

## SENSITIVITY OF NONLINEAR FRAMES TO MODELLING PARAMETERS AND EARTHQUAKE EXCITATIONS

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### ABSTRACT :

Modern seismic codes permit the use of response-history methods to assess buildings for adequate seismic resistance. However, before this procedure can be implemented, it is necessary to first develop a suitable structural model and then to subject this to a relevant earthquake acceleration. For the ultimate design earthquake, it is now usual to increase the efficiency of the design, by allowing the structure to exceed its elastic limit and hence dissipate energy in hysteretic damping. If the designer is using response-history analysis as the primary means of assessing structural adequacy, then this design strategy requires the structural model to be nonlinear. It has long been known in both mathematics and mechanical engineering that non linear dynamic models can be very sensitive to both modeling assumptions and initial conditions; however this is rarely investigated in structural engineering designs. This paper presents the results of a large number of nonlinear time history analyses that have been conducted on simple frames. The model used in the analyses considers both material and geometric non-linearities. Inelastic behavior of the structure is modeled by an extended perfectly elastic, perfectly plastic moment rotation relationship. The extension to the moment rotation relationships enables analysis up to complete collapse of the structure by allowing the connection to fracture once its deformation has exceeded its ultimate rotation. The results of these analyses are presented as a parameter space of modelling parameters and load parameters. A number of earthquakes are investigated and the sensitivity of the results to these is discussed.

### KEYWORDS:

Nonlinear dynamics, parameter space, structural dynamics,

## **1. INTRODUCTION**

As computer power increases and our structural models become more sophisticated, it is becoming increasingly desirable to design seismic resistant structures using response history analysis. The reason for this is that response history analysis has the potential to remove many of the uncertainties associated with response spectrum analysis; namely the approximations introduced when combining modes and the approximations made converting elastic behavior into inelastic behavior. We are now at the stage where it is possible to perform nonlinear dynamic analysis on fairly realistic computer models and hence drastically reduce these uncertainties; however, while response history analysis can drastically improve the structural modeling it introduces a new uncertainty; namely what is an appropriate acceleration record to subject our structural model to. Presently most codes such as (CEN 1994), (ICC 2003) require the most onerous design resulting from the use of three appropriate acceleration records or to use seven records and then design for use the average result. Which leads to the question of what is an appropriate acceleration record?

Recently there has been a great deal of research to try and develop procedures for selecting appropriate earthquake records. Most of these procedures start with a target design spectra and then either generate synthetic earthquake records, or scale real earthquake records.

Whether artificial records are generated or real earthquakes scaled, the objective of both these approaches is to produce a small number of earthquake records that will be able to accurately reconstruct the target design spectrum. These earthquake records are then used as appropriate acceleration records with which to design the building. However, if these acceleration records are used as the forcing function in a nonlinear dynamic model, it is possible that the results may be very dependant on the modeling parameters.

To assess how modeling uncertainties and different acceleration records affect the results of nonlinear dynamic analysis, this paper presents the results of many response history analyses of simple inelastic framed structures subjected to scaled, real earthquake. The idea is to present response spectra in a different form that includes the effects of nonlinearity.

## **2. THE MODEL**

The generalized version of this model uses the beam element of (Krenk 2001) and is presented in (Wilkinson and Hiley 2006) however the important elements are reproduced here by developing the stiffness matrix for a simple single-storey, single bay frame.

If the structure stiffness matrix of the frame shown in Figure 1 is assembled so that the degrees of freedom of each floor are grouped together, the resulting global structural stiffness matrix, shown as the first term in equation 1, is obtained. The degrees of freedom relating to rotation are given the subscripts 1 to 4 which correspond to the joints labeled in Figure 1.

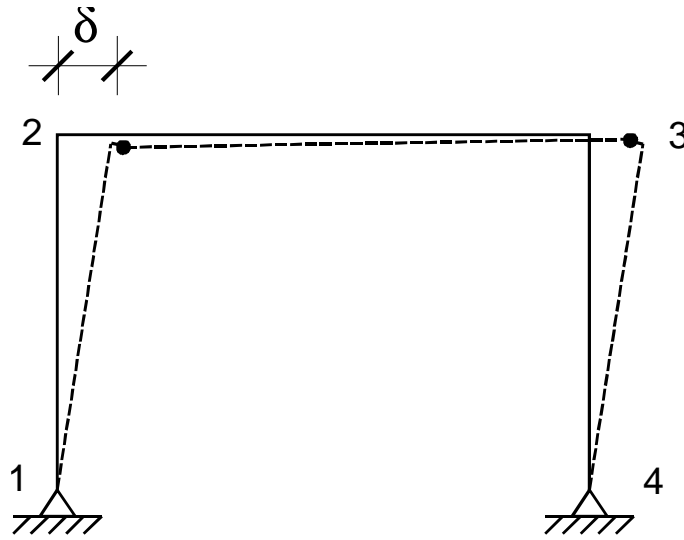


Figure 1 Single Storey Framed Structure

$$\begin{bmatrix}
 (3+\varphi')\frac{\psi EI_c}{L_c} & (3-\varphi')\frac{\psi EI_c}{L_c} & (6/L_c)\frac{\psi EI_c}{L_c} & 0 & 0 \\
 (3-\varphi')\frac{\psi EI_c}{L_c} & (3+\varphi')\frac{\psi EI_c}{L_c} + \frac{4EI_B}{L_B} & (6/L_c)\frac{\psi EI_c}{L_c} & \frac{2EI_B}{L_B} & 0 \\
 (6/L_c)\frac{\psi EI_c}{L_c} & (6/L_c)\frac{\psi EI_c}{L_c} & (24\varphi/L_c^2)\frac{\psi EI_c}{L_c} & (-6/L_c)\frac{\psi EI_c}{L_c} & (-6/L_c)\frac{\psi EI_c}{L_c} \\
 0 & \frac{2EI_B}{L_B} & (-6/L_c)\frac{\psi EI_c}{L_c} & (3+\varphi')\frac{\psi EI_c}{L_c} + \frac{4EI_B}{L_B} & (3-\varphi')\frac{\psi EI_c}{L_c} \\
 0 & 0 & (-6/L_c)\frac{\psi EI_c}{L_c} & (3-\varphi')\frac{\psi EI_c}{L_c} & (3+\varphi')\frac{\psi EI_c}{L_c}
 \end{bmatrix}
 \begin{Bmatrix}
 \vartheta_1 \\
 \vartheta_4 \\
 \delta \\
 \vartheta_2 \\
 \vartheta_3
 \end{Bmatrix}
 =
 \begin{Bmatrix}
 0 \\
 0 \\
 P \\
 0 \\
 0
 \end{Bmatrix}
 \quad (1)$$

Where:  $EI$  is flexural rigidity;  $L$  is member length;  $\varphi \equiv \alpha \cot \alpha$  and  $\psi \equiv \frac{1}{3}\alpha^2/(1-\varphi)$  are the symmetric and anti-symmetric bending stiffness coefficients;  $\varphi' \equiv \varphi/\psi$ ; and  $P$  is the axial compression force in the column (due to gravity in our case).

$$\alpha \equiv \frac{1}{2}L\sqrt{P/EI} \quad (2)$$

It should be noted that in equation 1 superscript  $C$  refers to properties associated with columns, while superscript  $B$  refers to properties associated with beams,

The inelastic properties of the frame are defined in terms of the rotational capacities,  $\mu_\theta$ , of individual connections. The moment rotation relationship of the individual connections is the same as adopted in (Wilkinson and Hiley 2004) and is shown in Figure 2 and defined in equation 3,

$$\mu_\theta = \frac{\theta_{ult}}{\theta_y} \quad (3)$$

where  $\theta_{ult}$  = the ultimate rotation of the connection and  $\theta_y$  = the yield rotation of the connection. The mass of the system is lumped at the degree of freedom corresponding to displacement, as is the damping. The stiffness matrix is partitioned and condensed prior to solution so that the problem to be solved becomes a single degree of

freedom. The equation of motion for this system was solved using a Runge-Kutta 4th order scheme with adaptive time step.

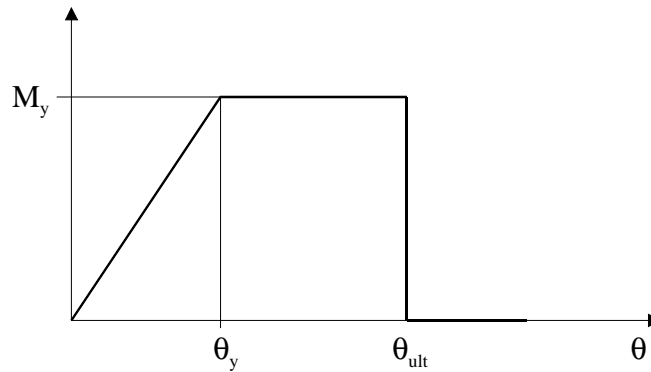


Figure 2 Moment Rotation Relationship for Beam Connections.

### 3. EARTHQUAKE EXCITATION

Earthquake excitations were selected from The European Strong-Motion Database (Ambraseys, Douglas et al. 2004). The earthquakes and their associated properties are given in Table 1, while the accelerograph and associated elastic response spectra are given in Figure 3.

Table 1 Earthquakes

Name	Country	Date	mb	Site intensity	Source Mechanism	Station name & Component
Bucharest	Romania	04/03/77	6.1	VIII	thrust	Bucharest-Building Research Institute, N-S
Duzce	Turkey	12/11/99	6.5	NA	oblique	Duzce-Meteoroloji Mudurlugu, WE
Erzincan	Turkey	13/03/92	6.1	NA	strike slip	Erzincan-Meteorologij Mudurlugu, N-S
Friuli	Italy	06/05/76	5.9	VIII	thrust	Tolmezzo-Diga Ambiesta, E-W
Gazli	Uzbekistan	17/05/76	6.2	IX	thrust	Gazli, E-W
Montenegro	Yugoslavia	15/04/79	6.1	IX	thrust	Bar-Skupstina Opstine, E-W
Tabas	Iran	16/09/78	6.4	IX+	thrust	Tabas, N74E

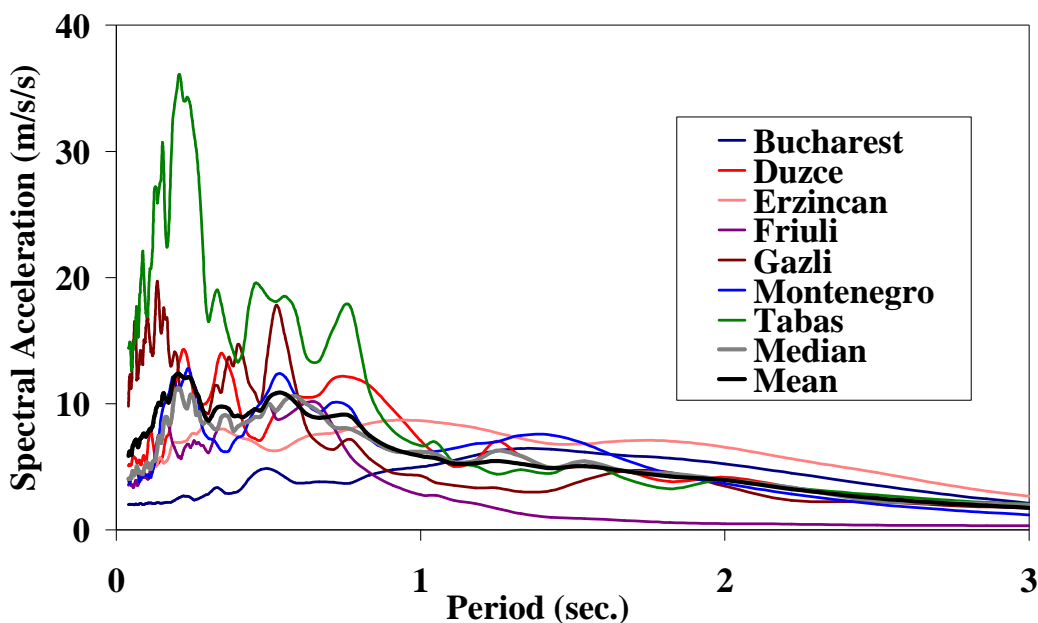


Figure 3 Acceleration Spectra

#### 4. EXAMPLES

A number of generic frames described by the equations above were subjected to different earthquake records. The frames were chosen to investigate how the characteristics of the earthquake excitation affected structures with different periods, and strengths. The oscillator described in this paper has three degrees of freedom (namely a deflection at the top of the structure and a rotation at the tops of each column). To simplify the problem further, the structure was converted to a two degree of freedom system by setting the yield moment at right-hand end of the beam to zero.

For this simple model, there are only five parameters that uniquely define the dynamic response of the system. These are the natural period of the structure, the percentage of critical damping, the strength of the structure, the rotational ductility of the structure, and the earthquake excitation. These can be reduced to four, by normalising the intensity of the earthquake excitation by the strength of the structure.

The structure was subjected to each of the earthquake accelerations shown in Table 1. As we are investigating the reserve capacity of buildings, the earthquakes acceleration records were scaled until they caused the structure collapse. The scaling factor was then normalized by dividing by the scaling factor required to initiate yielding of the structure.

The ratio of the mass to the stiffness of the structure was altered to vary the natural period. Periods from 0.3 second to 3 seconds were investigated in increments of .01 second, while the normalized amplitude of the earthquake acceleration was scaled in increments of 0.1. The damping was kept constant with a critical damping ratio of 2.0%. The rotational ductility was set to two (i.e.  $\mu_{\theta} = 2$ ).

#### 5. RESULTS

The results of the simulations are presented in Figure 4 as a parameter space, in which the period of the structure is plotted on the abscissa and the normalized earthquake intensity is plotted on the ordinate. In this figure, the structure either survives (plotted as green in the diagram) or collapses (plotted as red in the diagram). In this paper, failure is defined as the fracturing of the connection (i.e.  $\theta > \theta_{ult}$ ). The various figures, labeled a – h, show the response of the oscillator for different earthquakes.

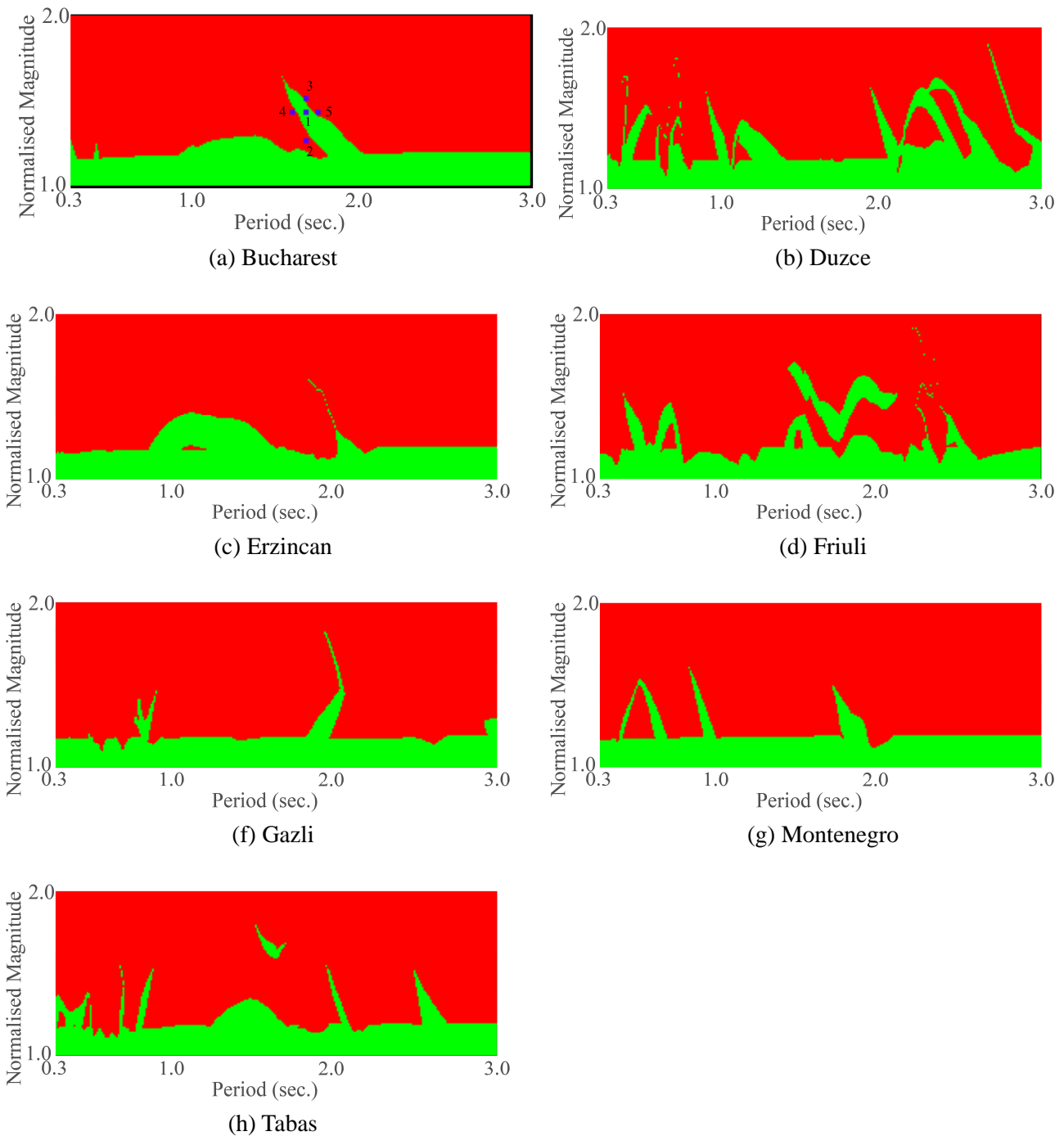


Figure 4 Parameter Space for  $\mu_0=2$

## 6. DISCUSSION

The first thing to notice about the diagrams is that there are areas of green (i.e. simulations that do not collapse) occurring above areas of red (simulations that do collapse). This means that an earthquake identical in all aspects except of a smaller intensity may make the structure collapse, whereas a bigger one does not; or alternatively, a slightly weaker structure may survive whereas a stronger structure collapses. The second thing to notice is the tongues of green that ascend into regions of red. To investigate the reasons for these anomalies, the parameter space has been sampled at five locations and the displacements of these simulations have been plotted in Figure

4a. The five locations are in the middle of the tongue, slightly above the tongue, slightly below the tongue, slightly to the left of the tongue and slightly to the right (labeled 1 to 5 in Figure 4a).

The reason for this is due to small differences in the onset of yielding between different simulations and thus different simulations following different futures. Sensitivity to modeling parameters and initial conditions is a common occurrence in nonlinear dynamical systems and even though the excitation of this system is non-periodic, it still displays considerable sensitivity to the modeling parameters.

In Figure 6 the displacement of each of the simulations has been plotted and the initiation of yielding has been indicated with a point. With the exception of simulation 2, every simulation yields at approximately 4 seconds and with a negative magnitude (simulation 2 first yields when the oscillator subsequently deflects in the opposite direction). Points 1, 4 and 5 have the same intensity earthquake but different periods and follow virtually the same path until yield. For this earthquake, longer period structures are more vulnerable and therefore simulation 5 yields first, followed by simulation 1 and then simulation 4. As simulation 5 is most susceptible to the earthquake, it fails shortly after yield, whereas simulation 1 and 4 survive the first excursion into yield. At the end of the first yield cycle, Simulation 1 has a larger plastic strain than Simulation 4 and therefore, in the next cycle, deflects less. During this next cycle both Simulation 1 and 4 yield; however, this second yielding proves fatal for Simulation 4, whereas simulation 1 survives.

Simulation 3 is subjected to a greater intensity earthquake and therefore yields and then quickly fails. Simulation 2, which has the lowest intensity of earthquake, does not yield when the other simulations do; however it yields and subsequently fails on the next cycle. Thus the border of the tongues can be explained as a zone where the reducing susceptibility of the structure due to its period is matched by the increasing intensity of the earthquake

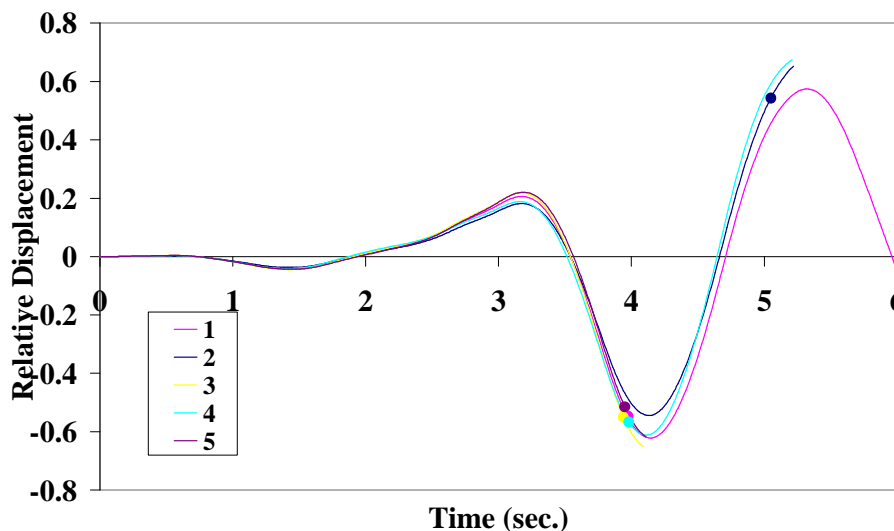


Figure 6 Displacement Response Histories and the Onset of Yielding

Each of the parameter spaces presented in Figures 4 depicts how sensitive the analysis is to both modeling parameters and the earthquake excitation. As the parameter space has been presented in terms of earthquake intensity rather than building strength, a good earthquake for checking the collapse performance limit of a building, is one where the parameter space contains no tongues of green and/or regions of red surrounded by green. For example for a building that did not have a natural period greater than 2 seconds, Bucharest would be a good choice to determine excess inelastic capacity, whereas Duzce would not.

require structures to be assessed for either the worst response resulting from three earthquakes or the average response resulting from seven earthquakes To compare the suitability of averaging seven earthquakes as a design

strategy, as opposed to considering the worst response of three earthquakes the median of the seven parameter spaces is displayed in Figure 7a, while three parameter spaces have been laid on top of each other In Figure 7b. To obtain Figure 7b, if any one of the simulations associated with a particular point in the parameter space resulted in collapse, then the point was colored red. By inspection, the three earthquakes chosen for Figure 7b were Duzce, Friuli and Tabas. Figure 7a shows that when the response of the seven earthquakes are averaged, all the tongues and regions of red contained within regions of green disappear – meaning, that for this simple system at least, and these earthquakes, averaging seven earthquakes removes uncertainty due to the modeling parameters. For the case of taking the worst response of three earthquake records, figure 7b shows that nearly all of the irregularities disappear; however there are a few places where specs of green are contained within regions of red, meaning that if a designer was to chose the worst of these three earthquakes as the design criteria, they may end up with an unsafe design, although they would be unlucky if they did.

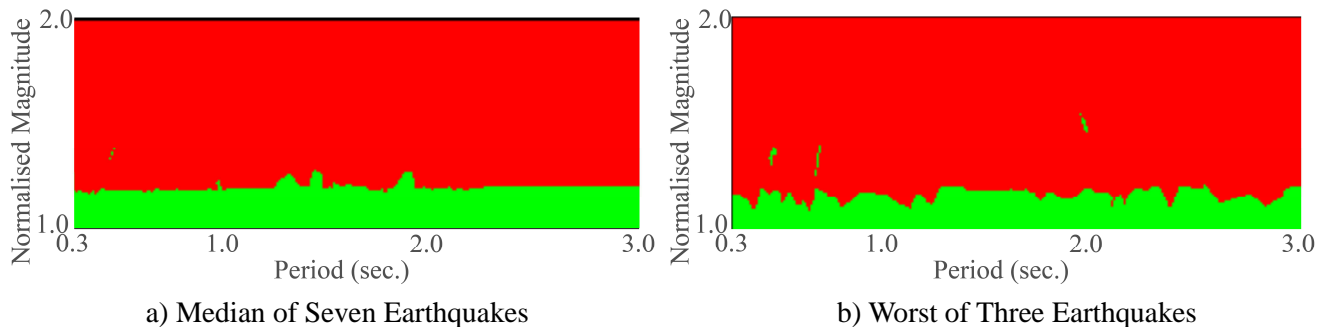


Figure 7 Parameter Space for multiple Earthquakes

## 7. CONCLUSION

The sensitivity of a simple nonlinear oscillator to earthquake excitations has been investigated. The results were presented in terms of a parameter space and it is suggested that this is a good mechanism for considering how indifferent an earthquake is to modeling parameters. The investigation showed that the behavior of nonlinear structures subjected to real earthquake records is complicated and that specifying a minimum strength is not necessarily sufficient to achieve a safe design. When designing for the collapse performance limit, adopting the average of seven earthquake excitations is preferable to designing for the worst case of three. In addition, when using nonlinear response history analysis, engineers should investigate the sensitivity of their model to both modeling parameters (strength, stiffness, period and ductility) as well as the excitation. Finally the use of elastic target spectra as the only criteria for developing design earthquake ground accelerations should be questioned.

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