EFFECT OF UNCERTAINTIES IN STRUCTURAL PARAMETERS ON EARTHQUAKE RESPONSE OF TORSIONALLY-COUPLED BUILDINGS

P. Banerji[1] and M. Barve[2]

[1] *Professor, Dept. of Civil Engineering , Indian Institute of Technology, Mumbai India*
[2] *Research Scholar, Dept. of Civil Engineering, Indian Institute of Technology, Mumbai, India*
Email: pbanerji@iitb.ac.in

ABSTRACT :

The effect of uncertainty in structural parameters on the response of a single-storey torsionally-coupled building subjected to stationary stochastic ground motion is studied here. A reliability-based formulation, where failure is defined as the exceedance of a predetermined value of the response quantity of interest, is used and solved using a first-order-reliability method (FORM). The responses of interest are the displacements on the stiff and flexible side edges, and the base shear and torque at the center of mass. The structural parameters considered uncertain are the uncoupled lateral and torsional frequencies, and the eccentricity, which are assumed to be statistically independent of each other. However, the structural response is also a random variable that depends on the structural parameters, and this requires special consideration in the reliability analysis. It is seen that the 84-percentile and 95-percentile responses are significantly affected by the uncertainty in structural parameters, especially when lateral-torsional coupling is significant. These effects are more for small eccentricity and damping ratios. The uncertainty in parameters significantly increases responses, especially when lateral-torsional coupling in large. These effects are larger for displacements on the flexible side.

KEYWORDS: Coupled system, Torsional coupling, Reliability, Uncertainty, Stochastic Analysis

1. INTRODUCTION

A structure is considered to be lateral-torsionally coupled, when it undergoes torsional motions while subjected to a purely translation ground motion. Such torsional motion occurs due to eccentricity between center of mass (CM) and center of stiffness (CS), rotational excitation at base due to earthquake ground motion and spatial variation of earthquake ground motion. It is found that due to these reasons a symmetric structural system may also exhibit lateral-torsional coupling. Studies on torsionally-coupled structures were initiated by Newmark (1969) and continued by Kan and Chopra (1977) and others. These studies only considered the effect of structural eccentricity and not the effect of torsional ground motions. De. La. Llera and Chopra (1994) used actual recorded ground motions from three earthquakes in USA and studied the symmetrical structure to understand the effects of base rotation and stiffness uncertainity. This study reveals dependence of structural response on stiffness uncertainity, when frequency ratios approach one. This study only concentrated on one aspect of uncertainty in structural parameters. A more detailed study of the effect of structural parameter uncertainty is attempted in this paper. A reliability-based formulation similar to that used by Igusa and Der Kiureghian (1988), based on the method proposed by Hohencichler and Rackwitz (1981), is used here.

2. ANALYSIS OF TORSIONAL COUPLED STRUCTURE

2.1 Equations of Motion

A single-storey torsionally-coupled structure (see Figure 1) with circular footing and rigid circular deck supported by inextensible, massless columns is considered. This system has three degrees of freedom viz. translation along x and y-axis and rotation about vertical axis. With the assumption of perfect symmetry about Y-Y axis and eccentricity '$e=e_y$' along Y-Y axis, the translation along x axis also causes the rotation about vertical axis. The structure is assumed to be subjected to an uniform earthquake motion modeled as a stationary, filtered white noise process given by the modified Kanai-Tajimi function (Clough and Penzien 1993).

$$S_g = S_o \frac{\omega_{fk}^4 + 4\xi_{fk}^2 \omega_{fk}^2 \omega^2}{(\omega_{fk}^2 - \omega^2)^2 + 4\xi_{fk}^2 \omega_{fk}^2 \omega^2} \quad \frac{\omega^4}{(\omega_{gk}^2 - \omega^2)^2 + 4\xi_{gk}^2 \omega_{gk}^2 \omega^2} \tag{2.1}$$

The first (ω_{fk}) and second filter (ω_{gk}), frequencies are considered, corresponding to firm ground, as ω_{fk} = 2.5 Hz and ω_{gk} = 0.25 Hz, with equal damping ratios for both the filters. ($\xi_{fk} = \xi_{gk} = 0.6$). White noise intensity and duration of strong ground shaking are not required as only normalized response quantities are considered here.

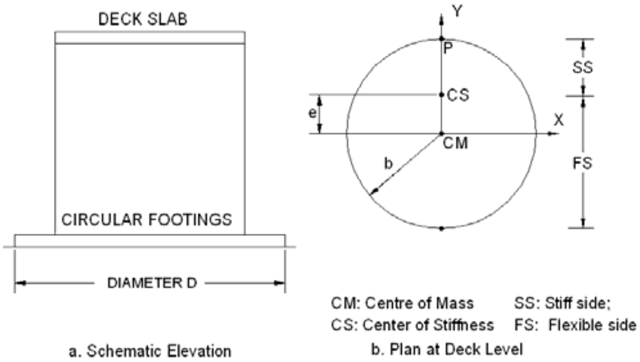

Figure 1 Schematic of a single storey torsionally-coupled structure

For the linear structure under consideration, the equations of motion can be written, corresponding to the degrees of freedom defined as the relative lateral displacement and torsional rotation of the floor centre of mass, as

$$\begin{Bmatrix} m\ddot{u}_x(t) \\ J\ddot{u}_\theta(t) \end{Bmatrix} + \begin{bmatrix} K_{xx} & K_{x\theta} \\ K_{\theta x} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_x(t) \\ u_\theta(t) \end{Bmatrix} = \begin{Bmatrix} -m\ddot{u}_{fx}(t) \\ -mr\ddot{u}_{f\theta}(t) \end{Bmatrix} \tag{2.2}$$

where $\ddot{u}_{fx}(t)$ is the lateral component and $\ddot{u}_{f\theta}(t)$ is the torsional component (in this paper it is zero) of foundation input motion, which is assumed here to be uniform over the foundation with no torsional component; K_{xx}, $K_{\theta\theta}$ are the stiffness values for lateral translation and torsional rotation about vertical axis, respectively; $K_{x\theta}$ and $K_{\theta x}$ denote the stiffness coupling terms. The static eccentricity of the system is normalized with respect to radius of gyration 'r' to obtain a nondimensional eccentricity **δ =e/r**. The uncoupled frequencies (i.e. for e = 0) in lateral and torsional modes are, respectively, defined as:

$$\omega_x = \sqrt{\frac{K_{xx}}{m}} \qquad \omega_\theta = \sqrt{\frac{K_{\theta\theta}}{mr^2}} \tag{2.3}$$

and the tuning ratio of the uncoupled torsional frequency to the uncoupled lateral frequency, defined as the frequency ratio β_t, and a corresponding detuning parameter β, are given as

$$\beta_t = \frac{\omega_\theta^2}{\omega_x^2} = \frac{K_{\theta\theta}}{r^2 K_{xx}} \qquad \beta = \beta_t - 1 \tag{2.4}$$

Free vibration characteristics can be obtained in closed form. Assuming that the coupled system frequencies are normalized as $\Omega = \omega / \omega_x$, the normalized coupled structure frequencies can be written as

$$\Omega_{1,2} = \sqrt{1+\frac{\beta}{2} \mp \frac{\beta}{2}\sqrt{1+\left(\frac{2\delta}{\beta}\right)^2}} \qquad (2.5)$$

The corresponding mode shapes can be derived as

$$\phi = \begin{Bmatrix} 1 & -\alpha \\ \alpha & 1 \end{Bmatrix} \qquad (2.6)$$

where the parameter α is given by

$$\alpha = \frac{2\,\delta}{\beta\left\{1+\sqrt{1+\left(\frac{2\,\delta}{\beta}\right)^2}\right\}} \qquad (2.7)$$

The solution of Eqn. 2.2 is done using the standard mode superposition method, where the displacement vector time history **u**(t) is given in terms of the modal amplitudes $Y_j(t)$ as

$$\boldsymbol{u}(t) = \sum_{j=1}^{2} \phi_j \; Y_j(t) \qquad (2.8)$$

and the corresponding modal equations are given as

$$\ddot{Y}_j(t) + \omega_j^2\, Y_j(t) + 2\xi\,\omega_j\,\dot{Y}_j(t) = \frac{L_{jx}}{M_j}\ddot{u}_{fx}(t) \qquad j=1,2 \qquad (2.9)$$

Solving Eqn. 2.9 for the modal amplitudes and then substitution into Eqn. 2.8 and incorporating Eqn. 2.6, the displacements corresponding to the degrees of freedom can be written as

$$u_x(t) = Y_1(t) - \alpha\; Y_2(t) \qquad (2.10a)$$
$$r u_\theta(t) = \alpha Y_1(t) + Y_2(t) \qquad (2.10b)$$

The lateral deformation of any resisting element at a distance 'y' from the centre of mass along the y-axis can be written, in terms of these displacements corresponding to the degrees of freedom, as

$$z(t) = u_x(t) - \frac{y}{r}[r\,u_\theta(t)] \qquad (2.11)$$

Substituting Eqn. 2.10 into Eqn. 2.11, the expression for z(t) can be written as

$$z(t) = \sum_{j=1}^{2} B_j \; Y_j(t) \qquad (2.12)$$

where B_j is defined as the modal response coefficient for the j^{th} mode. Values for different response coefficients using 'r' as radius of gyration and 'b' as half plan width can be expressed as follows:

$$B_{1V} = \frac{\omega_1^2}{1+\alpha^2} \quad \text{and} \quad B_{2V} = \frac{\alpha^2 \omega_2^2}{1+\alpha^2} \qquad \text{For base shear at CM } (V_B), \qquad (2.13a)$$

$$B_{1T} = \frac{\alpha\omega_1^2}{1+\alpha^2} \quad \text{and} \quad B_{2T} = \frac{-\alpha\omega_2^2}{1+\alpha^2} \qquad \text{For base torque at CM } (T_B) \qquad (2.13b)$$

$$B_{1P} = \frac{1-\alpha(b/r)}{1+\alpha^2} \quad \text{and} \quad B_{2P} = \frac{\alpha^2+\alpha(b/r)}{1+\alpha^2} \qquad \text{For displacements at P, } Z_P \qquad (2.13c)$$

$$B_{1Q} = \frac{1+\alpha(b/r)}{1+\alpha^2} \quad \text{and} \quad B_{2Q} = \frac{\alpha^2-\alpha(b/r)}{1+\alpha^2} \qquad \text{For displacement at Q, } Z_Q \qquad (2.13d)$$

Using standard stochastic analysis procedures in frequency domain approach, PSD function for any response can be written as (Der Kiureghian 1980)

$$S_R(\overline{\omega}) = \sum_{j=1}^{2}\sum_{k=1}^{2} B_j \; B_k \; H_j(-i\overline{\omega}) \; H_k^*(-i\overline{\omega}) \; S_{P_jP_k}(i\overline{\omega}) \qquad (2.14)$$

It is shown that many statistical properties of a Gaussian process can be expressed in terms of the first few moments of its spectral density function, called spectral moments (Vanmarcke 1972). Therefore, the spectral moments are calculated by following set of equations:

$$\lambda_i = \int_0^\infty \overline{\omega}^i \left(\sum_j \sum_k B_j \; B_k \; H_j(\overline{\omega}) \; H_K^*(\overline{\omega}) \; S_z(\overline{\omega}) \right) d\overline{\omega} \qquad (2.15)$$

2.2 Analysis With Deterministic Parameters

After computing values of the first three spectral moments, the cumulative probability that response will not exceed a threshold value r_0, can be computed from the expression (Vanmarcke, 1972).

$$F_{R_t}(r_0) = \left[1-\exp(-r_0^{\,2}/2\lambda_0)\right] \exp\left[-\nu\tau \frac{1-\exp\left\{-q_e r_0 \sqrt{\pi/2\lambda_0}\right\}}{\exp(r_0^{\,2}/2\lambda_0)-1} \right] \qquad (2.16)$$

where τ is the effective duration of ground motion in seconds. Parameter ν defines the mean zero crossing rate of the response and is given by,

$$\nu = \frac{\sqrt{\lambda_2/\lambda_0}}{\pi}, \qquad q = \sqrt{1-\frac{\lambda_1^2}{\lambda_0 \; \lambda_2}} \quad \text{and} \quad q_e = q^{1.2} \qquad (2.17)$$

q is a nondimensional parameter, in terms of spectral moments, which takes the values between zero to one and defines the spread of $S_R(\overline{\omega})$ about the central frequency. The probability that response of the structure will exceed the threshold value r_o i.e. the probability of structural failure (P_f) is given by,

$$p_f(r_0) = 1 - F_{R_t}(r_0) \qquad (2.18)$$

(1- P_f) is the confidence level suggesting that even if structural response takes a maximum value equal to r_o, one had the confidence level that failure would not occur. Therefore the structural response corresponds to a specified level of confidence is considered as r_o. To calculate desired threshold value, spectral moments are calculated and a set of different probability of failure can be computed by changing r_o values for the same set of spectral parameters. Then, using inverse interpolation technique r_o value corresponds to the specified probability of failure can be obtained.

2.3 Reliability Analysis for Uncertain Parameters

For the analysis presented here, the structural parameters ω_θ, ω_x and δ are considered as independent normal random variables. The peak structural response is also a random variable whose distribution is given by Eqn. 2.17 and which is dependent on the other three parameters. For reliability analysis, the set of random variables is required to be transformed from the original correlated non-normal space into the standard normal space,. Thus the vector of random variables $X^T = \{\omega_x \quad \omega_\theta \quad \delta \quad R\}$ is transformed into standard **u** space using the Rosenblatt transformation (Hohenbichler and Rackwitz 1981).

$$u_1 = \frac{\omega_x - \mu_{\omega x}}{\sigma_{\omega x}}, \quad u_2 = \frac{\omega_\theta - \mu_{\omega\theta}}{\sigma_{\omega\theta}}, \quad u_3 = \frac{\delta - \mu_\delta}{\sigma_\delta}, \quad u_4 = \Phi^{-1}\left[F_{R|\omega_x,\,\omega_\theta}\right] \tag{2.19}$$

A linearized transformation at the design point on the failure surface into standard normal variables is given as $u_i = a_i + B(X_i \quad R_i)^T$. And the inverse transformation to calculate design point from standard normal space (i.e., element of u_i) back into original space, is obtained by rewriting the above equation for u_i as:

$$\{x_i \quad R_i\}^T = B^{-1}\{u_i - a_i\} = J\,\{u_i - a_i\} \tag{2.20}$$

Here 'B' is a lower triangular transformation matrix and 'J' is the Jacobian matrix given by

$$B_{ij} = \frac{\partial u_i}{\partial x_j} \qquad \text{For } j < i \qquad \& \; B_{ij} = 0 \text{ elsewhere} \tag{2.21}$$

$$J_{ij} = \frac{\partial x_i}{\partial u_j} = \frac{1}{B_{ii}} \quad \text{For } i = j, \quad = 0 \text{ for } i < j \quad \text{and} \quad J_{ij} = \frac{-1}{B_{ii}} \sum_{k=j}^{i-1} B_{ik} J_{kj} \quad For\ i > j \tag{2.22}$$

The parameters ω_x, ω_θ and δ are assumed to be normal defined as $N(\mu_{\omega x}, \sigma_{\omega x})$, $N(\mu_{\omega\theta}, \sigma_{\omega\theta})$, $N(\mu_\delta, \sigma_\delta)$. Knowing the distribution of the structural parameters and the structural response, the probability of failure of torsionally coupled system can be written as

$$p_f = 1 - F_{R|X}(r_0) = \int_{\{g<0\}} F_{R|X}(r_0, X)\,\phi(\delta)\phi(\omega_x)\phi(\omega_\theta)\,d\omega_x d\omega_\theta d_\delta \tag{2.23}$$

The evaluation of the probability of failure using the above equation is a computationally intensive task. Here this is done using by approximately estimating the probability of failure using the modified Rackwitz-Fiessler FORM method (Hohenbichler and Rackwitz 1981). Failure of the torsionally-coupled structure is defined to occur when response (R) exceeds a level (r_o), defined as maximum threshold response of the structure. Thus, the equation of limit state of reliability can be written as,

$$g(X, R) = Z = r_0 - R \tag{2.24}$$

Where $Z > 0$ is the safe zone, $Z < 0$ defines the failure zone and $r_0 = R$ represented the failure surface. Using the gradient vectors (G_u) and direction cosines ($(\hat{\alpha})$the reliability index (β) is calculated using an iterative procedure.

$$u_{i+1} = -\hat{\alpha}\,\beta_i, \qquad G_u^{\,T} = \left\{ \frac{\partial g}{\partial u_1} \quad \frac{\partial g}{\partial u_2} \quad \frac{\partial g}{\partial u_3} \quad \frac{\partial g}{\partial u_4} \right\} \text{ and } \hat{\alpha} = \frac{1}{\sqrt{G_u^T\,G}}\,G_u \tag{2.25}$$

Then using reliability index the probability of failure by First Order Reliability Methods (FORM) can be calculated as $P_f = \Phi\left(-\beta\right)$. Where Φ(.) denotes the standard normal cumulative distribution function.

3. RESULTS AND DISCUSSIONS

As discussed in Section 2, the response quantities considered here are the displacements on the stiff side edge (Z_P) and the flexible side edge (Z_Q), and the base shears (V_b) and base torque at the center of mass (T_b). Two different coefficients of variations - 7% and 10% - are considered for all the basic random variables, i.e. the two uncoupled frequencies and the normalized eccentricity ratios. The reliability analysis is done repetitively to find the 95-percent confidence level values for each of the above responses, i.e. find the threshold value corresponding to a 5% probability of failure. The plots of the variation of the 95-percent confidence level values for all the four response quantities of interest with frequency ratio are presented in Figures 2-5. Two values of mean normalized eccentricity ratios, one corresponding to a small eccentricity and the other corresponding to a large eccentricity, are considered in the analysis results presented here.

It should be noted that the responses presented in the figures below are all normalized with respect to the corresponding responses computed using standard stochastic analysis for a torsionally uncoupled structure subjected to the stochastic ground motion defined in Eqn. 2.1 with all structural parameters defined by only their mean values, except that the eccentricity is considered to be zero.

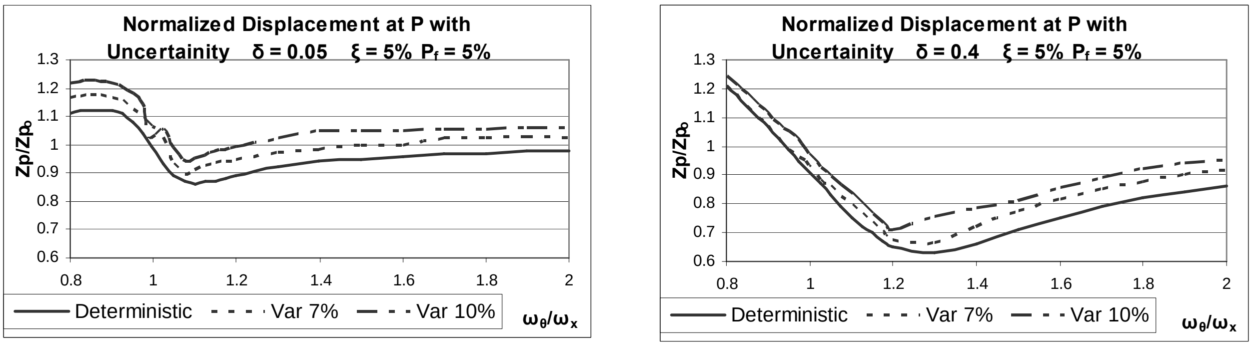

Figure 2 Normalized displacement Z_P variation with frequency ratio for two values of mean eccentricity

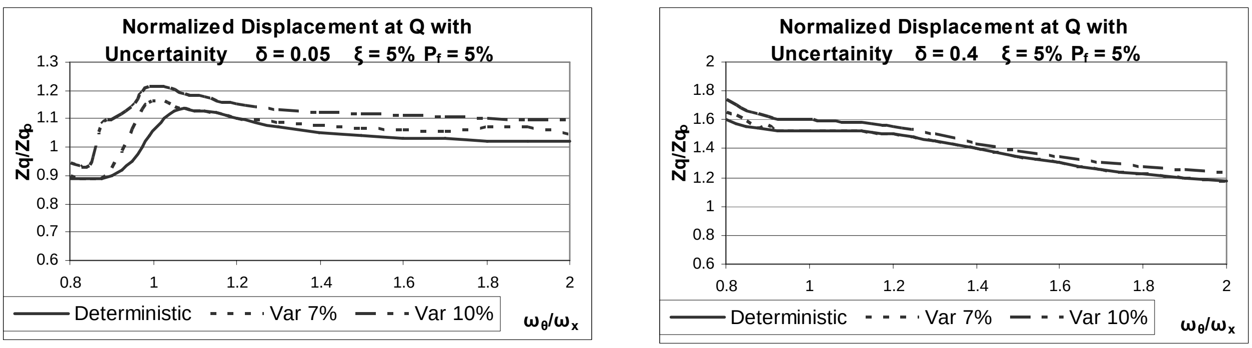

Figure 3 Normalized displacement Z_Q variation with frequency ratio for two values of mean eccentricity

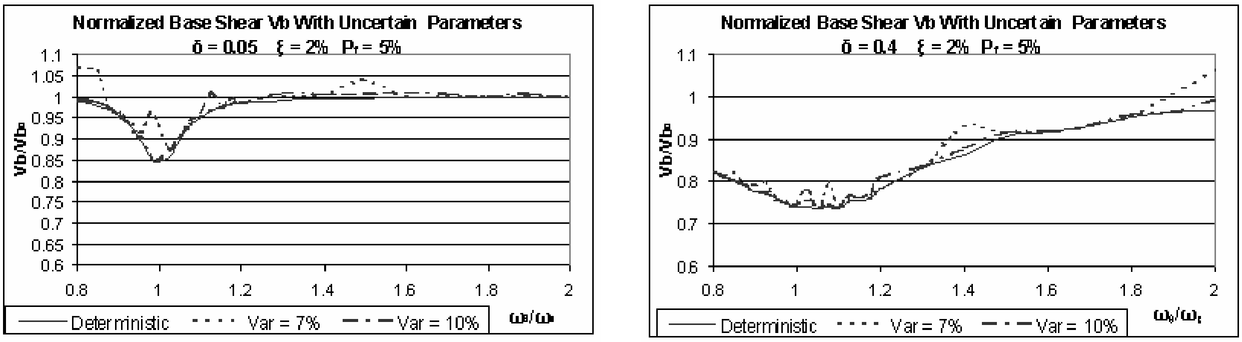

Figure 4 Normalized base shear V_b variation with frequency ratio for two values of mean eccentricity

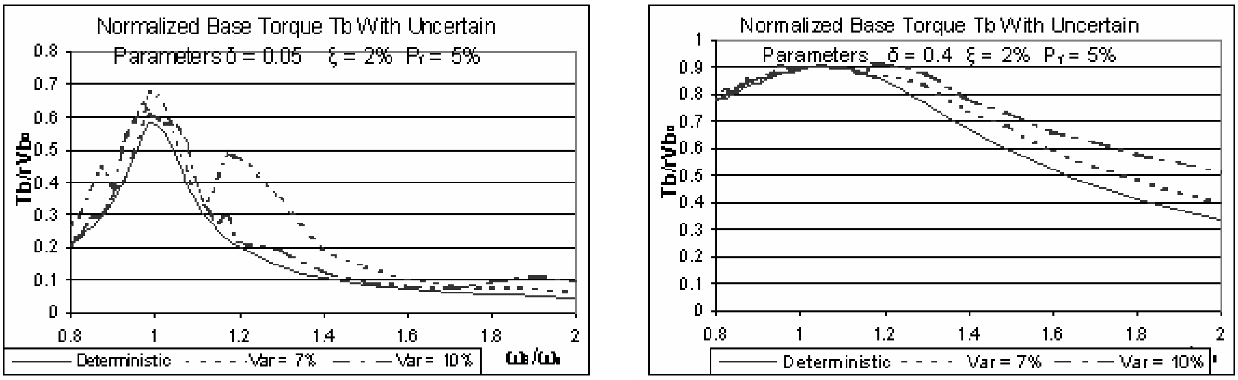

Figure 5 Normalized base torque T_b variation with frequency ratio for two values of mean eccentricity

It can be seen that except for base shear, where the effect is negligible, uncertainty in structural parameters have a significant effect on the response values. It can also be seen that uncertainty tends to increase the values of the response quantities, and this increase is greater as the uncertainty is increased, i.e. the coefficient of variation for the structural parameters is increased. For displacement responses, it is noted that the effect of uncertainty in structural parameters seems to be more for smaller eccentricity values, especially when the lateral-torsional coupling is large; although the results are not presented due to restrictions on paper length, it is seen that the effect of uncertainty is also larger for smaller damping ratios. Furthermore, the effect of the uncertainty is seemingly more for displacements on the flexible side than for those on the stiff side. It can also be said that the effect of uncertainty in the uncoupled frequency parameters is more when the frequency ratio is between 0.8 and 1.2, i.e. when the lateral-torsional coupling is large. However, when the lateral-torsional coupling is small, the effect of uncertainty in eccentricity governs. Thus, when uncertainties are considered in both frequencies and eccentricity, as in the results presented here, the effect of uncertainty is found to be significant across the spectrum of frequency ratios.

4. SUMMARY AND CONCLUSIONS

In this paper the effect of uncertainty in structural parameters, namely the uncoupled lateral and torsional frequencies and the static eccentricity, on the response of a single-storey torsionally-coupled structure to stochastic but uniform foundation base motion is studied. Typical wisdom, based on past research, suggests that the effects of uncertainty in structural parameters on the structural response are of a second-order effect to that due to the uncertainty associated with the base motion (the reason for defining it as a stochastic process). However, this study conclusively proves that the effects of uncertainty in structural parameters of a torsionally-coupled structure on its response are significant and cannot be neglected in the response analysis.

The effects of uncertainty are greater for smaller eccentricity and damping ratio values, especially when the frequency ratios are close to tuning, and the lateral-torsional coupling is large. The uncertainty in structural parameters tends to increase structural response to base motion, and this increase is greater when the uncertainty in the parameters is more. The effect of parametric uncertainty is more on displacement response on the flexible side than that on the stiff side. This is primarily because lateral-torsional coupling has a greater effect on the flexible side response rather than the stiff side displacements.

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