

# **A DESIGN METHOD FOR THE SEISMIC UPGRADE OF EXISTING R.C. FRAMES BY BUCKLING RESTRAINED BRACES**

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# **ABSTRACT:**

A large part of existing buildings in the world has been designed without considering seismic provisions. These buildings are generally not adequate to resist seismic force and need to be upgraded. The insertion of buckling restrained braces (BRBs) is believed a promising and innovative technique for the seismic upgrading of existing r.c. frames. These elements provides the r.c. structure with both lateral stiffness, thus reducing damage in reinforced concrete members during medium intensity seismic events, and dissipating capability, thus preventing collapse during strong seismic events. Furthermore, stiffness and strength of a BRB may be defined, practically independently, by choosing appropriate values of cross-section area, length of the yielding segment and yield stress of the steel. This can make the use of BRBs a very powerful and flexible tool for the seismic rehabilitation of a structure, if proper design tools were available. In this paper a method is proposed for the design of BRBs. It aims at obtaining a widespread distribution of the plastic deformation along the height of the building. The parameters which rule the design are determined by the analysis of the seismic response of a six-story r.c. frame, representative of structures designed to resist only gravity loads according to the Italian code of the early seventies.

**KEYWORDS:** buckling restrained braces, existing r.c. frames, seismic upgrading

# **1. INTRODUCTION**

In spite of the seismic activity present in many regions of the world, a large part of the r.c. structures presently in use has been designed without considering seismic provisions. As it is known, main structural elements, columns and beams, are usually disposed along a single direction in r.c. framed structures designed to sustain only gravity loads. This makes these structures very flexible and weak in the orthogonal direction. Furthermore the distributions of the lateral stiffness and shear strength along the height of these frames are not suitable to make the displacement/deformation demand widespread along the height and consistent with the capacity of the frame. All these deficiencies can be overcome by inserting Buckling Restrained Braces (BRBs) within the r.c. frames. For instance, the insertion of BRBs can increase to proper values both the total lateral stiffness and the shear strength of the structure. Furthermore, it can modify the distribution of the shear strength along the height so as to promote a widespread yielding of the structure and therefore a more favorable collapse mechanism during strong ground motions. Finally, it can modify the distribution of the lateral stiffness along the height so that the displacements demanded by the ground motion can better fit the displacement capacity of the structure.

BRBs (Uang and Nakashima, 2004; Watanabe *et al*., 1988; Xie, 2005) can be very effective in the seismic rehabilitation but, unfortunately, seismic codes do not provide provisions for their design. In this paper a design method for the seismic rehabilitation of r.c. frames designed to sustain only gravity loads using BRBs is proposed. According to this method two requirement have to be met by the r.c. frame upgraded by BRBs. The first, which will be called displacement requirement, aims at increasing the lateral stiffness of the structure and at modifying its distribution along the height to reduce the inter-story drift demand to values which are compatible with the capacity of the structure. The second, which will be called strength requirement, aims at increasing the shear strength of the structure and at making its distribution along the height more favorable for seismic purposes. The effectiveness of the proposed method is demonstrated comparing the seismic performance of a sample r.c. frame upgraded by BRBs to that assumed as target in design. Finally the proper values that has to be assigned to the parameters which control the proposed design method are determined.



# **2. PROPOSED DESIGN METHOD**

A proper design method should allow the determination of the properties that have to be provided to the structural members of the structure to attain the most dissipative collapse mechanism and avoid the achievement of the design limit state (corresponding to a given level of damage or, eventually, to the collapse of the structure) in occurrence of the relevant ground motion. Based on this consideration, the design method proposed in this paper allows the determination of the stiffness and the strength of the BRBs, which have to be inserted within the existing r.c. frame to achieve the desired seismic behavior, by two requirements. The first is a displacement requirement and aims at reducing the displacement demand below the value corresponding to the considered limit state. The second is a strength requirement and aims at providing the r.c. frame upgraded by BRBs with sufficient lateral strength which is distributed along the height proportionally to the story shear demanded by the design ground motion. This promotes the simultaneous yielding of the BRBs at all the stories and, therefore, the achievement of the most dissipative collapse mechanism.

#### *2.1. Displacement requirement*

According to the displacement requirement the inter-story drift demanded by the design ground motion has to be not larger than that corresponding to the considered limit state. This requirement allows the determination of the stiffness of BRBs by a procedure that may be resumed in the following steps.

*1. Determination of the displacement demand* 

A reliable evaluation of the displacement demand of the frame due to strong ground motions should be obtained by a nonlinear method able to predict the possible concentration of the inelastic deformation. But, because of the insertion of properly designed BRBs, all the stories will yield almost simultaneously and the story displacement demand will be widespread along the height. Based on this consideration the displacement demand is determined as the inter-story drift ∆*<sup>i</sup>* obtained by the elastic analysis of the structure based on the elastic (unreduced) spectrum of the reference ground motion. In this paper the modal response spectrum analysis is used, but the lateral force method of analysis can be used instead.

*2. Determination of the displacement capacity* 

According to a displacement based approach a selected limit state is achieved when the plastic deformation of one or more structural element attains the corresponding limit value. In this paper the provisions of the European seismic code (EC8, 2004) are adopted. This code stipulates the limit value of chord rotation, i.e. the angle between the tangent to the axis at the yielding end and the chord connecting this end with the point of contraflexure. For instance, it stipulates that the chord rotation capacity of the cross-section θ*u* (chord rotation at yielding θ*y* plus plastic rotation at member failure) has to be used for the Collapse Prevention limit state, while the Life Safety limit state is achieved when a chord rotation equal to θ*y* plus 75% of the plastic rotation at member failure is attained somewhere in the structure. The value of the total chord rotation capacity (elastic plus inelastic part) can be evaluated by the following equation stipulated in the EC8

$$
\theta_u = \frac{1}{\gamma_{el}} 0.016 \left( 0.3^\circ \right) \left[ \frac{\max(0.01; \omega)}{\max(0.01; \omega)} f_c \right]^{0.225} \left( \frac{L_v}{H} \right)^{0.35} \tag{2.1}
$$

where  $\gamma_{el}$  is equal to 1.5 for primary seismic elements and to 1.0 for secondary seismic elements,  $f_c$  is the mean value of the concrete compressive strength, ω and ω´ are the mechanical reinforcement ratio of the tension and compression, respectively, longitudinal reinforcement, ν is the normalized axial force (ratio of the axial force  $N/A<sub>c</sub> f<sub>c</sub>$ , *H* is the depth of cross-section, and  $L<sub>V</sub> = M/V$  is the ratio moment/shear at the end section. In such expression the positive effect of the confinement due to the reinforcements (longitudinal bars and stirrups) has been neglected because it is believed rather ineffective in columns of r.c. existing buildings. The value of the plastic part of the chord rotation capacity (plastic rotation capacity) of concrete members under cyclic loading may be calculated by considering the difference between the value given by Eqn. 2.1 and the values of the yield rotation  $θ$ <sup>*v*</sup> evaluated by the following equation:

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$$
\theta_{y} = \frac{\phi_{y}L_{v}}{3} + 0.0013\left(1 + 1.5\frac{H}{L_{v}}\right) + 0.13\phi_{y}\frac{d_{b}f_{y}}{\sqrt{f_{c}}}
$$
(2.2)

where  $\phi_y$  is the yield curvature of the end section,  $d_b$  is the (mean) diameter of the tension reinforcement and  $f<sub>v</sub>$  is the steel yield stress.

Because in r.c. existing buildings the yielding occurs mostly in the columns and in the BRBs, and because the BRBs are very ductile members, it is assumed that the frame achieves the selected limit state when the plastic rotation of the columns attains the corresponding limit value θ*l*. Therefore, the *i*-th inter-story drift capacity ∆*l,i* is calculated multiplying the smallest limit value of the plastic rotation of the columns of the *i*-th story for the story height (plastic part of ∆*l,i*) and adding the inter-story drift at yielding.

*3. Required stiffness of BRBs* 

If the inter-story drift demand ∆*i* is larger than the inter-story drift capacity ∆*l,i*, BRBs have to be added to increase the lateral stiffness of the frame and reduce the inter-story drift below the limit value ∆*l,i*. The lateral stiffness that has to be provided by the BRBs is equal to the total required lateral stiffness  $K_{Tot,i}$  minus the available lateral stiffness of the r.c. frame  $K_{Frame,i}$ . The required lateral stiffness  $K_{Tot,i}$  can be determined as the ratio of the total story shear force  $V_{d,i}$  determined by the elastic analysis of the structure to the inter-story drift capacity ∆*l,i*. The inter-story drift capacity should be reduced with respect to its nominal value to take into account that the real inter-story drifts may be larger than those obtained by the elastic analysis due to some concentration of the plastic deformation in a few stories. The available lateral stiffness of the frame *K<sub>Frame,i</sub>* can be determined as the ratio of the story shear force induced by the earthquake in the *i*-th story columns (sum of the shear forces of the columns determined by the elastic analysis of the structure) and the interstory drift demand ∆*i*.

*4. Iteration of the design procedure* 

The BRBs modify the required stiffness. Indeed, the reactions transferred by the BRBs to the r.c. frame modify the axial force of the columns and the inter-story drift capacity, which depends on plastic rotation capacity of the columns. Furthermore, the BRBs modify the periods of the frame and, therefore, the story shear forces and the inter-story drifts due to the ground motion. Because the story shear forces and the inter-story drifts do not change proportionally, the available lateral stiffness  $K_{Frame,i}$ , which is given by their ratio, slightly changes. As a consequence, the design of the BRBs has to be based on an iterative procedure.

#### *2.2. Strength requirement*

According to the strength requirement, the lateral strength of the structure has to be not smaller than the seismic design force. Furthermore, the ratio of the story shear force demand to the corresponding lateral strength has to be uniform along the height of the frame. This requirement allows the determination of the plastic strength of BRBs by a procedure that may be resumed in the following steps.

#### *1. Determination of the required lateral strength of the frame upgraded by BRBs*

The required lateral strength of each story of the frame is determined as the story shear  $V_{Sdi}$  evaluated by the elastic analysis of the structure based on the design spectrum. In this paper the modal response spectrum analysis is used, but the lateral force method of analysis can be used instead. The design spectrum is obtained reducing the ordinates of the reference elastic spectrum by means of the behaviour factor *q*. The adopted value of *q* has to be calibrated taking into account the ductility expected for the structural system.

*2. Determination of the available lateral strength of the r.c. frame*  Based on the assumption that in the r.c. existing buildings the yielding occurs mostly in the columns, the available lateral strength of the r.c. frame at the *i*-th story  $V_{f R d,i}$  is obtained as the total story shear transmitted by the columns *VRd,ji* of this story assuming that all the *i*-th story columns are yielded. By the equilibrium of the frame at collapse shown in Figure 1, the following equation for the determination of  $V_{f R d,i}$  is determined

$$
V_{f\,Rd,i} = \sum V_{Rd,ji} = \frac{\sum (M_{Rd,ji}^{B} + M_{Rd,ji}^{T})}{h_{i}}
$$
(2.3)





where,  $h_i$  is the story height, while  $M_{Rd,ji}$  is the column moment resistance, evaluated considering the interaction between bending moment and axial force (the superscripts *B* and *T* refer to the bottom and top cross-sections, respectively).

Alternatively, the lateral strength of the r.c. frame at the *i*-th story can be determined by the pushover analysis of the structure assuming an infinitive strength for all the members except the columns of the *i*-th story.

#### *3. Determination of the required plastic strength of BRBs*

BRBs are inserted where the available lateral strength of the r.c. frame is smaller than the total required lateral strength. The lateral strength that has to be provided by the BRBs  $V_{bRd,i}$  is given by the difference between  $V_{Sd,i}$  and  $V_{fRd,i}$ .

*4. Iteration of the procedure* 

When BRBs are inserted the periods of the frame decrease and the required lateral strength of the frame increases. Furthermore the reactions transferred by BRBs to the r.c. frame modify the axial force of the columns and also their moment resistance. As a consequence also the available lateral strength of the r.c. frame is modified by BRBs. Therefore, the determination of the plastic strength of BRBs has to be based on an iterative procedure.

### **3. ANALYSED STRUCTURE**

The proposed design method is used for the seismic upgrading of a r.c. frame designed to resist only gravity loads according to the regulations in force in the seventies in Italy. This frame is representative of many r.c. framed structure constructed in Italy before the adoption of seismic provisions in the Italian building code and is here used as a sample structure. The details of the design may be found in (Bosco *et al*, in press). The frame is six-story high and its geometrical scheme is shown in Figure 2 together with the size of the structural members. Figure 2 also shows the arrangement of the BRBs within the r.c. frame. The floor mass of the frame is 102.37 kNsec²/m. The seismic input is given by the elastic spectrum proposed by the EC8 for soil type C, characterized by a peak ground acceleration  $a<sub>e</sub>$  equal to 0.35 g. This earthquake level is suggested by the Italian national annexes to EC8 for the design of structures located in high seismicity areas. Four values of the design inter-story drift ∆*d,i* are considered for the determination of the required lateral stiffness of the BRBs (*KTot,i* – *KFrame,i*): 1.00, 0.75, 0.50 and 0.25 ∆*l,i*. For each value of ∆*d,i*, four values of *q* (6.7, 5.0, 4.0 and 3.3) are considered for the determination of the required lateral strength of BRBs  $V_{b R d,i}$ . Then, assuming that a pair of BRBs is used at the *i*-th story, the axial stiffness  $K_{b,i}$  and the axial plastic strength  $N_{b,v,i}$  required for the single BRB is calculated by the following equations

$$
K_{b,i} = \frac{K_{Tot,i} - K_{Frame,i}}{2 \cos \alpha}
$$
\n(3.4)

$$
N_{by,i} = \frac{V_{bRd,i}}{2\cos\alpha} \tag{3.5}
$$

where  $\alpha$  is the angle of inclination of the BRBs with respect to the beam longitudinal axis. The stiffness and the strength assigned to the BRBs are exactly the same to those required by the design.





Figure 2 Analyzed frame: (a) geometrical scheme of the r.c. frame, (b) cross-sections dimensions (B x H) of the r.c. frame members, (c) arrangement of the BRBs within the r.c. frame

For  $\Delta_{d,i} = \Delta_{l,i}$  and 0.75  $\Delta_{l,i}$ , the inter-story drifts evaluated by the elastic seismic analysis of the frame were always smaller than the design values. As a consequence, in these cases, the design of the BRBs is controlled only by the strength requirement for all the considered values of *q*. In this case, the design requires a minimum strength of the BRBs, while the stiffness of the BRBs may be assigned arbitrarily. In order to determine the stiffness of the BRBs, it is assumed that the length and the yield stress of the yielding core of the BRBs are equal to whole length of the brace (5.12 m) and 275 MPa, respectively. This hypothesis leads to BRBs with minimum stiffness for the given yield stress. When the design inter-story drifts are assumed equal to 0.50  $\Delta_{l,i}$  and 0.25  $\Delta_{l,i}$ , instead, the design is controlled also by the displacement requirement. In Figure 3 and Figure 4 the lateral stiffness and strength of the r.c. frame and those provided by the BRBs designed by the proposed method for the considered values of ∆*d,i* and *q* are shown and compared. It is evident that, the required stiffness of BRBs increases when the design inter-story drift is reduced (Fig. 3). For  $\Delta_{di} = 0.50 \Delta_{di}$  (Fig. 3b) the required stiffness of BRBs is slightly larger than that obtained when only the strength requirement applies (∆*d,i* > 0.50 ∆*l,i*, Fig. 3a) for all the considered values of *q*. As a consequence, due to the smaller periods of the upgraded frames, also the required lateral strength of BRBs is slightly larger (Figs. 4b vs. 4a). Finally, when the most strict requirement on inter-story drift is assumed (∆  $d_i = 0.25 \Delta_{li}$  Figs. 3c and 4c), the proposed design method requires BRBs which are much stiffer and stronger that those obtained in the other cases.



Figure 3 Lateral stiffness of the r.c. frame and that provided by the BRBs designed with the considered values of  $\Delta_{di}$  and *q*: design by  $\Delta_{di}$  > 0.50  $\Delta_{li}$  (a),  $\Delta_{di}$  = 0.50  $\Delta_{li}$  (b), and  $\Delta_{di}$  = 0.25  $\Delta_{li}$  (c)



Figure 4 Lateral strength of the r.c. frame and that provided by the BRBs designed with the considered values of  $\Delta_{d,i}$  and *q*: design by  $\Delta_{d,i}$  > 0.50  $\Delta_{l,i}$  (a),  $\Delta_{d,i}$  = 0.50  $\Delta_{l,i}$  (b), and  $\Delta_{d,i}$  = 0.25  $\Delta_{l,i}$  (c)



#### **4. NUMERICAL ANALYSES**

The seismic response of all the considered frames (the sample r.c. frame and the r.c. frames upgraded by BRBs) was evaluated by nonlinear dynamic analysis. Numerical analyses were carried out by means of the frame analysis program DRAIN-2DX (Powell, 1993; Prakash et al., 1993). A member-by-member modeling with plastic hinges assigned at member ends is adopted for beams and columns and an elasto-plastic behavior was assumed for BRBs. A Rayleigh viscous damping was used and set at 5% for the first two modes of vibration. Strain hardening was not considered, while the geometrical nonlinearity, i.e., the *P*-∆ effect, was considered. According to the EC8, nominal dead loads plus quasi-permanent live loads are assumed as initial gravity loads in the analyses. A set of ten artificial accelerograms, generated by the SIMQKE computer program (SIMQKE, 1976) and compatible with the elastic response spectrum used in design, is used for nonlinear dynamic analysis. By the numerical analyses the inter-story drift capacity ∆*l,i*, the maximum inter-story drift demand ∆*i*, and their ratio were determined. The inter-story drift corresponding to the achievement of the Life Safety limit state of the frame is assumed here as inter-story drift capacity ∆*l,i*. Note that the inter-story drift capacity ∆*l,i* depends also on the axial force of the columns and, therefore, changes during the earthquake. In this study the minimum value of ∆*l,i* is determined for each accelerogram. The results are plotted in Figure 5, Figure 6, and Figure 7 for the frames upgraded by BRBs. Here the value in each story is the mean of the ten values obtained for the ten accelerograms. Results obtained for the r.c. frame are not reported because it experienced collapse in most of the cases (six times out of ten) and very large inter-story drift in the other cases. This demonstrates that the analyzed r.c. frame provides extremely poor seismic performance for the considered earthquake level and needs to be upgraded.

Figure 5 shows that the inter-story drift capacity decreases when smaller values *q* are used in design, that is when stronger BRBs are used for the upgrading of the r.c. frame. For instance, when Δ<sub>*d,i*</sub> > 0.50 Δ<sub>*l,i*</sub> is used for the design of BRBs, the inter-story drift capacity of the r.c. frame at the first story for  $q = 3.3$  ( $\Delta_{l,l} = 33$  mm) is about 30% smaller that that obtained for  $q = 6.7$  ( $\Delta_{l,l} = 46$  mm). Similar differences may be observed when ∆*d,i* = 0.50 ∆*l,i* and 0.25 ∆*l,i* are used in design. This occurs because stronger BRBs transmit larger reaction forces to the r.c. frame determining a larger increase of the compressive axial force of the columns and a decrease of their plastic rotation capacity. Figure 5 shows also the comparison between the inter-story drift capacity determined by the results of nonlinear dynamic analysis and that used in design. This comparison shows a good agreement between the two sets of values for all considered cases and demonstrates the effectiveness of the method proposed for the estimation of the inter-story drift capacity.

When comparing the maximum inter-story drifts ∆*i* sustained by the analyzed frames, it is notable that the distribution of  $\Delta_i$  along the height of the frames upgraded by BRBs designed by  $\Delta_{di} = 0.50 \Delta_{li}$  and 0.25  $\Delta_{li}$  is almost constant regardless of the value of *q* adopted in design (Figs. 6b and 6c). Therefore, the BRBs designed by these values of ∆*d,i* are able to avoid the inter-story drift concentration at the lower stories of the frame even though they are provided with the smallest strength considered in this study (BRBs design by  $q = 6.7$ ). Instead, when Δ<sub>*d,i*</sub> > 0.50 Δ<sub>*l,i*</sub> is used in design, the frame is prone to develop a soft story mechanism except if a behavior factor not larger than 4 is adopted.



Figure 5 Distribution along the height of the inter-story drift capacity: design by ∆*d,i* > 0.50 ∆*l,i* (a), ∆*d,i* = 0.50 ∆*l,i* (b), and  $\Delta_{di} = 0.25 \Delta_{Li}$  (c)





Figure 6 Distribution along the height of the inter-story drift demand: design by  $\Delta_{di}$  > 0.50  $\Delta_{li}$  (a),  $\Delta_{di}$  = 0.50  $\Delta_{li}$ (b), and  $\Delta_{di} = 0.25 \Delta_{li}$  (c)



Figure 7 Distribution along the height of the inter-story drift demand to capacity ratio: design by ∆*d,i* > 0.50 ∆*l,i* (a),  $\Delta_{d,i} = 0.50 \Delta_{l,i}$  (b), and  $\Delta_{d,i} = 0.25 \Delta_{l,i}$  (c)

Note that the use of large values of *q* for the design of the BRBs has a beneficial effect on the inter-story drift capacity of the r.c. frame as shown in Figure 5. As consequence the maximum inter-story drift demand to capacity ratio  $\Delta$ <sub>*i*</sub> / $\Delta$ <sub>*l,i*</sub> along the height of the frames upgraded by BRBs designed by the  $\Delta$ <sub>*d,i*</sub> ≤ 0.50  $\Delta$ <sub>*l,i*</sub> and *q* = 6.7 is quite similar (Figure 7c) or even slightly smaller (Figure 7b) than that of the frames designed by larger values of *q*. In conclusion, the proposed design method allows the achievement of suitable seismic performance of the r.c. frame, even though BRBs are provided with the minimum strength considered in this study (design by  $q = 6.7$ ), if the value of the design inter-story drift is sufficiently small. In the analyzed case a design inter-story drift not larger than 0.50 ∆*l,i* has to be adopted to satisfy the Life Safety limit state verification. Furthermore the use of BRBs designed by a large value of *q* is more rational because, if a proper value of ∆*d,i* is also used, it drastically decreases the inter-story drift demand but does not reduce significantly the inter-story drift capacity.

### **5. CONCLUSIONS**

A new design method for the seismic rehabilitation of r.c. frames by BRBs is proposed in this paper. According to this method the r.c. frame upgraded by BRBs has to meet two requirements. A minimum lateral stiffness of the frame is stipulated by means of the displacement requirement, which is controlled by the design inter-story drift ∆*d,i*. While a minimum lateral strength is stipulated by means of the strength requirement, which is controlled by the behavior factor *q*. The sample frame analyzed in this study demonstrates that it is possible to overcome the typical deficiencies of existing r.c. structures designed to sustain only vertical load by BRBs designed by the proposed method if proper values of ∆*d,i* and *q* are adopted. In particular, when ∆*d,i* is taken larger than 0.5 ∆*l,i* the considered r.c. frame upgraded by BRBs meet the Life Safety limit state verification only if a value of *q* not larger than 4 is used. Alternatively, if ∆*d,i* is taken not larger than 0.5 ∆*l,i* even the largest



considered value of *q* (6.7) can be adopted. This second alternative, which leads to stiffer but weaker BRBs, is believed by the authors the most effective and is suggested for practical design purpose.

Further studies will investigate a wide set of r.c. structures, representative of r.c. existing buildings designed to sustain gravity loads only, upgraded by BRBs designed by the proposed method to generalize the results presented in this paper.

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