NON-LINEAR STATIC METHODS FOR SEISMIC FRAGILITY ANALYSIS AND RELIABILITY EVALUATION OF EXISTING STRUCTURES

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ABSTRACT :

Probabilistic approaches are more and more popular for seismic assessment of existing structures and specific methods have been properly developed for earthquake engineering. However, such methods generally require a huge number of non-linear time-history analyses which can be very time-consuming. Consequently, the present paper proposes a practical method based on static (pushover) analyses; a sample application shows the comparison about fragility and reliability evaluation obtained through static and dynamic approaches.

KEYWORDS: Structural Safety, Reliability, Non-linear Analysis

1. INTRODUCTION

Probabilistic approaches are rather popular nowadays for seismic assessment of existing structures and can be even utilized for a more comprehensive evaluation of the effects induced by seismic events before and after retrofitting. Since general purpose statistical approaches (Pinto & Al., 2004), possibly extended to time variant problems such as the simulation of seismic behaviour of structures, are usually time-consuming, specific methodologies properly developed for application in earthquake engineering (Cornell & Al. 2002) can be much more cost-effective in evaluating the seismic reliability of structures. Nevertheless, even those methods are based on the results of a huge number of non-linear time-history analyses which can be as time-consuming as the level of detail of the numerical models increases, as usually required for achieving a sufficient accuracy in simulating the structural response of the existing structures.
Starting form these considerations, the present paper deals with the possible application of non-linear static procedures (instead of time-history analyses) for evaluating seismic reliability of structures with respect to the Limit States of interest in seismic assessment and retrofitting. In particular, N2-method (Fajfar, 1999) instead of non-linear time-history analysis is utilized for obtaining seismic demand, resulting in huge savings in elaboration time. A similar method, the so-called IN2, has been already proposed by Dolsek & Fajfar (2004) for deriving the relationship between seismic intensity and structural demand by using (incremental) N2-Method instead of the well-known IDA (Vamvatsikos & Cornell, 2002). The present paper addresses also the key aspect of evaluating dispersion measures in that relationship always using static analyses. After an outline of the proposed procedure a sample application on an existing RC building is also proposed as preliminary validation

2. INSIGHTS ABOUT SEISMIC RELIABILITY OF STRUCTURES

Since, as a matter of principle, both seismic demand D and capacity C should be defined in probabilistic terms, the probability for a structure to attain a given Limit State P_{LS} can be defined in one of the two following ways:

$$P_{LS} = \int_0^{\infty} \left[1 - F_D(\alpha)\right] \cdot f_{C,LS}(\alpha) \cdot d\alpha \qquad (2.1)$$

or

$$P_{LS} = \int_0^{\infty} F_{C,LS}(\alpha) \cdot f_D(\alpha) \cdot d\alpha \qquad (2.2)$$

where F is Cumulative Distribution Function (CDF) and f is the Probability Density Function (PDF) referred to seismic demand D and structural capacity C_{LS} corresponding to a given Limit State LS.

2.1. The SAC-FEMA Methodology

Considering Eq. (2.1), a first hypothesis can be introduced for relating a seismic intensity measure *i* with a consistent demand parameter D defined on the structure. The pseudo-acceleration value S_a of the structure at hand is often assumed as intensity measure for seismic signals which can be scaled for covering the entire relevant integration domain in Eq. (2.1). As far as demand, various alternative parameters have been proposed in the scientific literature for quantifying seismic response of structures; as a matter of principle, two wide categories of such parameters can be recognized considering on one hand those based on the maximum value achieved by a given displacement quantity (interstorey drift, plastic rotation, chord rotation and so on) and on the other one the parameters considering the cyclic nature of seismic response in terms of dissipated energy or number cycles. Even combination of such parameters have been also proposed, but in practical applications demand measures belonging to the first class and based on kinematic parameters are usually adopted. While a comparative analysis of those parameters can be found in Faella & Al. (2008), in the present paper interstorey drift is considered as demand measure as usually adopted in similar applications. The probabilistic nature of the demand function can be reproduced by assuming the following expression for D

$$D = \hat{D} \cdot \varepsilon \tag{2.3}$$

being $\hat{D}$ the median value of demand D for a given intensity value s_a and ε a log-normal random variable with unit median and dispersion β_D (Cornell & Al., 2002). The relationship between the intensity measure s_a and the median of the demand measure $\hat{D}$ in eq. (2.3) can be assumed as follows

$$\hat{D} = a \cdot s_a{}^b \tag{2.4}$$

where the constants a and b have to be determined through dynamic analyses using a sufficiently wide number of recorded accelerograms (Pinto & Al., 2004).
Since seismic hazard is the key parameter to be taken into account, it can be determined through the hazard curve $H(S_a)$ providing the annual probability for seismic events to exceed the intensity value s_a; Such curve can be reasonably approximated as follows:

$$H(S_a) = \Pr(S_a \geq s_a) = k_0 \cdot s_a{}^{-k} \tag{2.5}$$

which, in the corresponding log-log plane, represents a linear relationship between seismic intensity and probability of exceeding that intensity value in one year (Cornell & Al., 2002).
Based on the above hypotheses and possibly assuming that even capacity is log-normally distributed around the median value $\hat{C}_{LS}$ and dispersion $\beta_{C,LS}$, the following expression can be derived for the probability of the structure to achieve the limit state LS:

$$P_{LS} = H\left(S_a\left(\hat{C}_{LS}\right)\right) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\left(\beta_D{}^2 + \beta_{C,LS}{}^2\right)} \;. \tag{2.6}$$

Such a probability is basically obtained as the product of two factors whose meaning can be easily understood:
- $H\left(S_a\left(\hat{C}_{LS}\right)\right)$ is the annual probability of occurrence of an earthquake whose intensity S_a is larger than the value s_a which in Eq. (2.6) corresponds to a median demand $\hat{D} = \hat{C}$; hence, this term represents the probability for the structure to achieve the LS of interest considering both capacity and demand as deterministic variables, the former one defined as a function of the geometric and mechanical model of the structure members and the latter one basically defined as a function of the seismic intensity measure;
- $e^{\frac{1}{2}\frac{k^2}{b^2}\left(\beta_D{}^2 + \beta_{C,LS}{}^2\right)}$ represents an amplification factor accounting for the probabilistic nature of both seismic demand and structural capacity. Further details on the theoretical derivation of Eq. (2.6) and its practical implementation within the FEMA-350 Guidelines (FEMA, 2000) can be found in Cornell & Al. (2002).

2.2. Fragility Analysis of Structures

The procedure described in the previous paragraph provides a closed-form solution for integrals in Eq. (2.1) and

(2.2) at the price of introducing some simplifying hypotheses such as those in Eq. (2.3) and (2.5). Alternative procedures are also available for the same purpose removing some of those hypotheses, but losing the attractive feature of a solution in closed-form. Indeed, starting from Eq. (2.2), the probability $P_{LS}(i)$ of exceeding a given Limit State LS conditioned to a consistent seismic intensity parameter i (namely, pseudo-acceleration S_a) can be defined considering the PDF of demand on the structure conditional to the same intensity measure:

$$P_{LS}(i)=\int_0^{\infty} F_{C,LS}(\alpha)\cdot f_D(\alpha|i)\cdot d\alpha \quad , \tag{2.7}$$

which, by definition, represents the fragility of the structure with respect to the considered Limit State LS. The unconditional probability of exceeding that same Limit State (namely, the seismic risk) can be easily evaluated as seismic hazard is known:

$$P_{LS}=\int_0^{\infty}\left|\frac{dH}{di}\right|\cdot P_{LS}(i)\cdot di \quad . \tag{2.8}$$

Seismic Fragility of the structure is usually evaluated through non-linear time-history analyses considering a group of recorded accelerograms, possibly scaled to the same value of the intensity measure *i*. The probability of failure of the structure under the k-th record can be estimated as follows

$$P_{LS,k}(i)=\Pr\left[C\le D_{k,i}\right]=F_C\left(D_{k,i}\right) \quad , \tag{2.9}$$

and seismic fragility can be finally derived

$$P_{LS}(i)=\frac{1}{n}\cdot\sum_{k=1}^{n}P_{LS,k}(i) \quad . \tag{2.10}$$

Finally, seismic risk P_{LS} can be determined by calculating the integral in Eq. (2.8) through well-known numerical methods.

3. NON LINEAR ANALYSES FOR SEISMIC RELIABILITY OF STRUCTURES

Both procedures shortly outlined above provides structural engineers with somehow simplified methodologies for evaluating seismic risk, namely the mean annual probability P_{LS} of a structure to achieve a given Limit State LS as exposed to a seismic hazard fully described by hazard curves. The complement of PLS is a measure of seismic reliability. Although their simple conceptual framework they both require to carry out a huge number of non-linear time history analyses that are as time-consuming as the structural model aims to closely reproduce the actual behavior of real structures. In particular, Incremental Dynamic Analyses (IDA in the following as in Vamvatsikos & Cornell, 2002) have to be carried out on structures for calibrating parameters a and b in Eq. (2.4) or determining seismic fragility according to in Eq. (2.10). For what concerns the former problem, Dolsek & Fajfar (2004) proposed a simple alternative procedure to IDA based on applying N2-Method considering demand spectra characterized by an increasing value of the intensity parameter (namely, the pseudo-acceleration at the fundamental period of vibration of the structure) with the aim of directly obtain the relationship between seismic demand D and intensity measure S_a. Consequently, an Incremental Static Analysis procedure has been proposed in that paper (Incremental N2 Method, namely IN2) for deriving the coefficient a and b in Eq. (2.4). According to that method, only one capacity curve (of course, for each relevant direction and horizontal force distribution) is needed for obtaining the relationship between the intensity measure S_a and the corresponding demand parameter D. However, since design spectra have been utilized in that case, no estimation of the record-to-record variability can be derived by the analyses and the mentioned authors just taken somehow typical values of dispersion β_D based on the assumption that top displacement values are affected by a 0.7 CoV as they derived by non-linear time-history analyses in similar cases.

The first innovation introduced by the present paper for improve the meaningfulness of Incremental Static Analyses consists in utilizing natural spectra, directly derived by the same recorded accelerograms which could have been utilized for performing time-history analyses for IDA. Natural spectra which their rough shape can be directly utilized for applying N2-method with the aim of reproducing the record-by-record variability which is typical of time-history analyses. Figure 1 shows the possible application of N2-Method with natural spectra. In particular, two spectra referred to the same natural accelerogram and scaled by different factors are represented in the mentioned figure and displacement demand can be easily derived by applying the equal-displacement rule as assumed by N2-Method (Fajfar, 1999) as the period of vibration T* of the SDOF equivalent to the given MDOF structure is larger than the period $T_0 \approx T_C$ which represents the intersection between the constant pseudo-acceleration branch and the constant pseudo-velocity one according to the Newmark & Hall (1982) idealization of seismic spectra. Since T* is usually longer than T_C for existing RC structures as a result of their lack in lateral strength and stiffness and provided that the interest of these authors is mainly focused on such structures rather than new ones, equal-displacement rule can be generally applied; nevertheless, for each spectra T_C could be even estimated (Newmark & Hall, 1982) and complete formulation of the demanded ductility according to N2-Method can be easily applied for estimating displacement demand (Fajfar, 1999).

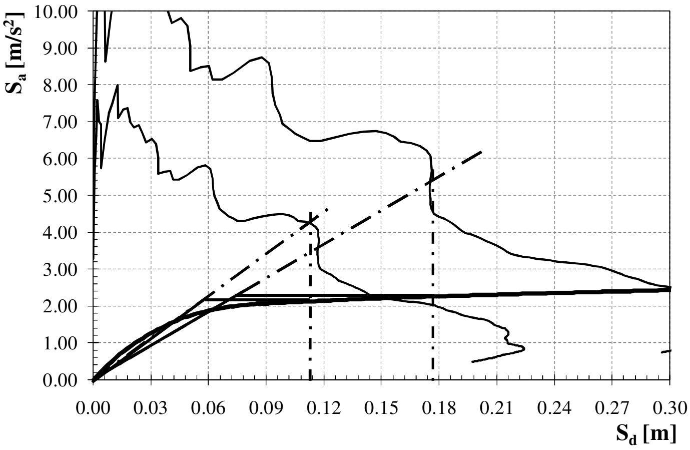

Figure 1 – Schematic application of the IN2 method

Furthermore, the probability $P_{LS,k}(i)$ could be even estimated by Eq. (2.9) considering the natural spectrum corresponding to the k-th accelerogram and suitably scaled to the intensity *i*; the displacement demand $D_{k,i}$ can be even estimated through N2-method and the corresponding probability $F_C(D_{k,i})$ can be also determined as a result of the probabilistic model assumed for describing the capacity of the structural members. In particular, if capacity is kept deterministic and denoted by the constant value C, $P_{LS,k}(i)$ is a Boolean variable taking zero if $D_{k,i}<C$ and one otherwise. Consequently, incremental N2-Method can be also utilized for determining fragility curves of the given structure through equation (2.10).
Finally, reliability analyses carried out by applying N2-Method, in the incremental formulation utilizing natural spectra directly derived by recorded accelerograms shortly outlined above, needs to be compared in terms of results which can be obtained by applying non-linear time-history analyses as usually assumed in the original formulations of SAC-FEMA method and Fragility Analysis outlined in section 2. It is easy to understand and possibly quantify how cost-effective would be static-analysis-based methodologies in terms of computational costs with respect to the usual procedures based on dynamic analyses. Indeed, reasonably controlled differences arising by static analyses with respect to dynamic ones, especially if on the safe side, would be also tolerated considering the cost-effectiveness of the former methodology.

4. SAMPLE APPLICATION

Besides the methodological consistency of the proposed methods which has been shortly outlined within the previous paragraph, a preliminary comparison between the results of a reliability analysis carried out through non-linear static analyses has to be carried out. Far from obtaining a complete validation of the procedure outlined in section 3, which would need a wider parametric field, especially in terms of structural typologies to be investigated, the present section will present a sample application of reliability analysis carried out by both non-linear time-history analyses and static pushover ones utilizing the N2 Method with natural spectra.

4.1. Analyzed structure and considered ground motions

The sample structure is a four storey building ideally designed to gravitational load only according the old

Italian Code for concrete structures (Regio Decreto, 1939). Its plan view is represented in Figure 2, while member dimensions as well as rebar amount is omitted herein for the sake of brevity and can be drawn by Faella & Al. (2008) along with other relevant information about the numerical model which has been developed in OpenSEES (Fenves & Al., 2004).

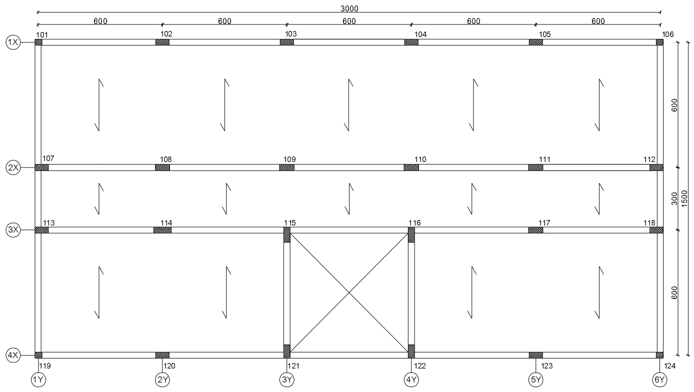

Figure 2 – Plain view of the structure

The seismic input for IDA has been obtained from a set of 45 recorded accelerograms, whose corresponding response spectra have been utilized in static analysis carried out through N2; intensity measure S_a has been assumed ranging between 0.25 m/s^2 and 5.00 m/s^2, with 0.25 m/s^2 acceleration steps.

4.2. Static versus Dynamic pushover curves

Since the key aspect to be investigated deals with comparing the results of static and dynamic analyses, a first comparison is shown in terms of the relationship between top displacement Δ_{top} and base shear force V_{base} in both the main direction of the structure. Static pushover analyses have been performed by assuming the two horizontal force distributions either proportional to the storey masses or to the product of those masses by the lateral displacement deriving by the first relevant mode of vibration of the structure.

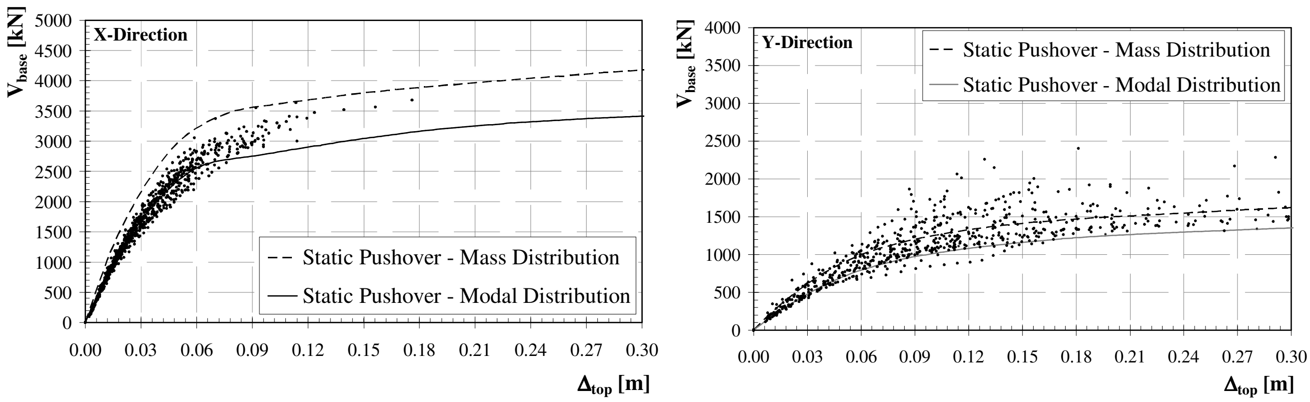

Figure 3 – Capacity curves obtained through Static and Dynamic Pushover Analysis.

Figure 3 confirms the substantial agreement between static pushover and dynamic analyses. Indeed, initial lateral stiffness is always underestimated by static pushover analyses, even in the case on the so-called modal force distribution, as a result of the participation of higher modes of vibrations whose effect is reproduced in dynamic analyses and somehow neglected in static ones. On the contrary, static analyses results in a rather accurate prediction of the ultimate base shear especially in X-direction in which complete frame structures are present; yet, response is slightly more irregular in Y-direction where no complete frames are present as often occurs in existing RC structures designed for only gravitational actions.

4.2. Incremental Analysis: Static versus Dynamic Procedure

Incremental Dynamic Analysis (IDA) has been carried out with reference to the 45 accelerograms deriving by seismic records in sites of different soil characteristics. The results obtained in terms of IDA curves are represented in Figure 4 in which the difference between the two main directions of the structure is clearly pointed out.

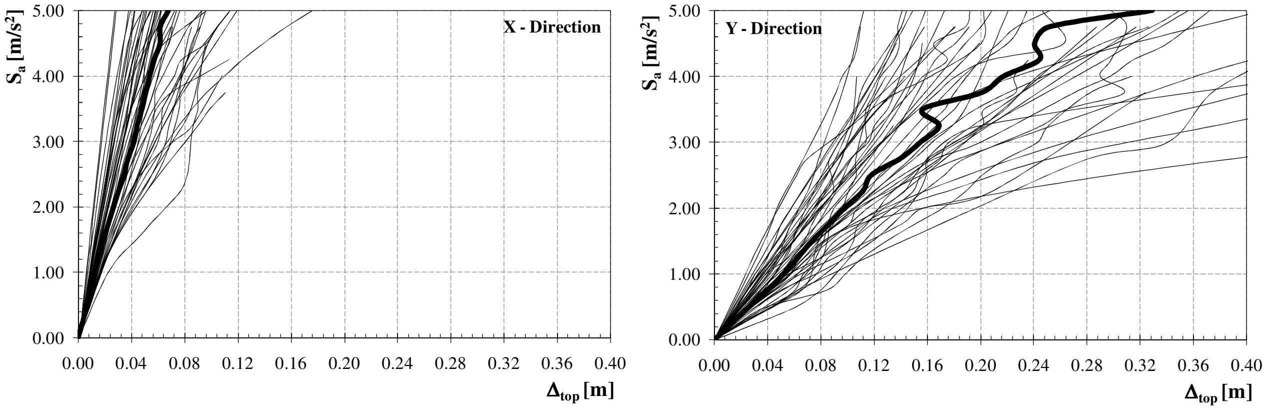

Figure 4 – Incremental Dynamic Analyses: 45 accelerograms and median value

Similar curves can be also derived through pushover analysis and the N2-Method considering for both directions the 45 response spectra derived by the mentioned accelerograms and the two horizontal force distributions applied at the various level of the structure.

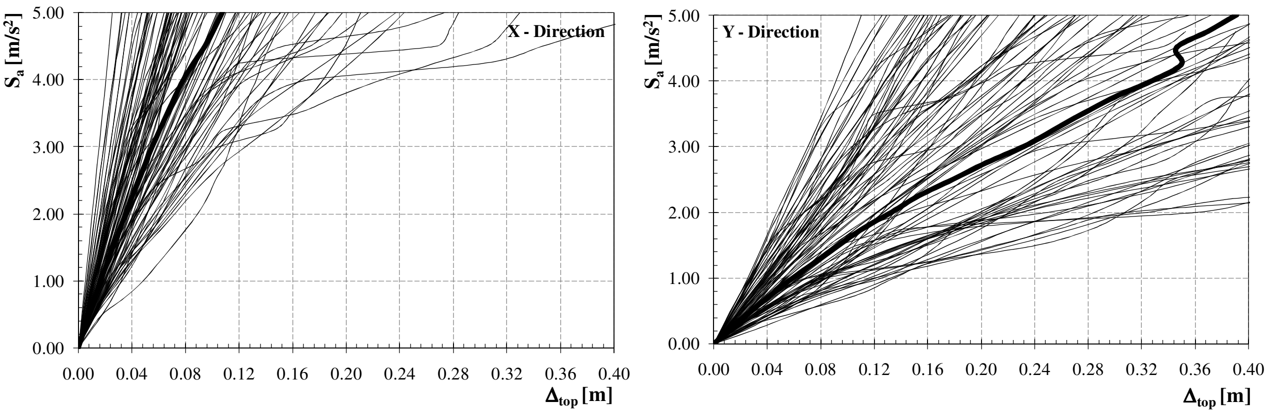

Figure 5 – Incremental Static Analysis: 45 accelerograms and median value

A direct comparison between the median curves obtained in both directions by means of the two procedures is shown in Figure 6 in which the median of the results obtained by static analysis is evaluated both considering all the results obtained by N2-Method and splitting such results depending on the distribution of lateral force utilized for deriving the capacity curve. Such figure shows that, in both directions, static analysis results in a conservative estimation of the displacement demand median value $\hat{D}$ (in terms of top displacement Δ_{top}). Furthermore, no significant difference can be observed if one looks after the results obtained by pushover based on either "modal" force distribution or "mass" one.
Finally, the record-to-record variability of the results in terms of demand D can be considered for both static- and dynamic-based procedures. Figure 7 shows the values derived for β_D in both the main directions of the structures pointing out that no relevant difference arises between the dispersion in static and dynamic analyses for low values of pseudo acceleration S_a, namely as the structure completely responds within the linear elastic range. Variation of β_D as a function of the intensity measure S_a is similar under the qualitative standpoint, regardless the analysis method, while static analyses provide the most conservative estimation even in terms of dispersion β_D; these somehow surprising result can be explained if one considers that results of pushover analyses carried out under two lateral force distributions are taken into account for evaluating both the median demand value $\hat{D}$ and its dispersion β_D. Moreover, values of dispersion ranging 0.4 and 0.6 confirms the

results obtained by other authors and are also close o those assumed *a priori* by Dolsek & Fajfar (2004).

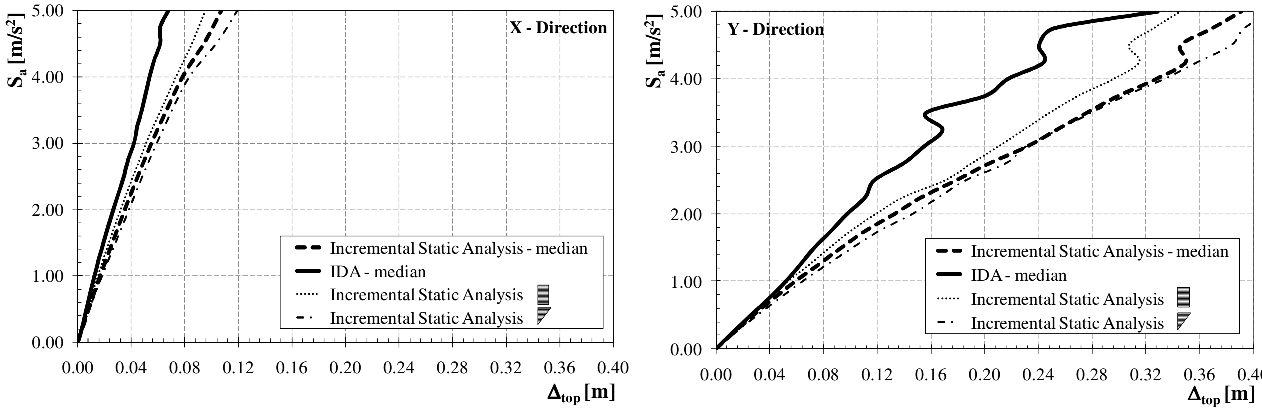

Figure 6 – Comparison between Incremental Static and Dynamic Analysis.

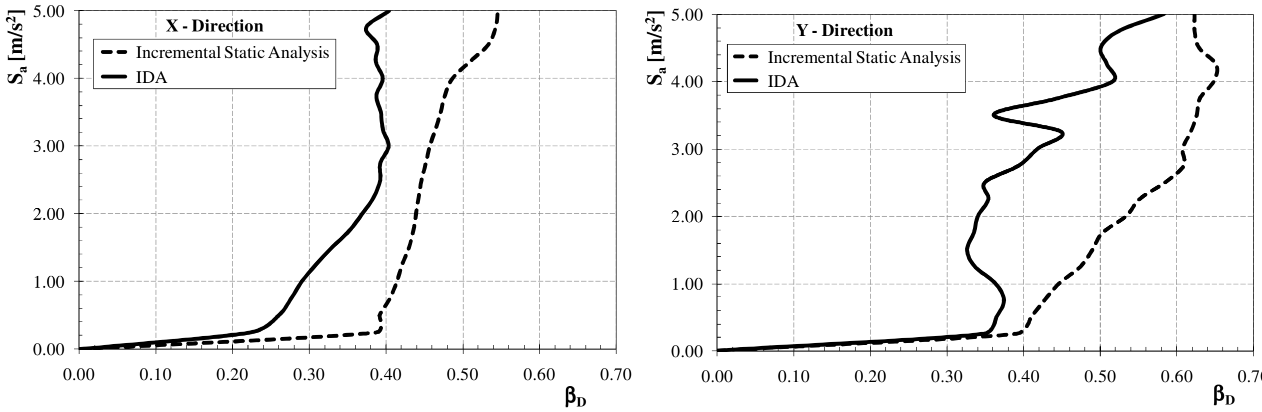

Figure 7 – Comparison between Static and Dynamic Analysis in terms of dispersion parameter β_D.

As a final remark, static analyses provide a conservative estimation of seismic risk P_{LS} evaluated according to Eq. (2.6) being both its factors larger than the corresponding values derived through dynamic analyses.

4.2. Fragility Curves: Static versus Dynamic Approach

Deriving fragility curves by non-linear static analyses is also straightforward as described in paragraph 2.2. Figure 8 show the results obtained on the considered case-study with reference to the two main directions.

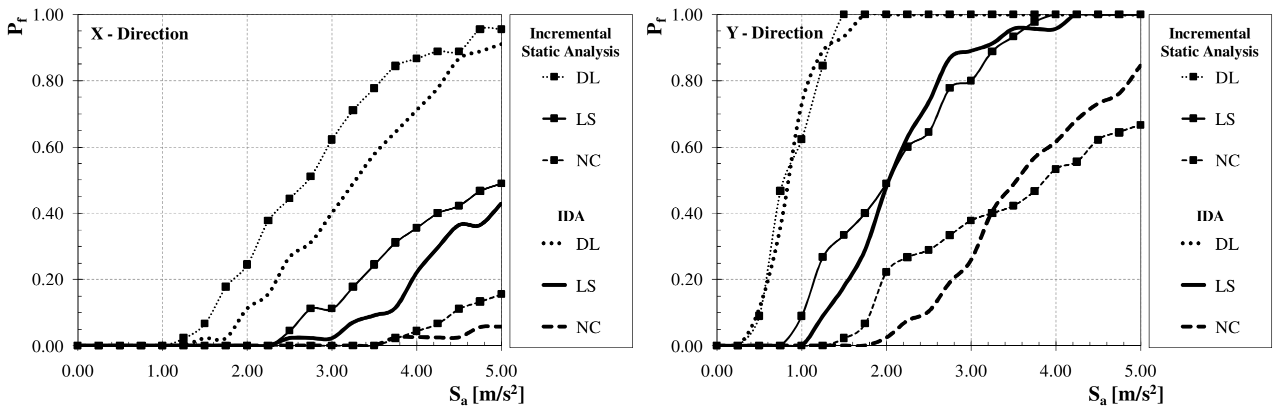

Figure 8 – Comparison between Static and Dynamic Analysis in terms of Fragility Curves

The three relevant limit states (namely, Damage Limitation, Life Safety and Near Collapse) like those mentioned within EC8 (2003, 2004) and other codes of standards are considered in the analyses assuming displacement deterministic capacity described by interstorey drift values of 0.4%, 1.0% and 2.0% as suggested by Kappos (1991). The mentioned figure points out that non-linear static analyses usually provide conservative estimation of fragility $P_{LS}(i)$ confirming the general trend observed in the previous section for the SAC-FEMA method. Indeed, only fragility curves in Y-directions with respect to the Limit States of LS and NC do not follow the mentioned trend probably as a result of the possible loss of accuracy of the equal-displacement rule in cases of very low design ratio.

5. CONCLUSIONS

In the present paper, two procedures aimed at applying well-known methods for seismic reliability evaluation of structures are outlined as possible generalization of the so called IN2 method proposed by Dolsek & Fajfar (2004). Using response spectra directly derived by recorded accelerograms is the key feature of the two proposed generalizations of that method. Consequently, direct estimation of dispersion of seismic demand parameter with respect to the assumed intensity measure can be directly estimated through N2 Method in its incremental version described in section 3. Finally, a sample application of both procedures has been shown and commented in the final section of the paper, confirming that static analyses can be proficiently applied, usually resulting in conservative evaluation of seismic risk and reliability of structures. A wider validation of such procedures is however needed for e mode comprehensive proof of this finding.

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