & \mathrm{I} \\ -\mathrm{M}^{-1}\mathrm{K} & -\mathrm{M}^{-1}\mathrm{C} \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} 0 \\ \mathrm{M}^{-1}\mathrm{D} \end{bmatrix} U \tag{4.2.a}$$

$$y = [C_{\mathrm{x}} \quad \mathrm{C}_{\dot{\mathrm{x}}}] \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \tag{4.2.b}$$

$$U = -K_C.x \tag{4.2.c}$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix}, q = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} \tag{4.2.d}$$

$U_{(k)} \in \mathbb{R}^{n\times1}$ is control force vector and K_c is gain matrix. q is state vector of system. Transforming the above equation into discrete state space form as follow,

$$\begin{cases} q_{(k+1)} = A.q_{(k)} + B.U_{(k)} \\ y_k = C.q_k \end{cases} \tag{4.3}$$

In structural vibration control, a controller is usually used to help a structure to achieve certain precise positioning or tracking requirements. These requirements should be satisfied for the structure with natural frequencies within the controller bandwidth and within the disturbance spectra. A Linear Quadratic Gaussian (LQG) controller can meet these requirements and is often used for tracking and disturbance rejection purposes.
In this paper, state space model of equivalent fixed-base system is in discrete form with time-variant state matrices hence, general form of this type equation is defining Eqn. 4.4.

$$\begin{cases} q_{(k+1)} = A.q_{(k)} + B.U_{(k)} + G_{(k)}.w_{(k)} \\ y_{(k)} = C.q_{(k)} + \upsilon_{(k)} \end{cases} \tag{4.4}$$

$$k=1,2,3,\ldots i,\ldots, \ T/\Delta t$$

In the above equation, $w_{(k)} \in \mathbb{R}^{1\times1}$ is applied free-field motion and $G_{(k)}$ is identified vector from system identification. T is time duration of applied motion and Δt is simple time of motion.
The controller is driven by the plant output y. The controller produces the control signal U that drives the plant. This signal is proportional to the plant estimated state denoted $\hat{x}$, and the gain between the state and the controlled signal U is the controller gain (K_c). We use the estimated state $\hat{x}$ rather than the actual state x, since typically the latter is not available from measurements. The estimated state is obtained from estimator, which is part of the controller, as shown in Fig. 3.

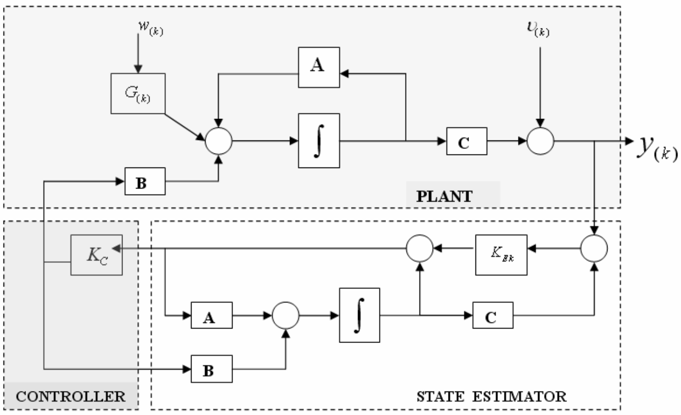


Figure 3. The inner structure of the LQG closed-loop system.

The estimator equations follow from the block-digram in Fig. 3:

$$\hat{x}_{(k+1|k+1)} = \hat{x}_{(k+1|k)} + K_{E(k+1)}[y_{(k+1)} - C.\hat{x}_{(k+1|k)}] \tag{4.5}$$

where

$$\hat{x}_{(k+1|k)} = A.\hat{x}_{(k|k)} + B.U_{(k)} \tag{4.6}$$

In Eqn. (4.5) $K_{E(k+1)}$ is the optimal estimator gain matrix given by solution of the filter (or estimator) algebraic Riccati equation.

$$K_{E(k+1)} = P_{(k+1|k)}.C^T.[C.P_{(k+1|k)}C^T + V]^{-1} \tag{4.7}$$

where

$$P_{(k+1|k)} = A.P_{(k|k)}.A^T + G_{(k)}.W.G^T{}_{(k)} \tag{4.8}$$

and

$$P_{(k+1|k+1)} = [I - K_{E(k+1)}.C].P_{(k+1|k)} \tag{4.9}$$

In the above equations, P is the solution of the estimator algebraic Riccati equation. The noise $\upsilon_{(k)}$, called measurement noise, has covariance $V = E(\upsilon.\upsilon^T)$, the noise $w_{(k)}$ is called free-field motion, and its covariance is $W = E(w.w^T)$.Both noises are uncorrelated. In the above equations, the controller gain (K_c) and estimator gain ($K_{E(k)}$) are unknown quantities. We determine these gains such that the performance index, J,

$$J = \sum_{k=0}^{N} [(C.\hat{x}_{(k|k)})^T.Q.(C.\hat{x}_{(k|k)}) + (U_{(k)})^T.R.(U_{(k)})] \tag{4.10}$$

is minimized. In the above equation, R is a positive definite input weight matrix and Q is a positive semidefinite stete weight matrix. It is well known that the minimum of J is obtained for the feedback

$$U = -K_c.\hat{x}_{(k|k)} \quad 4.11)$$

$$K_c = [B^T.P_{(k+1)}.B + R]^{-1}.B^T.P_{(k+1)}.A \tag{4.12}$$

where $P_{(k+1)}$ is defined by the Riccati equation

$$P_{(k)} = A^T.P_{(k+1)}.A - A^T.P_{(k+1)}.B.[B^T.P_{(k+1)}.B + R]^{-1}.B^T.P_{(k+1)}.A + C^T.Q.C \tag{4.13}$$

5. Numerical Examples

5.1. Five-story building with shallow foundation at ground surface subjected to earthquake with and without SFSI

A five-story building is employed in this study with all parameters for the structural model, foundation and soil. Table 5.1 summarizes all the parameters.

Table 5.1. System parameters for numerical examples

Model	Parameter	Value	Unit
Structure	Floor Mass (M_i)	9100	kg
	Column Stiffness (K_i)	10000	kN/m
	Foundation-to-Storey Height	2	m
	Floor Moment of Inertia(J_i)	10000	$kg.m^2$
	Damping	2%	
Foundation	Half-side length (B)	1.8	m
	Mass (M_0)	11400	kg
	Moment of Inertia (J_0)	25000	$kg.m^2$
Soil	Mass density	1700	kg/m^3
	Poisson's Ratio	1/3	
	Shear Velocity	180	m/s

As a first step, the computation of SFSI state vector is conducted.In this example ,complex foundation impedance functions investigated by Wong and Luco (1987) is used. Displacement of 5th floor of structure for both embedded and shallow foundations is plotted in Fig. 5. In this example, kinematic interaction is excluded by shallow foundation.

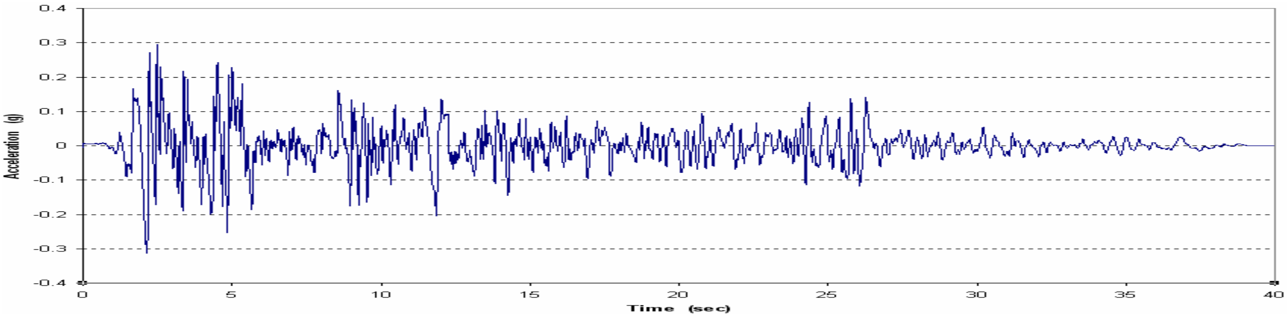
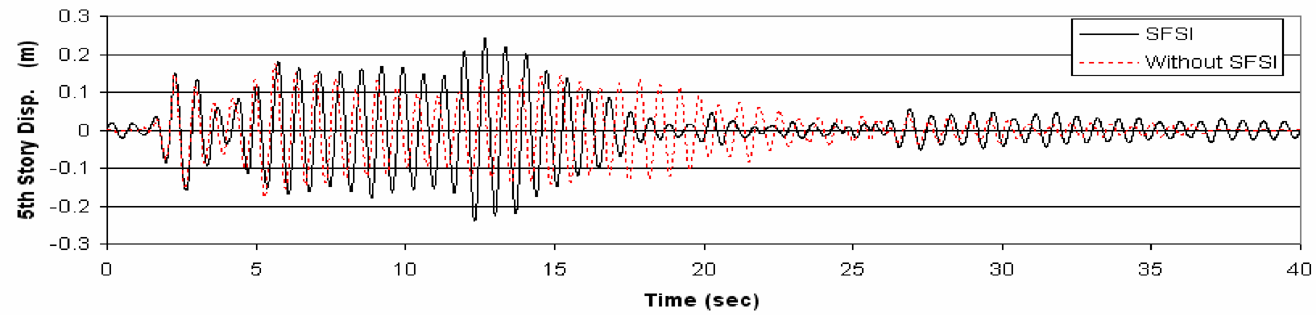
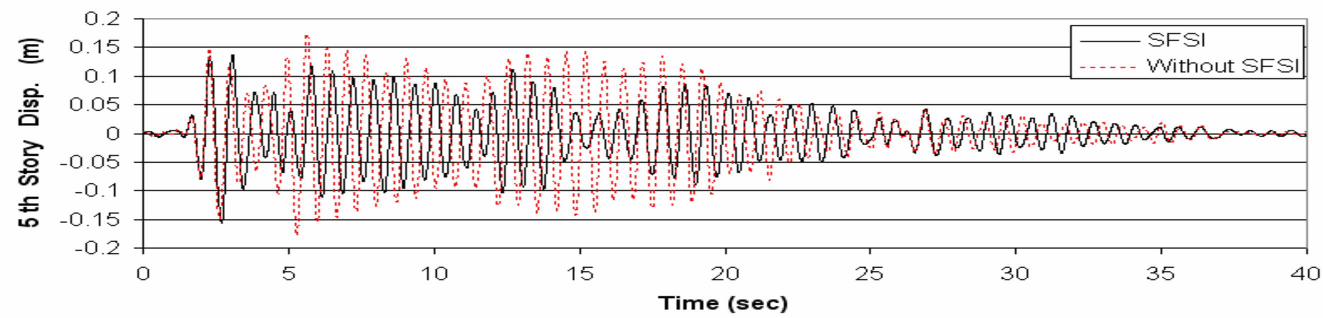


Figure 4. Acceleration time history, free-field motion on site categories, D, from 117 station, ELcentro Earthquake, 1940

Ground motion data from the 1940 Elcentro Earthquake (PGA=0.313 g) (Fig.4)was used in all studies. The ground motion recording was obtained from a database provided from PEER on the site categories D (in NEHRP provisions).



a)Shallow foundation at ground surface



b)Embedded foundation ($e/B = 1$)

Figure 5. Comparison of response time history of 5th floor.

Optimal control algorithm is based on the minimization of a quadratic performance index. External control forces are assumed to be applied on the total roots of the structures. For simplicity, the weighting matrices for the quadratic performance index are chosen as

$$Q = \beta \begin{pmatrix} K & 0 \\ 0 & M \end{pmatrix}, R = I_5 \qquad (5.1)$$

where I_5 is a 5×5 identity matrix and the coefficient β determines the relative importance of minimizing control energy versus maximizing control effectiveness. Smaller values of β indicate that minimization of control energy is considered more important, whereas larger values of β refer to an increased importance of control effectiveness. In this study, the coefficient $\beta = 10^6$ is chosen.

In Figures 5, 6, the time history of the displacement and control force 5th story are plotted.

It is seem that in the without SFSI case, control forces result in a greater reduction of the structural response than with SFSI, and this forces are smaller than with SFSI case.

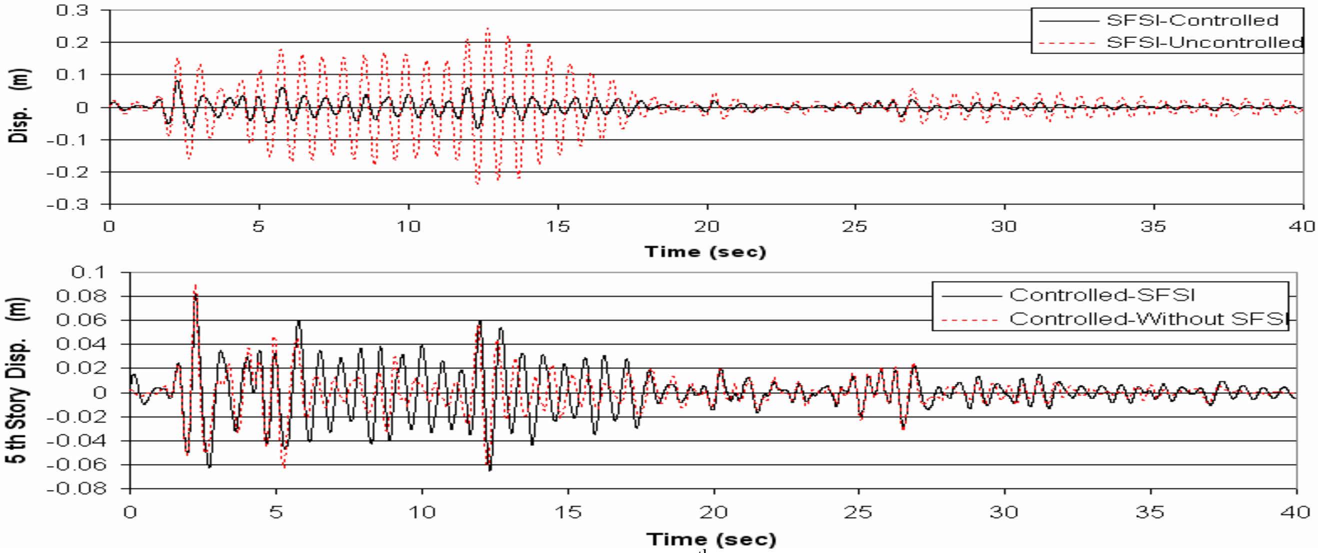


Figure 5. Comparison of response time history of 5th floor, Shallow foundation at ground surface

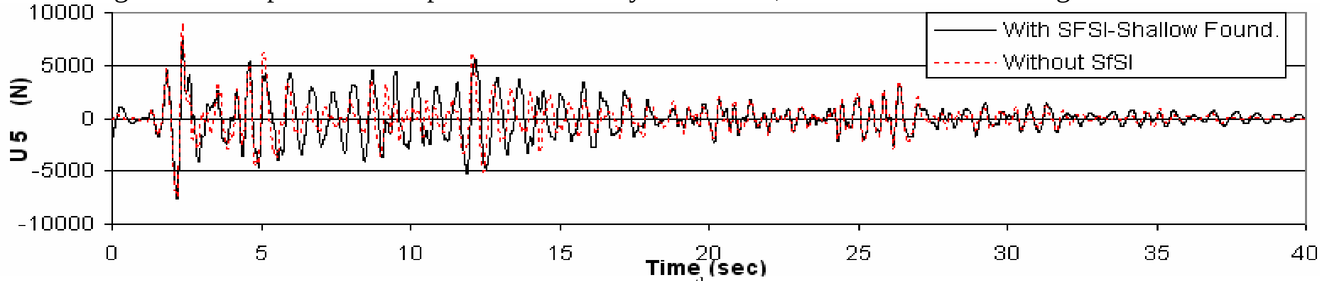


Figure 6. Comparison of control forces applied in 5th story, Shallow foundation at ground surface

5.2. Five-story building with embedded foundation subjected to earthquake with and without SFSI

All parameters of the structural model, foundation, and soil are illustrated in Table 5.1. Embedded depth of foundation, in this study is chosen 5 m (e / B =1). Impedance of embedded foundations differs from that of shallow foundations in several important ways. For this example, impedance functions of embedded foundation investigated by Luco and Wong is used. For Embedded foundations, the dominant mechanism affecting base slab motions are embedded effects associated with ground motion reductions that occur below the original ground surface. Foundation input motions for an incident wave field consisting vertically propagating, coherent SH waves, investigated by Apsel and Luco (1987) is used. In Figures 7, 8, the time history of the displacement and control force of 5th story are plotted.

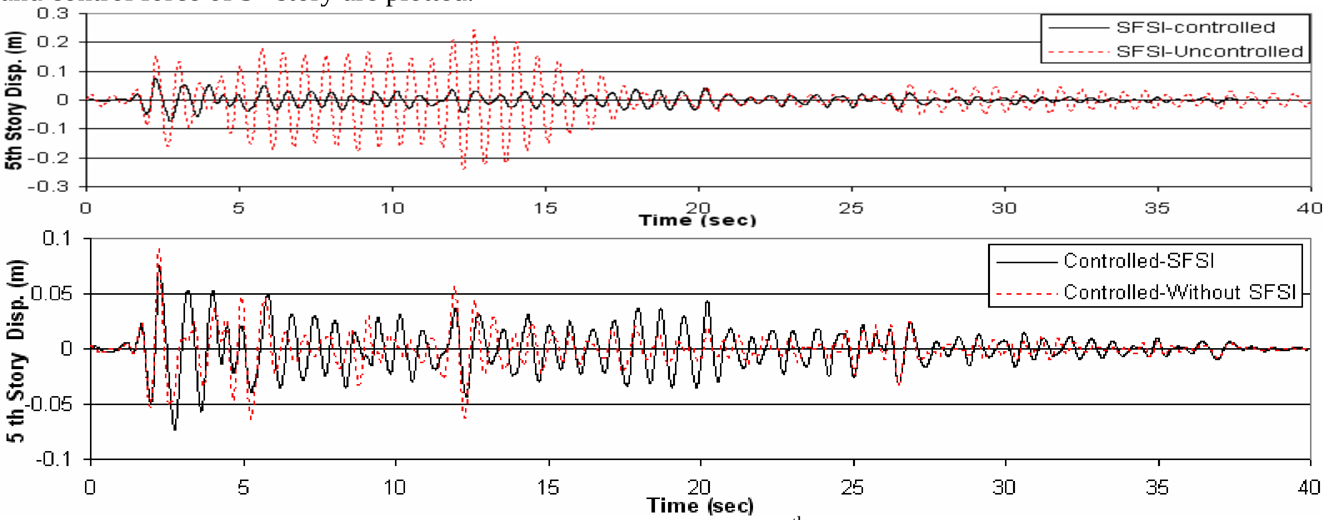


Figure 7. Comparison of response time history of 5th floor,Embedded foundation

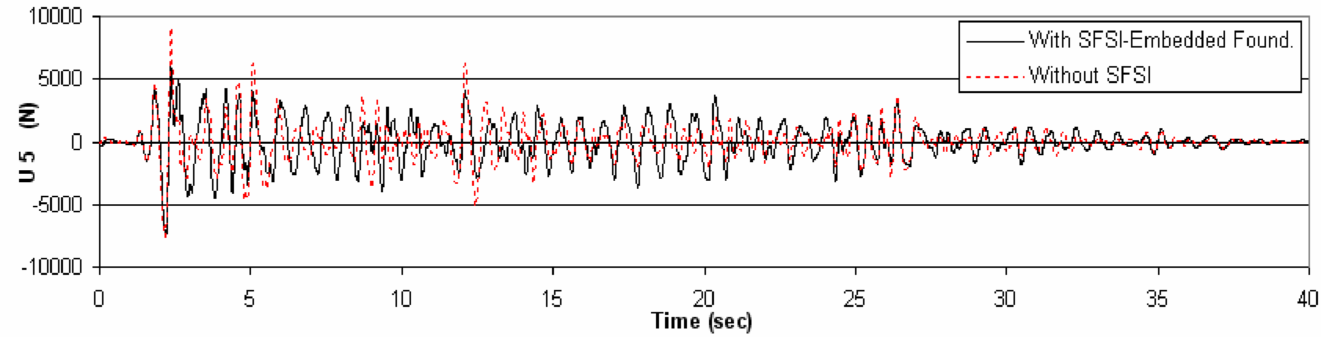


Figure 8. Comparison of control forces, Embedded foundation

In this case, it is seem that in the without SFSI case, control forces result in a greater reduction of the structural response than with SFSI, and this forces are greater than with SFSI case.

6. Conclusion

Dynamic soil-foundation-structure interaction under earthquake loads is a complicated phenomenon. Unless the fundamental frequency of the structure is near that of its supporting soil strata, SFSI generally results in a reduction of the structural deformation and shear force at base.

In this study, it is shown that assuming fixed-base for a structure with shallow foundation is not conservative, but for a structure with embedded foundation is conservative.

This study has focused on the evaluation of LQG controller considering Soil-Foundation-Structure Interaction effects. An equivalent fixed-base system, with modified lateral force of stories and the same mass and damping and stiffness of structure is shown to adequately approximate the actual SFSI system with excellent accuracy.

Two illustrative examples of a five-storey building resting on and in elastic half-space are used to investigate the effectiveness of the developed control algorithm LQG. It is shown that the LQG controller considering SFSI effects is more effective in suppressing the structural response in both embedded and surface footing but requires more control force when foundation is shallow at surface ground. It is shown that the applied system identification in this study is useful for other analyses of structure considering SFSI effects.

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