# A SIMPLE DESIGN METHOD FOR YIELDING STRUCTURES SUBJECT TO TORSION

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**ABSTRACT :**

Single story structures with different in-plane wall strength and stiffness, rotational inertia, and out-of-plane wall stiffness are subjected to impulse ground motions to obtain their dynamic response considering torsion. A simple algorithm is developed to model this behaviour. It is shown that the median increase in response of the critical component considering torsion from earthquake records is similar to that from impulse records. A simple design methodology is then proposed which enables the likely critical element earthquake response considering torsion to be obtained from building analyses not considering torsion.

**KEYWORDS:** Torsion, Impulse, Building Inelastic Response

## 1. INTRODUCTION

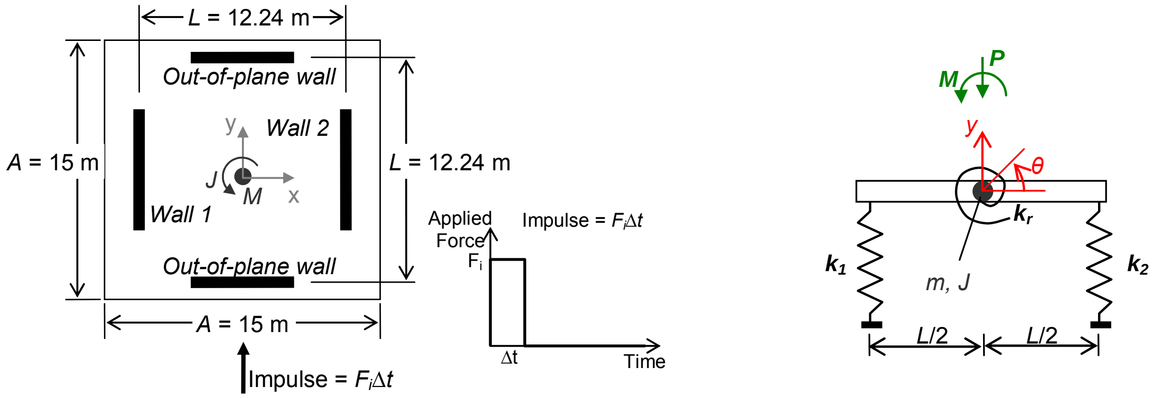
Torsional response of structures during earthquake shaking is caused by stiffness and strength eccentricities relative to the centre of mass or by torsional ground motions. The significant body of research on the asymmetric response of these structures has been summarized by Rutenburg (1998), De Stefano and Pintucchi (2006) and Au et al. (2008). While much has been done, there is still a need for a simple method to understand the total response of general structures which may deform torsionally during earthquake motion. This method should consider rotational inertia, out-of-plane walls, and both high and low ductility demands. It should be appropriate for structures with both high and low levels of torsional sensitivity, for different assumptions related to strength and stiffness dependency, and it should have a strong fundamental (rather than empirical) basis. The method should also be able to be transformed into a design/assessment method so that important effects of torsion can be anticipated and applied appropriately to design methods to mitigate these effects.

This paper is a step towards satisfying this need for single story structures considering dynamic effects using impulse loading, or a nonlinear impulse procedure (NIP). In particular, answers are sought to the following questions:

i) Can a closed-form solution be derived for a single-storey system subject to an impulse based on two-degree-of-freedom free vibration concepts?
ii) What is the applicability of the non-linear impulse procedure (NIP) in the context of a design approach for earthquake excitation?
iii) Can a design approach based on impulse response be developed?

## 2. BENCHMARK STRUCTURE AND MODELLING

The benchmark single story building, shown in Figure 1 below, was used in analyses by Castillo (2004). Benchmark parameters used in this study are given in Table 1. All analyses used these parameters unless expressed otherwise. The floor slab was assumed to be a perfectly rigid diaphragm.




(a) Plan View (ii) Schematic for Modelling

Figure 1. Benchmark building (Castillo, 2004)

***Table 1. Benchmark Values***

| Parameter | Symbol | Benchmark value |
|---|---|---|
| Mass | $M$ | 1766 kN |
| Rotational mass | $J$ | $J_r$= 66225 kNm$^2$ |
| Wall 1 stiffness | $k_1$ | 14212 kN/m |
| Wall 2 stiffness | $k_2$ | $KR\times k_1$ (where $KR$=1) |
| Wall 1 yield strength | $F_{y1}$ | 220.3 kN |
| Wall 2 yield strength | $F_{y2}$ | $SR\times F_{y2}$ (where $SR$=1.36) |
| Out-of-plane wall stiffness | $k_{out}$ | $KR_{out}\times k_1$ (where $KR_{out}$=0) |
| Bilinear factor | $r$ | 0.0001 |

The mass of the square diaphragm was represented as both a point mass, $M$, and a rotational mass, $J$, coincident with the centre of the rigid diaphragm. The rotational mass associated with $J_r$ was that for a uniformly distributed mass over a square floor slab. An in-plane wall stiffness ratio, $KR$, of 1.0 and a strength ratio, $SR$, of 1.36 (i.e. $F_{y2}$ = 1.36×220.3 = 300 kN) were adopted as benchmark values. A bilinear factor, $r$, of 0.0001 was chosen to provide post-yield stability for the computer model and approximate elastic perfectly plastic response. The out-of-plane walls were modelled as an equivalent rotational spring located at the centre of mass. Herein, out-of-plane wall stiffness is specified as a ratio, $KR_{out}$, of Wall 1's benchmark stiffness. For all analyses it was assumed that these walls would remain elastic throughout the entire response. No out-of-plane walls ($KR_{out}$=0) were adopted as the benchmark.

All NIP analyses were performed using RUAUMOKO-2D (Carr, 2005) using Newmark's constant average acceleration integration method. The impulse, $F_i\Delta t$, shown in Figure 2, was a constant force over one time-step, $\Delta t$, of 0.001 s. Damping was ignored in all impulse analyses to allow a simple closed form solution to be developed. The impulse force magnitude was that required to produce a benchmark translational ductility demand of 5 for the building with no twist about the vertical axis. This was found to be 105,210 kN, and produced an ultimate displacement of 77.5 mm, five times the yield displacement of 15.5mm.

## 3. TWO DEGREE OF FREEDOM MODEL

A generalised 2DOF analytical model was developed to investigate single storey building torsional response to impulse using the plan view schematic in Figure 1a. The two degrees of freedom are translation, $y$, and rotation, $\theta$, at the centre of mass. To keep this solution as general as possible, allowance for out-of-plane walls was made, given by an equivalent rotational spring of stiffness $k_r$.

The equations of motion for the above system are given in Equation 3.1. This set of equations can be solved (Au, 2007) to give Equations 3.2 to 3.5.

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & (k_2 - k_1)L/2 \\ (k_2 - k_1)L/2 & k_r + (k_1 + k_2)L^2/4 \end{bmatrix}\begin{Bmatrix} y \\ \theta \end{Bmatrix} = \begin{Bmatrix} -P \\ M \end{Bmatrix} \tag{3.1}$$

$$\begin{aligned} y = {} & A\Phi_{1y}\cos(\omega_1(t-t_0)) + B\Phi_{1y}\sin(\omega_1(t-t_0)) + \Phi_{1y}\frac{P_y*}{K_y*} \\ & + C\Phi_{2y}\cos(\omega_2(t-t_0)) + D\Phi_{2y}\sin(\omega_2(t-t_0)) + \Phi_{2y}\frac{P_\theta*}{K_\theta*} \end{aligned} \tag{3.2}$$

$$\begin{aligned} \theta = {} & A\Phi_{1\theta}\cos(\omega_1(t-t_o)) + B\Phi_{1\theta}\sin(\omega_1(t-t_o)) + \Phi_{1\theta}\frac{P_y*}{K_y*} \\ & + C\Phi_{2\theta}\cos(\omega_2(t-t_o)) + D\Phi_{2\theta}\sin(\omega_2(t-t_o)) + \Phi_{2\theta}\frac{P_\theta*}{K_\theta*} \end{aligned} \tag{3.3}$$

$$\begin{aligned} \dot{y} = {} & -A\Phi_{1y}\omega_1\sin(\omega_1(t-t_o)) + B\Phi_{1y}\omega_1\cos(\omega_1(t-t_o)) \\ & - C\Phi_{2y}\omega_2\sin(\omega_2(t-t_o)) + D\Phi_{2y}\omega_2\cos(\omega_2(t-t_o)) \end{aligned} \tag{3.4}$$

$$\begin{aligned} \dot{\theta} = {} & -A\Phi_{1\theta}\omega_1\sin(\omega_1(t-t_o)) + B\Phi_{1\theta}\omega_1\cos(\omega_1(t-t_o)) \\ & - C\Phi_{2\theta}\omega_2\sin(\omega_2(t-t_o)) + D\Phi_{2\theta}\omega_2\cos(\omega_2(t-t_o)) \end{aligned} \tag{3.5}$$

where:

$$K_y* = \Phi_{1y}{}^2(k_1+k_2) + \Phi_{1y}\Phi_{1\theta}(k_2-k_1)L + \Phi_{1\theta}{}^2(k_1+k_2)\frac{L^2}{4} + \Phi_{1\theta}{}^2 k_r \tag{3.6}$$

$$K_\theta* = \Phi_{2y}{}^2(k_1+k_2) + \Phi_{2y}\Phi_{2\theta}(k_2-k_1)L + \Phi_{2\theta}{}^2(k_1+k_2)\frac{L^2}{4} + \Phi_{2\theta}{}^2 k_r \tag{3.7}$$

$$P_y* = -\Phi_{1y}P + \Phi_{1\theta}M \tag{3.8}$$

$$P_\theta* = -\Phi_{2y}P + \Phi_{2\theta}M \tag{3.9}$$

From the initial conditions: $y(t_o) = y_o$, $\dot{y}(t_o) = \dot{y}_o$, $\theta(t_o) = \theta_o$ and $\dot{\theta}(t_o) = \dot{\theta}_o$, the constants are:

$$A = \frac{\theta_o\Phi_{2y} - y_o\Phi_{2\theta}}{\Phi_{2y}\Phi_{1\theta} - \Phi_{1y}\Phi_{2\theta}} - \frac{P_y*}{K_y*} \qquad B = \frac{\dot{\theta}_o\Phi_{2y} - \dot{y}_o\Phi_{2\theta}}{\omega_1(\Phi_{2y}\Phi_{1\theta} - \Phi_{1y}\Phi_{2\theta})}$$

$$C = \frac{y_o\Phi_{1\theta} - \theta_o\Phi_{1y}}{\Phi_{2y}\Phi_{1\theta} - \Phi_{1y}\Phi_{2\theta}} - \frac{P_\theta*}{K_\theta*} \qquad D = \frac{\dot{y}_o\Phi_{1\theta} - \dot{\theta}_o\Phi_{1y}}{\omega_2(\Phi_{2y}\Phi_{1\theta} - \Phi_{1y}\Phi_{2\theta})}$$

The frequencies, $\omega_i$, and mode shapes, $\Omega_i$, are found from a free-vibration analysis ($M$ = 0, $P$ = 0) where the fundamental frequency, $\omega_1$, is the lowest obtained using the negative sign in Equation 3.10.

$$\omega^2 = \frac{1}{2mJ}\left[ m\left(k_r + \frac{(k_1+k_2)L^2}{4}\right) + J(k_1+k_2) \pm \sqrt{\left(m\left(k_r + \frac{(k_1+k_2)L^2}{4}\right) - J(k_1+k_2)\right)^2 + mJL^2(k_2-k_1)^2}\right] \quad (3.10)$$

$$\{\Phi_i\} = \begin{Bmatrix} 1.0 \\ \dfrac{2m\omega_i^2}{L(k_2-k_1)} - \dfrac{2(k_1+k_2)}{L(k_2-k_1)} \end{Bmatrix} \quad (3.11)$$

The displacement of each wall in this case can then be calculated by summing the translational response at the centre of mass with the additional displacement due to rotation, such that:

$$y_{Wall1} = y - \frac{\theta L}{2} \quad (3.12)$$

$$y_{Wall2} = y + \frac{\theta L}{2} \quad (3.13)$$

The solution was found to become unstable when the strength and stiffness of both walls were identical and when rotational mass was zero – effectively a SDOF system. While the second degree of freedom could be condensed out, a small rotational mass of 1 kNm$^2$ and a 0.01 kN/m difference in wall stiffness was found to provide stability without loss of accuracy.

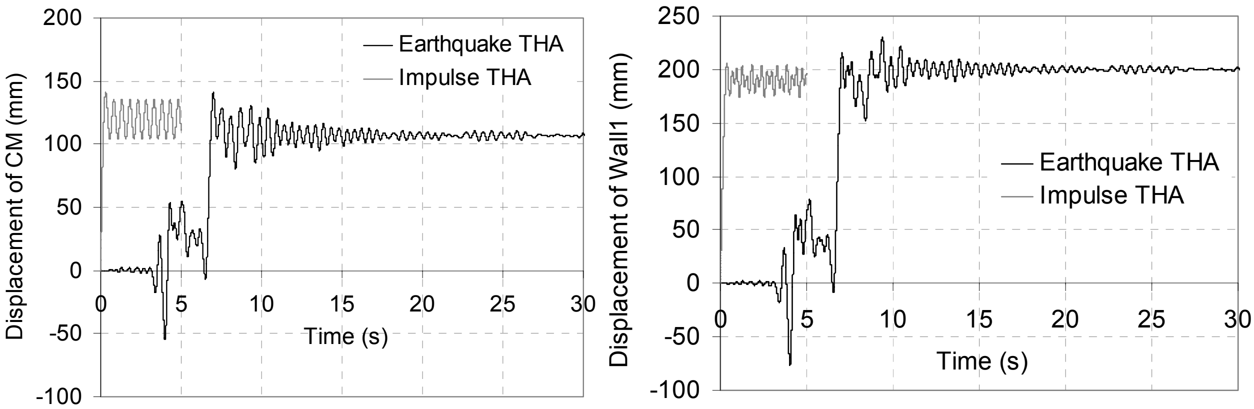
This solution, with specified loading, can be modified to consider elastic free vibration, yielding of the first wall, and yielding of the second wall. Specifying a rotational stiffness also allows out-of-plane walls to be considered. During the elastic portion of the free vibration response, the 2DOF system has $M = 0$ and $P = 0$. Both walls will have their respective elastic stiffness of $k_1$ and $k_2$, and the system will have the following initial conditions if the impulse duration is very small: $t_o = 0$, $y_o = 0$, $\dot{y}_o = \dot{u}_i$, $\theta_o = 0$ and $\dot{\theta}_o = 0$. Here, $\dot{u}_i$ is the velocity imparted to the structure by the impulse. At yielding of the first wall, Wall 1, its stiffness becomes $rk_1$. The yield force in Wall 1 is then represented as a constant force $P = F_{y1}$, and moment, $M = F_{y1}L/2$, applied at the centre of mass. The initial conditions are then the final conditions of the elastic response. When the second wall, Wall 2, yields, it now has its post-elastic stiffness, $rk_2$. Similarly, the yield force at Wall 2 can be represented by $P = (F_{y1} + F_{y2})$ and $M = (F_{y1} - F_{y2})L/2$ at the centre of mass. The initial conditions are taken as the values when the second wall reaches its yield displacement. It should be noted that there is no simple closed form solution for the peak response, but the solution can be obtained using simple numerical methods. For layouts with different numbers of walls and floor plans, a revised and more complex solution may be developed.

The equations described generally predict the peak wall displacement due to impulse well. However, if the second wall reaches its peak displacement before the weaker wall, Wall 1, attains its respective maximum displacement, the peak response is not predicted well by the 2DOF solution. This is because Wall 2 is unloading and this is not captured in the simple analytical solution. In this case, the peak displacements estimated by these equations may be slightly overestimated which would be conservative if they were used for design (Au, 2007, Au et al. 2008).

## 4. CONSIDERING EARTHQUAKE EFFECTS USING IMPULSE METHOD

To evaluate the effect of earthquakes on the response of structures, structures with a number of periods and configurations were analyzed. The response of the structure was then compared to the impulse response, using an impulse magnitude which produced the same translational displacement when torsion was restrained.

The impulse response was compared to an earthquake time history analysis using the 1994 Northridge Earthquake (Sylmar) NS record. Analyses were performed on the benchmark structure given in Figure 1a and Table 1 using the computer program RUAUMOKO-2D. To approximate the damping in real structures, an initial stiffness Rayleigh damping model was used in the earthquake analysis, with 5% of critical damping specified in modes 1 and 2. The fundamental period of the frame was 0.5s. Firstly, analysis of the torsionally restrained structure was carried out with the earthquake record to obtain the displacement as shown in Figure 2(a). This shows that the peak displacement occurs over a very short time, so it is reasonable to use an impulse approximation to predict the response. The structure required an impulse of 157 MN force acting for 0.001 s to push the structure to the same peak displacement as that obtained from the earthquake record (with the rotational degree of freedom still restrained). The earthquake and impulse time history analyses were then performed again with the rotational degree freedom unrestrained. Figure 2(b) shows that impulse underpredicts the peak critical wall displacement by 10%. In general, the impulse response would not be expected to be identical to that of the earthquake.



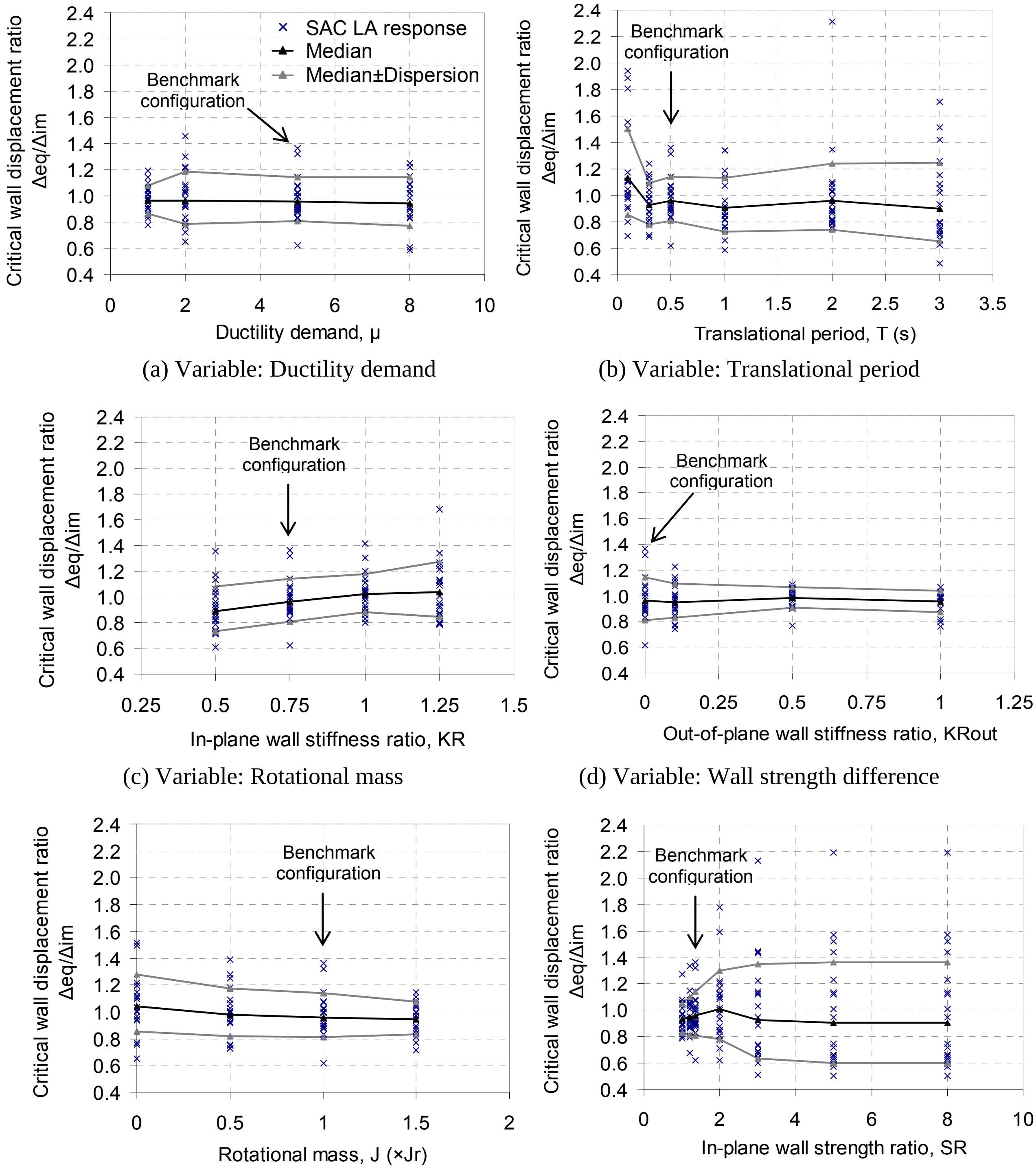

(a) Torsion Restrained (b) Torsion Unrestrained

Figure 2. Translational response of Critical Wall, Benchmark Structure – Sylmar NS Record

The single record methodology in (a) above was repeated using the 20 LA SAC design level earthquake records to statistically quantify the difference between predicted displacements from earthquake time history analyses and nonlinear impulse procedures for different structural configurations. Parameters addressed were: excitation scale factors for both the impulse and earthquake analyses to produce ductility demands of 1, 2, 5 and 8 in the critical wall when torsion is restrained; various masses to achieve periods of 0.1, 0.3, 0.5, 1, 2 and 3 seconds; rotational masses of 0, 0.5 $J_r$, $J_r$ and 1.5 $J_r$; wall strength ratios of 1, 1.2, 1.36, 2, 3, 5 and 8; out-of-plane wall stiffness ratios of 0, 0.1, 0.5 and 1; and lastly, in-plane wall stiffness ratios of 0.5, 0.75, 1 and 1.25.

Impulse and earthquake scale factors were found by iteration using the bisection method prior to each 2DOF time history analysis. Iteration continued until the translational (1DOF) ductility was within 1% of the target ductility. The subsequent 2DOF simulation, allowing twist, was carried out using the scale factor previously found to obtain the peak critical wall displacement. These analyses were automated. The results are plotted as a ratio of peak earthquake displacement to peak impulse displacement in Figure 3. The median ± dispersion lines represent the 16th and 84th percentile values found by fitting a log-normal distribution.

Figure 3 shows that the median earthquake response of the critical wall considering torsion is similar to the impulse response considering torsion for all parameters. Based on a log-normal distribution, the overall median of 0.96 and dispersion, which is the standard deviation of the natural logarithm of the data of 0.19, can be used to amplify results from NIP to give a final prediction of peak earthquake response with a specified statistical level of confidence. A procedure to do this is described in the following methodology.




(a) Variable: Ductility demand (b) Variable: Translational period

(c) Variable: Rotational mass (d) Variable: Wall strength difference

(e) Variable: Wall stiffness difference (f) Variable: Out-of-plane wall stiffness

Figure 3. Ratio of peak SAC LA earthquake response to peak NIP response
Benchmark structure: μ=5, $T$=0.5s, $J$=$J_r$, $SR$=1.36, $KR$=0.75 and $KR_{out}$=0

## 5. DESIGN APPROACH

The following step-by-step methodology is proposed for single storey structures with configurations similar to that in Figure 1a which are subject to earthquake excitations.

Step 1. Estimate the likely displacement response of the structure using standard methods assuming that no twist occurs about the vertical axis, $\Delta_{NoTwist}$.

Step 2. Find the impulse, $F\Delta t$, that would push the structure to $\Delta_{NoTwist}$, if twist is restrained and no damping is assumed, using the 2DOF methodology developed above to model the structure. This may be carried out using a very high value for $k_r$.

Step 3. Use $F\Delta t$ to obtain the response of the critical wall, $\Delta_{Twist}$, if twist about the vertical axis is not restrained, using the 2DOF methodology to model the structure. A realistic $k_r$ value should be used.

Step 4. Obtain the ratio of earthquake displacement to impulse displacement from the previous section, $R_{E|I}$ for the desired statistical level of confidence as 0.96 × exp(0.19 × NS), where NS is the number of standard deviations above the median in a normal distribution to provide a desired level of confidence.

Step 5. Multiply $\Delta_{Twist}$ by $R_{E|I}$ to estimate the demand in the critical element.

## 6. DESIGN EXAMPLE

Step 1. For the structure given in Figure 4, the fundamental period with no torsion is 0.7s. It is subject to an earthquake which produces a total displacement of $\Delta_{NoTwist}$ = 60 mm when twist about the vertical axis is restrained. Any method satisfying code requirements can be used for this displacement prediction.

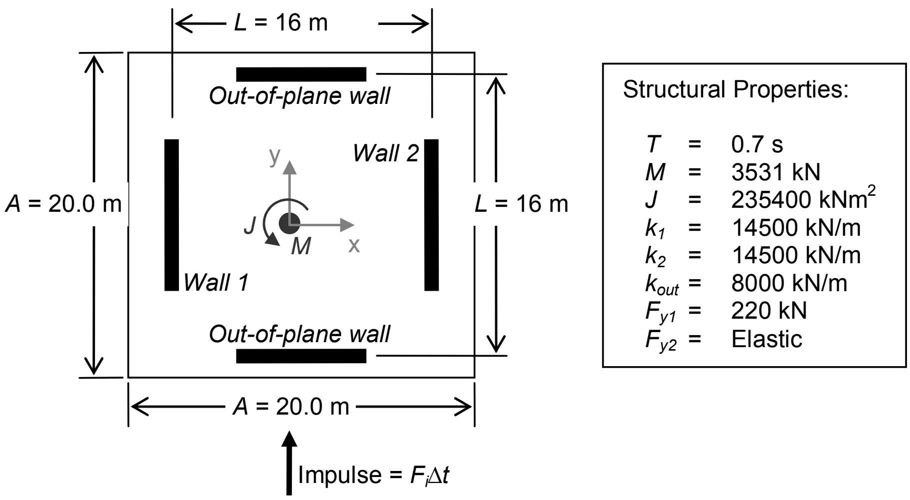



Figure 4. Plan and Properties

Step 2. By trial and error, the impulse, $F\Delta t$, that would push the structure to $\Delta_{NoTwist}$ is a force of 165000 kN acting for 0.001s.

Step 3. When the structure is permitted to twist about its vertical axis, under this impulse, the response of the critical wall, $\Delta_{Twist}$, is found to be 83 mm. This is a 38% increase due to torsion.

Step 4. The ratio of earthquake displacement to impulse displacement from Table 2, $R_{E|I}$, for an overall 84% corresponds to one standard deviation above the mean. The ratio is therefore 0.96 × exp(0.19 × 1.0) = 1.16.

Step 5. The estimated displacement demand on the critical element is therefore 1.16 × 83 mm = 96 mm. This corresponds to a ductility demand of 6.0.

## 7. CONCLUSIONS

This paper described the use of impulse loading to represent the behaviour of single story structures to earthquake shaking. It was shown that:

i) A simple closed-form solution was derived for a single-storey system subject to an impulse based on two-degree-of-freedom free vibration concepts.

ii) The impulse response provides a good indication of the median earthquake response.

iii) A design approach is developed for simple single story structures for different levels of uncertainty of torsional response. A design example is also provided.

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