CALCULATION ON DEFORMATION RATIO OF SHEAR TO BENDING FOR STEEL REINFORCED CONCRETE SPECIAL-SHAPED COLUMNS

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ABSTRACT:

Elastic stiffness should be calculated while developing a restoring force model of steel reinforced concrete (SRC) special-shaped column. Whether it is needed to take the shear deformation into account due to the irregular section of specially-shaped columns and its small shear span ratio was seldom studied in the literature. To this end, it is studied theoretically based on the mechanic model which can simulate real engineering using the classical mechanic theory. The results show that while the shear span ratio is equal to1, shear deformation is about 30%~50% of total deformation. While the shear span ratio is equal to1.5, the ratio is 20%~30%. While the shear span ratio is equal to2.5, the ratio is more than10%.

KEYWORDS: steel reinforced concrete, special-shaped columns, spring stiffness, shear deformation

1. INTRODUCTION

Steel reinforced concrete (SRC) special-shaped column is a new-type special-shaped column, which mainly contains steel with right amount longitudinal bars and stirrups in simple special-shaped section (L, T, + shape) [Chen Z.P., Zhang X.D. (2006)]. For one side, it keeps some advantages of ordinary special-shaped column, such as no corner angle, nice-looking, practical and increasing the real using areas; for another side, it inherits the advantages of SRC structure, such as high bearing capacity, good seismic behavior and deformability[Zhao H.T.(2001)]. It is an effective approach using SRC special-shaped column to overcome the weakness of ordinary special-shaped column whose bearing capacity is not high and seismic behavior is not good[Chen Z.P., Zhao H.T. (2006)].

In order to study the whole process reaction and failure mechanism of the frame construction with SRC special-shaped column under earthquake action, the restoring model of SRC special-shaped column must be determined, and the accuracy of analysis result of seismic response depends on the precision of the restoring model to a certain degree. While SRC special-shaped column restoring model is established, the calculation of elastic stiffness must be referred to[Shi J.(2000), Xue J.Y.(2000)]. Because the special-shaped column is not regular in section, whether the shearing deformation needs to be considered or not while calculating elastic stiffness, it is still a gap to now at home and abroad. In view of this, this paper does some work in theory for the first time, and offers some important reference data for the establishment of SRC special-shaped column restoring model.

2. CALCULATION THEORY ON DEFORMATION RATIO OF SHEAR TO BENDING FOR SRC SPECIAL-SHAPED COLUMNS

The deformation of SRC special-shaped column includes bending, shearing and axial deformation under external load. The axial deformation is very small, generally can be ignored. And the shearing deformation is also very small and can be ignored too, which is mentioned in classical material mechanics[Timoshenko S.(1978)] when the ratio of shear span is large comparatively. As the special-shaped column is mainly applied to the residential buildings, the height of floor is not large generally, and the SRC special-shaped column's leg has the characteristic of "long and narrow" therefore the ratio of shear span is not large. So whether the shearing deformation needs to be considered or not, in view of this, it is necessary to carry on the deep research in theory.

Supposing the inflection point is at the middle of the span for the SRC special-shaped column whose length is l and the following formula can be obtained.

Shearing deformation:

$$\delta_s = \frac{kPl}{GA} \tag{2.1}$$

Where, GA is shear stiffness; P is transverse shear force; l is the length of column; k is the nonuniform coefficient of shear stress distributing along the section.

$$k = \frac{A}{I^2}\int_A \frac{S^2}{b^2} dA \tag{2.2}$$

Where, A = the section area; I =the moment of inertia; b = the section width of the location where shear stress is requested; S =the area moment to the neutral axis z of the section above or below the location requested (Fig. 1).

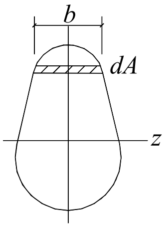

Fig.1 shape of cross section

Flexural deformation:

$$\delta_b = \frac{Pl^3}{12EI} \tag{2.3}$$

Where, EI = flexural stiffness; the other symbols have the same meaning as above.
The equation (2.1) Compare with (2.3):

$$\eta = \frac{\delta_S}{\delta_b} = \frac{12k(EI)}{(GA)l^2} \tag{2.4}$$

In order to calculate conveniently it can be simplified as the following:

$$\frac{EI}{GA} = \beta \frac{E_c I_c}{G_c A_c} \tag{2.5}$$

For the special-shaped column with kinds of layout steel the value of β changes from 1.03 to 1.1, while the steel ratio varies from 3% to 15%; according to the calculation results and referring to the Japan Code, the stiffness of steel section is less than the stiffness of reinforced concrete section for the ordinary SRC structure, so it can be approximately considered as the following:

$$\frac{EI}{GA} \approx \frac{E_c I_c}{G_c A_c} \tag{2.6}$$

Where, $E_c / G_c = 2(1+\mu)$, μ is Poisson ratio, and usually it is 0.2 for concrete.

Through the formula (2.2), (2.3), (2.6), the following equation can be obtained:

$$\eta = \frac{\delta_s}{\delta_b} = \frac{28.8\xi}{I \times l^2} \tag{2.7}$$

Where, $\xi = \int_A \frac{S^2}{b^2} dA$; Other symbols have the same meaning as above.

3. CALCULATION METHOD ON DEFORMATION RATIO OF SHEAR TO BENDING FOR SRC SPECIAL-SHAPED COLUMNS

Because special-shaped column section is not regular, both the nonuniform coefficient of shear stress k and the moment of inertia I are related to loading direction. In actual construction, the structure with special-shaped column may suffer from every direction's action. In this paper it is mainly researched along engineering axis, 45° and 135° direction, and every special-shaped column's leg has the same height, as Fig.2 shows.

For L and + section only one situation ((1), (4) in Fig.2) can be considered because of symmetry when loading along engineering axis direction. For T section two situations can be considered along wing ((2) in Fig.2) and web ((3) in Fig.2). When loading along 45° direction only one situation ((5), (6), (7) in Fig.2) of +, T and L section can be consider. The number (8) is only for L shape along 135° direction.

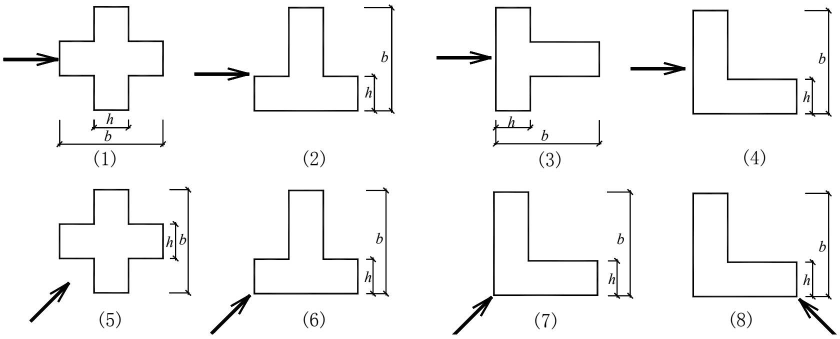

Fig.2 The section of special –shaped columns and the loading direction

Take + shape section as an example for the situation of loading along engineering axis direction, as Fig.3 shows.

When $h/2 \le y \le b/2$,

$$I_{z1} = \int_{h/2}^{b/2} y^2 h dy\,,\quad \xi_1 = \int_{h/2}^{b/2} \frac{1}{h^2}\{(b/2-y)h[y+1/2(b/2-y)]\}^2 h dy$$

When $0 \le y \le h/2$,

$$I_{z2} = \int_{0}^{h/2} y^2 b dy \quad \xi_2 = \int_{0}^{h/2} \frac{1}{b^2}\{1/2(b-h)h[h/2+1/4(b-h)] + b(h/2-y)[y+1/2(h/2-y)]\}^2 b dy$$

$$I_z = 2(I_{z1} + I_{z2}) \tag{3.1}$$

$$\xi = 2(\xi_1 + \xi_2) \tag{3.2}$$

$$l = 2\lambda h_0 \tag{3.3}$$

Where, λ = ratio of shear span; h_0 = effective depth of section;

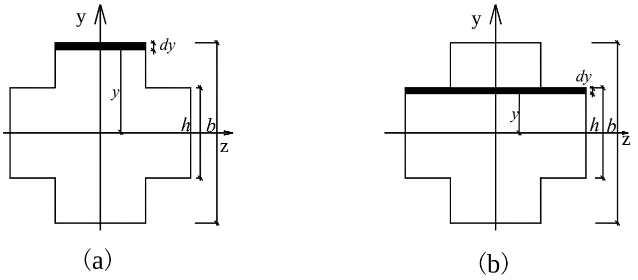

Fig.3 The figure of computation

Taking equation (3.1), (3.2), (3.3) into (2.7), the ratio of shearing deformation to bending deformation can be calculated. Similarly, other sections loading along the engineering axis direction can be calculated.

4. ANALYSIS OF THE CALCULATION RESULTS

In this paper, all special-shaped columns have the same section height of column leg; 200mm and 240mm commonly used in actual project are chosen as the section thickness of column leg. And 2.0, 2.5, 3.0, 3.5 and 4.0 are chosen as the ratio of section height to section thickness of column leg; 1.0, 1.5, 2.0 and 2.5 are chosen as the ratio of shear span λ. The calculation results are shown in Table1 to Table6. The calculation results is totally the same for L shape column loading along the engineering axis direction and for T shape column loading along the web direction, they share the Table1 together. Similarly, The calculation results is also the same for + shape column loading along the engineering axis direction and for T shape column loading along the flange direction, they share the Table2 together. Fig.4 to Fig.9 show the relationship among the deformable ratio of shear and bending, ratio of shear span and ratio of section height to section thickness of column leg for L shape with 200mm section thickness loading along the direction of engineering axis, 45° and 135°.

Table 1 the ratio of deformation of shear and bending in T shape loading along the flange

h(m)	b/h	I_z ($10^{-2}\times m^4$)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.1200	1.0917	0.491	0.218	0.123	0.079	0.055
0.2	2.5	0.2283	1.1154	0.458	0.204	0.114	0.073	0.051
0.2	3.0	0.3867	1.1772	0.455	0.202	0.115	0.073	0.051
0.2	3.5	0.605	1.2527	0.464	0.206	0.116	0.074	0.052
0.2	4.0	0.8933	1.3297	0.477	0.212	0.119	0.076	0.053

Table 2 the ratio of deformation of shear and bending in T shape loading along the web

h(m)	b/h	I_z ($10^{-2}\times m^4$)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.1467	1.2793	0.704	0.313	0.176	0.113	0.078
0.2	2.5	0.3127	1.4463	0.814	0.362	0.204	0.130	0.090
0.2	3.0	0.5787	1.5908	0.921	0.409	0.230	0.147	0.102
0.2	3.5	0.9696	1.7036	1.011	0.449	0.253	0.162	0.112
0.2	4.0	1.5105	1.7902	1.086	0.483	0.272	0.174	0.121

Table 3 the ratio of deformation of shear and bending in L shape loading along the direction of 45°

h(m)	b/h	I_z ($10^{-2}\times m^4$)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.0933	1.0766	0.377	0.167	0.094	0.060	0.042
0.2	2.5	0.1721	1.0723	0.332	0.148	0.083	0.053	0.037
0.2	3.0	0.2907	1.0722	0.312	0.138	0.078	0.050	0.035
0.2	3.5	0.4592	1.0795	0.303	0.135	0.076	0.049	0.034
0.2	4.0	0.6876	1.0903	0.301	0.134	0.075	0.048	0.033

Table 4 the ratio of deformation of shear and bending in L shape loading along the direction of 135°

h(m)	b/h	I_z ($10^{-2}\times m^4$)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.1999	1.1129	0.835	0.371	0.209	0.134	0.093
0.2	2.5	0.4531	1.1423	0.931	0.414	0.233	0.149	0.103
0.2	3.0	0.8661	1.1598	1.005	0.447	0.251	0.161	0.112
0.2	3.5	1.4791	1.1708	1.060	0.471	0.265	0.170	0.118
0.2	4.0	2.3319	1.1781	1.104	0.491	0.276	0.177	0.123

Table 5 the ratio of deformation of shear and bending in T shape loading along the direction of 45°

h(m)	b/h	I_z (10^{-2}×m^4)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.1333	1.1191	0.559	0.248	0.140	0.089	0.062
0.2	2.5	0.2705	1.1149	0.543	0.241	0.136	0.087	0.060
0.2	3.0	0.4987	1.5409	0.768	0.341	0.192	0.123	0.085
0.2	3.5	0.6976	1.0403	0.444	0.197	0.111	0.071	0.049
0.2	4.0	1.0782	1.0147	0.440	0.195	0.110	0.070	0.049

Table 6 the ratio of deformation of shear and bending in cross shape loading along the direction of 45°

h(m)	b/h	I_z (10^{-2}×m^4)	k	η				
				λ =1.0	λ =1.5	λ =2.0	λ =2.5	λ =3.0
0.2	2.0	0.1758	1.6984	1.120	0.498	0.280	0.179	0.124
0.2	2.5	0.2606	1.4101	0.661	0.294	0.165	0.106	0.073
0.2	3.0	0.3600	1.0527	0.379	0.168	0.095	0.061	0.042
0.2	3.5	0.5783	1.1393	0.403	0.179	0.101	0.065	0.045
0.2	4.0	0.8667	1.1937	0.416	0.185	0.104	0.067	0.046

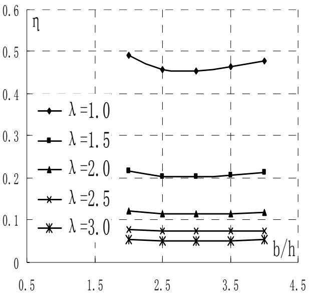

Fig.4 T column deformation ration loading in flange

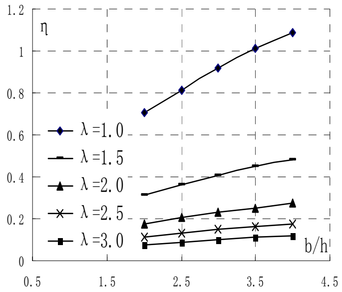

Fig.5 T column deformation ration loading in web

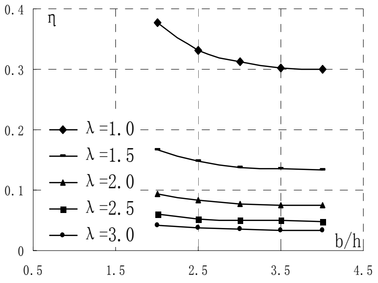

Fig.6 L column deformation ration loading in 45°

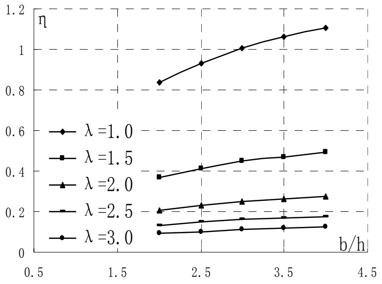

Fig.7 L column deformation ration loading in 135°

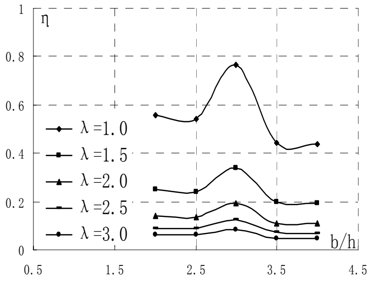

Fig.8 T column deformation ration loading in 45°

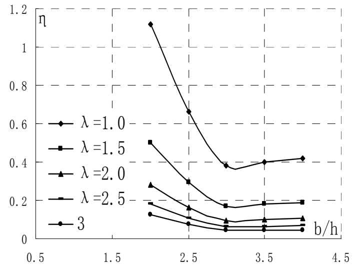

Fig.9 + column deformation ration loading in 45°

From Table1 and Table2, nonuniform coefficient of shear stress is influenced largely by section type and loading direction. The value of k is relatively small for T section loaded along the flange and + sections loaded along the engineering axis; and the value of k is relatively large for T section loaded along the web and L sections loaded along the engineering axis. According to documents, for the first situation, the maximum shearing stress in section reduce (compared with rectangular section under the same situation) because of the existence of flange, and namely shearing stress in section is distributed uniformly, so the value of k is smaller. For the second situation, the existence of flange doesn't play a corresponding role, so the value of k is larger. From the Table3 and Table4, for L section loaded along 45° and 135° direction the value of k is relatively small.

From Table1 to Table6, for the same section and loaded along the same direction, the deformation ratio increases as the value of k increases; from Fig.4 to Fig.9 the deformation ratio decreases and curves change slowly as the shear span ratio increases.

From Table1 and Fig.4, for T section loaded along the flange and + section loaded along engineering axis, when shear span is 1, the deformation ratio is nearly 0.5, namely the shear deformation account for 1/3 of the total deformation; when shear span ratio is 1.5, it is nearly 1/6; and deformation ratio has the tendency of decrease in the beginning then increase later as the ratio of section height to section thickness of column leg increases.

From Table2 and Fig.5, for T section loaded along the web and L section loaded along the engineering axis direction, when shear span ratio is 1, the deformation ratio is more than 0.7, and especial when the ratio of column leg height to column leg thickness is more than 3, the deformation ratio varies from 0.9 to 1.1, namely the shearing deformation accounts for 1/2 of the total deformation; when shear span ratio is 1.5, the deformation ratio is 0.4 and the shear deformation can't be neglected; in addition, the deformation ratio increases as the ratio of column leg height to column leg thickness increases.

From Table3 and Fig.6, for L section loaded along 45° direction the deformation ratio reduces as the ratio of column leg height to column leg thickness increases; when shear span ratio is 1, the deformation ratio is the largest approaching to 0.4.

From Table4 and Fig.7, the deformation ratio increases as the ratio of column leg height to column leg thickness increases for L section loaded along 135° direction. When shear span ratio is 1, the deformation ratio exceeds 0.8 and namely the shearing deformation accounts for 1/2 of the total deformation; when shear span ratio is 1.5, the average of deformation ratio is 0.44, and namely shear deformation accounts for 1/3 of the total deformation, which can't be neglected.

From Table5 and Fig.8, the deformation ratio increases in the beginning the decreases later as the ratio of column leg height to column leg thickness increases for T section loaded along 45° direction; when ratio of column leg height to column leg thickness 1.0, the deformation ratio is approaching 0.8.

From Table6 and Fig.9, the deformation ratio decreases as the ratio of column leg height to column leg thickness increases for + section loaded along the 45° direction; when shear span is 1.0 and ratio of column leg height to column leg thickness 2.0, the deformation ratio exceeds 1.0.

5. CONCLUSION

Through the theoretical analysis, the nonuniform coefficient of shear stress and deformation ratio under different shear span ratio are calculated, and the following conclusions can be obtained.

(1) The value of k decreases because of the existence of web when loading along engineering axis direction and shear stress distribute uniformly.

(2) The value of k is small relatively for L shape section loaded along 45° and 135° direction; and it is large relatively for T and + shape section loaded along 45° and 135° direction.

(3) While the shear span ratio is equal to 1 shear deformation accounts for 30% to 50% of the total deformation; while shear span is 1.5 it is about 20%~30%; while the shear span is 2.0 it is about 10%, so the shear deformation can’t be neglected during theory analysis.

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