STUDIES ON DESIGN METHOD FOR FRICTION ENERGY DISSIPATION BRACED-RC FRAME BASED ON ELASTIC STIFFNESS

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ABSTRACT :

Design method based on the elastic stiffness for friction energy dissipation braced frame structure was studied in this paper. Firstly, original multi-degree-of-freedom (MDOF) system without braces was transformed into an equivalent single-degree-of-freedom (E-SDOF) system by means of an assumed vibration mode, and the relationship between elastic stiffness of SDOF system and its seismic action was deduced according to the design spectrum. Then the equivalent seismic action of global structure with braces was calculated by drift limit, stiffness and mass layout of original structure, and the seismic action distribution of global structure was estimated according to the seismic distribution of the original system. The numerical example demonstrated the procedure of this design method. Finally, precision improving method by means of interim drift limit was proposed.

KEYWORDS: elastic stiffness, friction energy dissipation brace, original structure, design method, frequent earthquake

1. INTRODUCTION

The technology of friction energy dissipation, as a principal type of structure passive control, started from the end of 1970s. Recurring to friction energy components or devices installed in the original structure, by utilizing the relative displacement aroused by structure vibration, which make the energy dissipative devices to consume the earthquake input energy, thus transform the vibration energy of the structure into heat energy. Due to high efficiency absorption, simple construction, wide application scope and convenient maintenance, this kind of energy dissipation system has been widely applied in multi-story, high and super-high buildings, topping tower, long-span bridges and pipeline. It can also be used to improve the aseismic and wind-proof performance of old buildings [Zhou, 1997]. In recent years, concept of aseismic design & method and the project application is experiencing new revolutions, while the research and development of passive energy dissipation system becomes quite an active field among them [Ou, 1996]. However, a breakthrough in the design method has not been achieved.

2. EQUIVALENT SINGLE-DEGREE-OF-FREEDOM (E-SDOF)

As SDOF system has a rigorous theoretical basis and a simple mathematical expression, we usually transform MDOF system into E-SDOF on the basis of certain hypothesis for the convenience of research. Through the research on E-SDOF system, we transform its outcome into MDOF system back, consequently to get some practical conclusions. Based on the lateral displacement mode of the structure, this article transformed the MDOF system into E-SDOF system as shown in Fig.1. The specific equivalent processes are as following:

The complete dynamic equilibrium equation of MDOF system is:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=-[M]\{1\}\ddot{x}_g \tag{1}$$

$[M]$, $[C]$, $[K]$ is mass matrix, damping matrix and stiffness matrix of MDOF, respectively. $\{X\}$ is the displacement vector representing the displaced shape of the structure, $\ddot{x}_g$ is the acceleration of the ground.

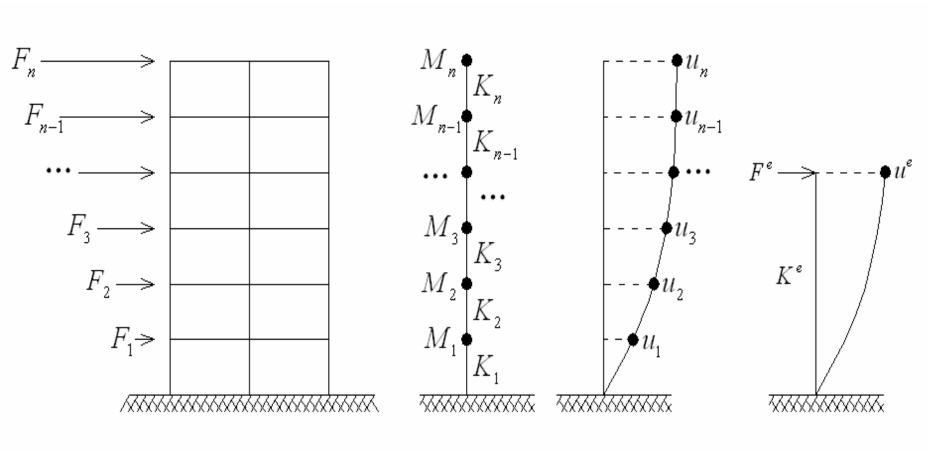

Fig.1 The equivalent single-degree-of-freedom (E-SDOF) system

Assumed the displaced shape of the MDOF system to be $\{f\}$, the maximum of the top displacement to be $x(t)=x_0\sin(wt)$, then:

$$\{X\}=x(t)\{f\}=x_0\sin(wt)\{f\} \tag{2}$$

Substituting Eq. (2) into Eq. (1):

$$[M]\{f\}\ddot{x}(t)+[C]\{f\}\dot{x}(t)+[K]\{f\}x(t)=-[M]\{1\}\ddot{x}_g(t) \tag{3}$$

Then,

$$\{F_0\}=[K]\{f\}x_0 \tag{4}$$

$$\{V\}=\{1\}^T\{F_0\} \tag{5}$$

$\{F_0\}$ is the inertial force of the MDOF system under earthquake action, V is the base shear of the MDOF system.

Multiplying Eq. (3) by $\{f\}^T$ gives:

$$\{f\}^T[M]\{f\}\ddot{x}(t)+\{f\}^T[C]\{f\}\dot{x}(t)+\{f\}^T[K]\{f\}x(t)=-\{f\}^T[M]\{1\}\ddot{x}_g(t) \tag{6}$$

From Eq. (6), the equivalent lateral stiffness K^e is:

$$K^e=\{f\}^T[K]\{f\} \tag{7}$$

The equivalent base shear V^e is:

$$V^e=\{f\}^T[K]\{f\}x_0=\{f\}^T\{F_0\} \tag{8}$$

Hence, the equivalent top displacement x^e is:

$$x^e=\frac{V^e}{K^e}=\frac{\{f\}^T[K]\{f\}x_0}{\{f\}[K]\{f\}}=x_0 \tag{9}$$

Eq. (9) shows that the equivalent displacement equals to the top displacement of the MDOF system.

2.1 The Relation between Elastic Lateral Stiffness and Period for E-SDOF System

Under the effect of frequent earthquakes, in order to make the equivalent globe structure (consist of major structure and braces) meet the desire of the story drift limitation, an equation must be as:

$$\frac{V^e}{K_f^e + K_b^e} \le [q_e]^e h \tag{10}$$

K_f^e is the equivalent lateral stiffness of the original structure, K_b^e is the equivalent lateral stiffness of the friction energy dissipation braces, $[q_e]^e$ is the equivalent story drift angle limitation, h is the height of the structure.

Owing to the relationship between seismic action and lateral stiffness, the equivalent lateral stiffness of braces K_b^e can be computed by the Eq. (10) according to the equivalent base shear V^e of the original structure. However, the equivalent base shear V^e will change when the stiffness of the structure system K_f^e comes to be $K_f^e + K_b^e$. In order to satisfy the Eq. (10), another K_b^e can be worked out. It is clear that this is an iterative process, the ultimate convergence value is the equivalent lateral stiffness of the braces K_b^e and its relative equivalent base shear V^e. As to figure out the ultimate convergence value, Eq. (10) can be expressed by

$$V^e\left(K_b^e\right) \le \left(K_f^e + K_b^e\right) \cdot [q_e]^e h \tag{11}$$

Due to the small weight of the braces compared with the original structure, its weight is neglected during the deducing process. Thus frequency equation for original structure and global structure can be given as:

$$\left|K_f^e - w_f^2 M^e\right| = 0 \tag{12}$$

$$\left|K^e - w^2 M^e\right| = 0 \tag{13}$$

M^e is the equivalent mass of the original structure, w_f is the equivalent cyclic frequency of the original structure, K^e is the equivalent lateral stiffness of the global structure, w is the cyclic frequency of the global structure. It is clear that $K^e = K_f^e + K_b^e$.

For the deducing convenience, the ratio of the equivalent stiffness is defined as

$$c = \frac{K_f^e + K_b^e}{K_f^e} \tag{14}$$

It is obvious that $c > 1$. Eq. (14) could be

$$K_b^e = (c-1)K_f^e \tag{15}$$

Combining Eq. (12), Eq. (13) and Eq. (14), gives

$$w^2 = c w_f^2 \tag{16}$$

namely

$$\frac{T}{T_f} = \frac{1}{\sqrt{c}} \tag{17}$$

T_f is the equivalent period of the original structure, T is the equivalent period of the global structure.

It shows that if the equivalent lateral stiffness is c times that of the original system, the equivalent period is to be $1/\sqrt{c}$ times, which leads to the changes of base shear.

2.2 The Relation between the Period of E-SDOF System and Its Seismic Action

If $T_f \le T_g$, increase of structural period will not lead to the accretion of seismic effect according to the design spectrum [Seismic Code, 2001], thus the minimum equivalent lateral stiffness of the braces K_b^e can be directly figured out by Eq. (10).

If $T_f > T_g$, period of most frame structures will be among $T_g \le T_f \le 5T_g$ and this article only aims to discuss the relationship between period and seismic action on the basis of response spectrum, accordingly the instance of $T_f > 5T_g$ will not be discussed in this paper.

If $T_g \le T_f \le 5T_g$, the design spectrum gives

$$a = \left(\frac{T_g}{T}\right)^g h_2 a_{\max} \tag{18}$$

a is the seismic influence coefficient, $a_{\max}$ is the maximum value of seismic influence coefficient, h_2 is the damping adjustment factor, g is the power index of the curvilinear decrease section.

The contribution of friction energy dissipation braces to the structure in elasticity stage is only to increase the stiffness but not to change the damping characteristics. On this condition, the damping adjustment coefficient of the original frame structure or the global structure $h_2 = 1$ will maintain the same. In the case of the damping ratio $x = 0.05$ of the normal reinforced concrete, its attenuation index is $g = 0.9$, from which we can deduce the ratio of the seismic influence coefficient between global structure and original structure is

$$\frac{a}{a_f} = \left(\frac{T_f}{T}\right)^g = \left(\frac{T_f}{T}\right)^{0.9} \tag{19}$$

a_f, a is the seismic influence coefficient of the original structure and global structure, respectively.

2.3 The Relation between the Elastic Lateral Stiffness of E-SDOF System and Its Seismic Action

According to the response spectrum, the seismic action of E-SDOF system can be expressed as:

$$F_{Ek} = aG_{eq} \tag{20}$$

F_{Ek} is the base shears of E-SDOF, G_{eq} is equivalent total gravity load of the structure.

Combine Eq.(17), Eq.(19) and Eq.(20):

$$\frac{V^e}{V_f^e} = \frac{a}{a_f} = \left(\frac{T_f}{T}\right)^{0.9} = c^{0.45} \tag{21}$$

V_f^e is the equivalent base shear of the original structure, V^e is the equivalent base shear of the global structure.

Eq.(21) means that if the equivalent elastic lateral stiffness of the global system is c times that of the original system, then its equivalent seismic action is accordingly comes to be $c^{0.45}$ times.

Combine Eq. (11) and Eq. (14), an equation gotten with:

$$V^e(c) \le cK_f^e \cdot [q_e]^e h \tag{22}$$

The initial values V_0^e and $c_0 = \frac{V_0^e}{[q_e]h} \Big/ K_f^e$ can be obtained from the design procedure of the original structure. To apply iterative solution for Eq. (21) and Eq. (22), its process is shown in Table 1 as following:

Table 1. Iterative process

$c = K_f^e \big/ (K_f^e + K_b^e)$	(21)(22) → ⇢	$V^e(K_b^e) = aG_{eq}$
c_0	→	$c_0^{0.45}V_0^e$
$c_0 \cdot c_0^{0.45} = c_0^{1.45}$	↙ →	$(c_0^{1.45})^{0.45}V_0^e = c_0^{0.6525}V_0^e$
$c_0 \cdot c_0^{0.6525} = c_0^{1.6525}$	↙ →	$(c_0^{1.6525})^{0.45}V_0^e = c_0^{0.743625}V_0^e$
…	↙ →	…
$c_0 \cdot c_0^{0.81813} = c_0^{1.81813}$	↙ →	$(c_0^{1.81813})^{0.45}V_0^e = c_0^{0.81816}V_0^e$

The iterative process, with the initial value of c_0 and V_0^e, will converge to Eq.(23):

$$c = c_0^{1.82}, \quad V^e = c_0^{0.82}V_0^e \tag{23}$$

It is also possible that $T_f > T_g$ in the beginning, but if this happens in iterative process, a condition to terminate iterative process and to determine a stiffness ratio are needed. From the above-mentioned derivation process, we can see that Eq. (19) implied the condition of $T \ge T_g$. From Eq. (17), the maximum value of c is $c_{\max} = (T_f / T_g)^2$. In other words, when the condition of $T_g \le T_f \le 5T_g$ happens,

$$c = \min\left(c_0^{1.82}, \left(\frac{T_f}{T_g}\right)^2 \right), \quad V^e = c^{0.45}V_0^e \tag{24}$$

3. DESIGN METHOD BASED ON ELASTIC STIFFNESS

Friction energy dissipation braced RC frame is composed of original frame structure and friction energy dissipation braces. During the process of structure design, the engineer should firstly design the original structure which is followed by the design of the friction energy dissipation. Considering to the characteristics of friction energy dissipation braced RC frame system, the engineer should proceed the design of original structure in terms of decreased seismic action in the first place, then design and distribute energy dissipation braces and verify the calculation.

Combined with the conclusion on lateral stiffness and base shear of E-SDOF system, taking the displacement vector of original frame structure to be the displacement mode of transforming MDOF system into E-SDOF system, and assuming that original frame structure obeys the same vibration mode with global structure, the design processes of friction energy dissipation braced RC frame based on the elastic stiffness are as following:

(1). Establish the story shear model for original system and calculate the lateral stiffness $\{K_f\}$;

(2). Calculate the inter-story drift $\{\Delta u_{f,j}\}$ of mode $j(j=1,\ldots,m)$ of the original system under seismic action, get the inter-story drift $\{\Delta u_f\}$ by mode-superposition;

(3). Calculate the displacement vector $\{f\}$ of original system under seismic action;

(4). According to Eq. (9), calculate the equivalent displacement u_f^e of original system and the maximum top displacement $[u]^e$ that satisfy the drift limitation;

(5). Calculate the relative story shear force $\{V_f\}$, the story seismic action $\{F_f\}$ and the based shear $V_f^e=\{f\}^T\{F_f\}$ of original structure;

(6). Calculate the initial stiffness $c_0=\left.\frac{V_f^e}{[u]^e}\middle/\frac{V_f^e}{u_{f,n}^e}\right.=u_{f,n}^e/[u]^e$ and figure out the c by Eq. (24);

(7). Calculate the equivalent base shear of the global system under the seismic action by $V^e=\frac{c}{c_0}V_f^e$;

(8). Calculate seismic action distribution $\{F\}$ of global structure according to that of original system;

(9). Calculate the minimum story lateral stiffness $\{K\}$ according to the inter-story drift limit specified in the code or expected by the engineer;

(10). Calculate the minimum story lateral stiffness required by friction energy dissipation braces through the equation $\{K_b\}=\{K-K_f\}$.

(11). Establish a model by proper distribution of the energy dissipation braces and then verify.

Step (1) and step (2) can be obtained automaticlly from the design process of the original structure. Almost all design software can output these essential data. During the design processes, the energy dissipation braces should be added into the calculation model at the very beginning to proceed the calculating verification of the elastic stage and to judge whether the accuracy of the design is acceptable or not. If the accuracy can meet the desire, then the calculating verification during the elasto-plastic stage should be done afterwards.

4. VERIFICATION

A 10 stories RC frame is used to demonstrate and verify the above-mentioned design method. With fortification intensity 8 and site classification 2 (the characteristic period is 0.4s), the structure shows 10 spans in the horizontal direction (X Direction) and 3 spans in the longitudinal direction (Y Direction). Three-dimensional model of the structure is shown as Fig.2. The strength grade of the concrete is C30. The height of the first story is 4.5 meter while top story is 4.2 meter and that of other is 3.6 meter. This article only analyses the structure in Y-direction.

Story ratio vector of original structure under seismic action is: $\{q_f\}=\{1/467,1/447,1/455,1/469,1/471,1/450,1/473,1/469,1/457,1/524\}$. All the story drift does not meet the limitation $1/550$ specified in the code. The

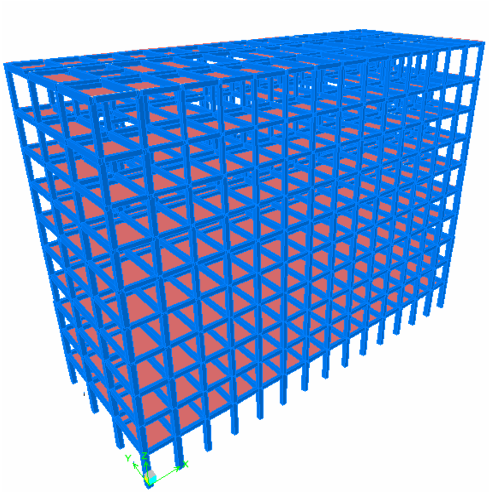

Fig.2 Three-dimensional model

further design of friction energy dissipation braces is required.

Establish a story shear model for original system by employing the stiffness and mass data outputted from the design program. All the design processes based on the elastic stiffness are as following:

(1). The story lateral stiffness of original structure is:

$$\{K_f\}=\{2.59E8,4.76E8,6.72E8,8.60E8,1.02E9,1.10E9,1.28E9,1.37E9,1.42E9,1.34E9\}^T$$

(2). The story drift $\{\Delta u_f\}$ of original structure under seismic action is

$$\{\Delta u_f\}=\{0.00899,0.00806,0.00791,0.00767,0.00765,0.00800,0.00761,0.00767,0.00788,0.00859\}^T$$

(3). The lateral displacement $\{f\}$ of the original structure is

$$\{f\}=\{0.08003,0.07104,0.06298,0.05507,0.04740,0.03975,0.03175,0.02414,0.01647,0.00859\}^T$$

(4). The equivalent displacement of the original structure is $u_f^e=0.08003$; the maximum top displacement limitation of the global structure is $[u]^e=\frac{1}{550}\times(4.5+8\times3.6+4.2)=0.06818$;

(5). The story shear of the original structure is:

$$\{V_f\}=\{K_{f,i}\times f_i\}^T=\{2.33E6,3.84E6,5.32E6,6.60E6,7.80E6,8.80E6,9.74E6,1.05E7,1.12E7,1.15E7\}^T$$

The story seismic action of the original structure is:

$$\{F_f\}=\{2.33E6,1.51E6,1.48E6,1.28E6,1.21E6,9.97E5,9.41E5,7.67E5,6.82E5,3.21E5\}^T$$

The equivalent base shear of the original structure is $V_f^e=\{f\}^T\{F_f\}=6.16E5$

(6). The initial ratio of stiffness is $c_0=\left.{V_f^e}\middle/{[u]^e}\right/\left.{V_f^e}\middle/{u_f^e}\right.=1.17377$

The ratio of stiffness is $c=\min\left(c_0^{1.82},\left(\frac{T_f}{T_g}\right)^2\right)=1.33860$

(7). The equivalent base shear of global structure is $V^e=\frac{c}{c_0}V_f^e=7.03E5$;

(8). The story seismic action of the global structure is:

$$\{F\}=\{2.66E6,1.72E6,1.69E6,1.46E6,1.38E6,1.14E6,1.07E6,8.75E5,7.77E5,3.66E5\}^T$$

(9). According to the story drift angle limitation $1/550$, the minimum lateral stiffness of the global structure is:

$$\{K\}=\{3.48E8,6.68E8,9.26E8,1.15E9,1.36E9,1.53E9,1.70E9,1.83E9,1.95E9,1.60E9\}^T;$$

(10). The minimum lateral stiffness that must be provided by the braces is:

$$\{K_b\}=\{8.87E7,1.92E8,2.54E8,2.89E8,3.40E8,4.33E8,4.17E8,4.61E8,5.30E8,2.64E8\}^T$$

Add $\{K_b\}$ into original structure, compute the elastic story drift angle by ETABS, get: $\{q\}=\{1/583,1/566,1/557,1/554,1/554,1/540,1/553,1/550,1/538,1/562\}$, as line 3 in Fig. 3.

The story drift in story 2 and story 5 exceeds the limitation but approaches the limit value of elastic story drift specified in the code. It can be finally determined by partial adjustment.

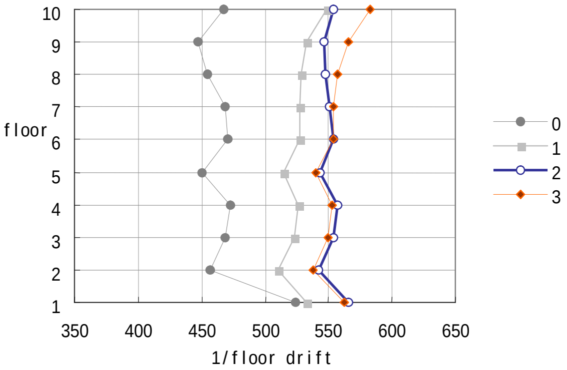

Fig.3 The story drift angle of the model

5. METHODS FOR IMPROVEMENT OF DESIGN ACCURACY

To do structural analysis from the minimal lateral stiffness computed on the basis of design method of elastic stiffness, we can see that story drift of each floor under seismic action is around the drift limit. Reasons for the errors are as following:

(1). Design method only takes the shear deformation into consideration. ETABS combines the both affect of the shear deformation and bending deformation;

(2). Assume the global system obeys the same vibration mode of the original system.

This deviation aroused by the design method can be decreased by employing various means. The following will illustrate an example for design precision improving by setting an interim drift limit.

Line 0 in Fig. 3 shows the story lateral displacement curve of original structure under seismic action. Line 1 represents the lateral displacement curve of the global structure which has the minimum story lateral stiffness computed through the design method based on the elastic stiffness while taking 1/524 as the story drift limitation and under seismic action. Take this global structure as a new original structure, the lateral displacement curve of the global displacement under seismic action take 1/550 as the story drift limitation as Line 2 demonstrates. By comparison of Line 2 and Line 3, we can see that the story drift angle of the global structure will be evenly distributed around the drift limit by setting an interim drift limit.

6. CONCLUSION

The relation (Eq. 24) between elastic lateral stiffness and seismic action of E-SDOF is accurate. The seismic action distribution of global structure can be estimated according to the seismic shear distribution of original system. The design method based on elastic stiffness for friction energy dissipation braced frame structure is efficient. The precision improving method by means of interim drift limit is effective.

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