CALCULATION OF DYNAMIC STRESS INTENSITY FACTORS IN THE MIXED MODE USING COMPLEX FUNCTIONS THEORY

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ABSTRACT:

An Analytical method for calculating dynamic stress intensity factors in the mixed mode (combination of opening and sliding modes) using complex functions theory is presented. The basis of the method is grounded on solving the two-dimensional wave equations in the frequency domain and complex plane. After solving the wave equations (considering appropriate boundary conditions), the stress and strain fields, also the J-integrals are obtained. Finally using the J-integrals, dynamic stress intensity factors are calculated. Numerical results including the values of dynamic stress intensity factors for a crack in an infinite domain subjected to P and SV waves are presented.

KEYWORDS: stress intensity factor; J-integral; dynamic fracture; mixed mode fracture

1. INTRODUCTION

Stress intensity factor is one of the most important parameters in the fracture mechanics. Calculation of this factor in the case of dynamic loading is required for analysis of cracked structures against earthquake. Many researches have been done about dynamic problems of cavity and crack. Pao and Mow [1] introduced the problem of diffraction of elastic waves and dynamic stress concentrations. Wells and Post [2] calculated dynamic stress distribution surrounding a running crack based on a photoelastic analysis. Also the unstable crack propagation was studied by Wells [3]. Achenbach [4] showed the effect of dynamic loading on the brittle materials. Freund [5] and Sih [6] described the problem of elastodynamics in the media including crack. Chirino and Abascal [7] calculated dynamic stress intensity factors based on the boundary elements method. The use of complex functions in elastostatic problems was expanded by Muskhelishvili [8]. Kassab and Hsieh [9] solved the static problem of doubly connected domain. Pengfei et al. [10] and Kohno and Ishikawa [11] described a method for solving infinite plate including cavity using complex functions theory. Verrut [12] calculated the deformation of a tunnel in an elastic half plane. Also Liu et al. [13] and Han et al. [14] solved the problem of scattering waves in infinite media based on complex variables theory. In addition, Bowie [15] solved the crack problems by mapping technique. In the present study an analytical method for solving dynamic problem of crack using complex functions theory is introduced. The power of this method lies in the conformal mapping of a complicated shape to a simple geometric shape (e.g. unit circle) in the mapped plane. Therefore a large number of unsolved problems or those with very complicated geometries can be solved in a very simple manner. Use of complex functions theory in dynamic problems of crack is different from elastostatic. In this way, potential functions are applied in two-dimensional elastic wave equations. Then these equations are transformed into complex plane. In this domain, solution of the resulting partial differential equations (calculation of the potential functions) is found in the series of the Hankel functions with unknown coefficients. Applying appropriate boundary conditions of the crack, a set of algebraic equations are achieved. Solving these equations, the unknown coefficients and consequently potential functions are evaluated. Using the potential functions, all desirable parameters such as stress, strain, displacement and the J-integrals are calculated. Finally, based on the relation between the J-integrals and stress intensity factors, these parameters are calculated.

2. GOVERNING EQUATIONS

The two-dimensional elastic wave equations in directions x and y are expressed as:

$$\begin{cases} \mu\nabla^2 u + (\lambda+\mu)\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 v}{\partial x \partial y}\right) = \rho\dfrac{\partial^2 u}{\partial t^2} \\ \mu\nabla^2 v + (\lambda+\mu)\left(\dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 u}{\partial x \partial y}\right) = \rho\dfrac{\partial^2 v}{\partial t^2} \end{cases} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2.1}$$

In these equations ∇^2 is two-dimensional Laplace operator and u, v are the displacements in directions x and y respectively. Also λ, μ are the Lamé elastic constants and ρ is the density of medium. The potential functions Φ (for P wave) and Ψ (for SV wave) are defined as below:

$$\begin{cases} u = \dfrac{\partial\Phi}{\partial x} + \dfrac{\partial\Psi}{\partial y} \\ v = \dfrac{\partial\Phi}{\partial y} - \dfrac{\partial\Psi}{\partial x} \end{cases} \tag{2.2}$$

Therefore Eqn. 2.1 is converted to:

$$\begin{cases} \nabla^2\Phi = \dfrac{1}{C_p^2}\dfrac{\partial^2\Phi}{\partial t^2} \\ \nabla^2\Psi = \dfrac{1}{C_s^2}\dfrac{\partial^2\Psi}{\partial t^2} \end{cases} \tag{2.3}$$

In which C_p and C_s are the propagation velocities of compressive and shear waves respectively:

$$\begin{cases} C_p = \sqrt{\dfrac{\lambda+2\mu}{\rho}} \\ C_s = \sqrt{\dfrac{\mu}{\rho}} \end{cases} \tag{2.4}$$

3. COMPLEX FUNCTIONS AND CONFORMAL MAPPING

The complex variables $\zeta, \overline{\zeta}$ are introduced as:

$$\zeta = x + iy \qquad \overline{\zeta} = x - iy \qquad \zeta = re^{i\theta} \tag{3.1}$$

Using the relations below,

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial \zeta}+\frac{\partial}{\partial \overline{\zeta}} \qquad \frac{\partial}{\partial y}=i\left(\frac{\partial}{\partial \zeta}-\frac{\partial}{\partial \overline{\zeta}}\right) \tag{3.2}$$

$$\frac{\partial^2}{\partial x^2}=\frac{\partial^2}{\partial \zeta^2}+2\frac{\partial^2}{\partial \zeta \partial \overline{\zeta}}+\frac{\partial^2}{\partial \overline{\zeta}^2} \qquad \frac{\partial^2}{\partial y^2}=-\left(\frac{\partial^2}{\partial \zeta^2}-2\frac{\partial^2}{\partial \zeta \partial \overline{\zeta}}+\frac{\partial^2}{\partial \overline{\zeta}^2}\right) \tag{3.3}$$

The Laplace operator in the complex plane is obtained as:

$$\nabla^2 = 4\frac{\partial^2}{\partial \zeta \partial \overline{\zeta}} \tag{3.4}$$

In the case of harmonic waves, Φ and Ψ are expressed as:

$$\begin{cases} \Phi(\zeta,\overline{\zeta},t)=\phi(\zeta,\overline{\zeta})e^{-i\omega t} \\ \Psi(\zeta,\overline{\zeta},t)=\psi(\zeta,\overline{\zeta})e^{-i\omega t} \end{cases} \tag{3.5}$$

where ω is the frequency of wave functions. Therefore for the harmonic case and in the complex plane, Eqn. 2.3 is changed to:

$$\begin{cases} \dfrac{\partial^2 \phi}{\partial \zeta \partial \overline{\zeta}}=\left(\dfrac{i\alpha}{2}\right)^2 \phi \\ \dfrac{\partial^2 \psi}{\partial \zeta \partial \overline{\zeta}}=\left(\dfrac{i\beta}{2}\right)^2 \psi \end{cases} \quad \text{with} \quad \begin{matrix} \alpha=\omega/C_p \\ \beta=\omega/C_s \end{matrix} \tag{3.6}$$

In the crack problems involving complicated boundaries, conformal mapping converts these boundaries to the simple shape (unit circle). The mapping function is expressed by:

$$\zeta = w(\eta) \tag{3.7}$$

Implying the mapping function $w(\eta)$, Eqn. 3.6 is written in the mapping plane as below:

$$\begin{cases} \dfrac{\partial^2 \phi}{\partial \eta \partial \overline{\eta}}=\left(\dfrac{i\alpha}{2}\right)^2 w'(\eta)\,\overline{w'(\eta)}\,\phi(\eta,\overline{\eta}) \\ \dfrac{\partial^2 \psi}{\partial \eta \partial \overline{\eta}}=\left(\dfrac{i\beta}{2}\right)^2 w'(\eta)\,\overline{w'(\eta)}\,\psi(\eta,\overline{\eta}) \end{cases} \tag{3.8}$$

The solution of these equations is found as:

$$\begin{cases} \phi(\eta,\bar{\eta}) = \sum_{n=-\infty}^{+\infty} a_n H_n^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n \\ \psi(\eta,\bar{\eta}) = \sum_{n=-\infty}^{+\infty} b_n H_n^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n \end{cases} \tag{3.9}$$

In the above equations, $H_n^{(1)}(\chi r)$ is the Hankel function of first kind and order n:

$$H_n^{(1)}(\chi r) = J_n(\chi r) + iY_n(\chi r) \tag{3.10}$$

Where $J_n(\chi r)$ and $Y_n(\chi r)$ are the Bessel functions of first and second kind respectively and of order n. This type of the Hankel function satisfies the radiation boundary condition. a_n, b_n are the unknown coefficients calculated from boundary conditions of the problem.

4. DISPLACEMENTS AND STRESSES

From Eqn. 2.2 displacement field in the mapping plane (η planes) is derived as:

$$\begin{cases} u_\rho + iu_\varphi = 2\frac{\bar{\eta}}{\rho|w'(\eta)|}\frac{\partial}{\partial\bar{\eta}}(\Phi - i\Psi) \\ u_\rho - iu_\varphi = 2\frac{\eta}{\rho|w'(\eta)|}\frac{\partial}{\partial\eta}(\Phi + i\Psi) \end{cases} \tag{4.1}$$

u_ρ and u_φ are the radial and tangential components of displacement vector respectively. (The second part of Eqn. 4.1 is the conjugated form of the first one). Using the relations below,

$$\begin{cases} \frac{\partial}{\partial\eta}\left[H_n^{(1)}\left(\xi|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n\right] = \frac{\xi}{2}H_{n-1}^{(1)}\left(\xi|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-1} w'(\eta) \\ \frac{\partial}{\partial\bar{\eta}}\left[H_n^{(1)}\left(\xi|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n\right] = -\frac{\xi}{2}H_{n+1}^{(1)}\left(\xi|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+1} \overline{w'(\eta)} \end{cases} \tag{4.2}$$

Eqn. 4.1 is converted to:

$$\begin{cases} u_\rho + iu_\varphi = -\alpha \sum_{n=-\infty}^{+\infty} a_n H_{n+1}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+1} + i\beta \sum_{n=-\infty}^{+\infty} a_n H_{n+1}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+1} \\ u_\rho - iu_\varphi = \alpha \sum_{n=-\infty}^{+\infty} a_n H_{n-1}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-1} + i\beta \sum_{n=-\infty}^{+\infty} a_n H_{n-1}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-1} \end{cases} \tag{4.3}$$

Based on the Hook law, the stress field in the η plane is evaluated as:

$$\begin{cases} \sigma_\rho + \sigma_\varphi = -2\alpha^2(\lambda+\mu)\Phi \\ \sigma_\varphi - \sigma_\rho + 2i\tau_{\rho\varphi} = -8\mu \frac{\eta^2}{\rho^2 \overline{w'(\eta)}} \frac{\partial}{\partial\eta}\left[\frac{1}{w'(\eta)}\frac{\partial}{\partial\eta}(\Phi + i\Psi)\right] \end{cases} \tag{4.4}$$

σ_ρ, σ_φ and $\tau_{\rho\varphi}$ are the radial, hoop and shear components of stress tensor respectively. Subtracting two parts of the above relations yields:

$$\begin{cases} \sigma_\rho - i\tau_{\rho\varphi} = -\alpha^2(\lambda+\mu)\Phi + \frac{4\mu\eta^2}{\rho^2 \overline{w'(\eta)}} \frac{\partial}{\partial\eta}\left[\frac{1}{w'(\eta)}\frac{\partial}{\partial\eta}(\Phi + i\Psi)\right] \\ \sigma_\rho + i\tau_{\rho\varphi} = -\alpha^2(\lambda+\mu)\Phi + \frac{4\mu\bar{\eta}^2}{\rho^2 w'(\eta)} \frac{\partial}{\partial\bar{\eta}}\left[\frac{1}{\overline{w'(\eta)}}\frac{\partial}{\partial\bar{\eta}}(\Phi - i\Psi)\right] \end{cases} \tag{4.5}$$

(Again the second part of Eqn. 4.5 is the conjugated form of the first one). Eqn. 4.5 is written in term of the Hankel functions as below:

$$\begin{cases} \sigma_\rho - i\tau_{\rho\varphi} = -\alpha^2(\lambda+\mu) \sum_{n=-\infty}^{+\infty} a_n H_n^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n} + \frac{\mu\alpha^2\eta^2 w'(\eta)}{\rho^2 \overline{w'(\eta)}} \sum_{n=-\infty}^{+\infty} a_n H_{n-2}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-2} \\ \qquad + \frac{i\mu\beta^2\eta^2 w'(\eta)}{\rho^2 \overline{w'(\eta)}} \sum_{n=-\infty}^{+\infty} b_n H_{n-2}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-2} \\ \sigma_\rho + i\tau_{\rho\varphi} = -\alpha^2(\lambda+\mu) \sum_{n=-\infty}^{+\infty} a_n H_n^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n} + \frac{\mu\alpha^2\bar{\eta}^2 \overline{w'(\eta)}}{\rho^2 w'(\eta)} \sum_{n=-\infty}^{+\infty} a_n H_{n+2}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+2} \\ \qquad - \frac{i\mu\beta^2\bar{\eta}^2 \overline{w'(\eta)}}{\rho^2 w'(\eta)} \sum_{n=-\infty}^{+\infty} b_n H_{n+2}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+2} \end{cases} \tag{4.6}$$

5. BOUNDARY VALUE PROBLEM

Now consider a crack in an infinite elastic medium subjected to the harmonic incident wave. The length of the crack is "2a" and on its surface the stress is vanished. All of the wave variables are the sum of incident and scattering components, therefore the potential functions are written as:

$$\begin{cases} \phi = \phi^{in} + \phi^{sc} \\ \psi = \psi^{in} + \psi^{sc} \end{cases} \tag{5.1}$$

and boundary conditions as:

$$\begin{cases} \left(\sigma_\rho - i\tau_{\rho\varphi}\right)^{in} + \left(\sigma_\rho - i\tau_{\rho\varphi}\right)^{sc} = 0 \\ \left(\sigma_\rho + i\tau_{\rho\varphi}\right)^{in} + \left(\sigma_\rho + i\tau_{\rho\varphi}\right)^{sc} = 0 \end{cases} \quad \text{at} \quad \rho = 1 \tag{5.2}$$

Superscripts "in" and "sc" are for incident and scattered components respectively. The incident components of stress are calculated from incident potential functions based on Eqn. 4.5 Also scattering component of this parameter is found from Eqn. 4.6 Finally the boundary value problem is written as below ($\rho = 1$):

$$\begin{cases} -\alpha^2(\lambda+\mu)\sum_{n=-\infty}^{+\infty} a_n H_n^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n + \frac{\mu\alpha^2\eta^2 w'(\eta)}{\overline{w'(\eta)}}\sum_{n=-\infty}^{+\infty} a_n H_{n-2}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-2} \\ +\frac{i\mu\beta^2\eta^2 w'(\eta)}{\overline{w'(\eta)}}\sum_{n=-\infty}^{+\infty} b_n H_{n-2}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n-2} = \\ \alpha^2(\lambda+\mu)\Phi^{in} + \frac{4\mu\eta^2}{\overline{w'(\eta)}}\frac{\partial}{\partial\eta}\left[\frac{1}{\overline{w'(\eta)}}\frac{\partial}{\partial\eta}(\Phi^{in}+i\Psi^{in})\right] \\ \\ -\alpha^2(\lambda+\mu)\sum_{n=-\infty}^{+\infty} a_n H_n^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^n + \frac{\mu\alpha^2\overline{\eta}^2\overline{w'(\eta)}}{w'(\eta)}\sum_{n=-\infty}^{+\infty} a_n H_{n+2}^{(1)}\left(\alpha|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+2} \\ -\frac{i\mu\beta^2\overline{\eta}^2\overline{w'(\eta)}}{w'(\eta)}\sum_{n=-\infty}^{+\infty} b_n H_{n+2}^{(1)}\left(\beta|w(\eta)|\right)\left[\frac{w(\eta)}{|w(\eta)|}\right]^{n+2} = \\ \alpha^2(\lambda+\mu)\Phi^{in} + \frac{4\mu\overline{\eta}^2}{w'(\eta)}\frac{\partial}{\partial\overline{\eta}}\left[\frac{1}{w'(\eta)}\frac{\partial}{\partial\overline{\eta}}(\Phi^{in}-i\Psi^{in})\right] \end{cases} \tag{5.3}$$

The incident potential function for P wave is as following:

$$\phi^{in} = \phi_0 e^{i\alpha[x\cos(\gamma)+y\sin(\gamma)]} \tag{5.4}$$

In which γ is the angle between direction of the incident wave and x-axis. Also ϕ_0 is a coefficient regarded as unity. This equation in ζ plane is written as:

$$\phi^{in} = e^{i\alpha\left[\frac{\zeta+\bar{\zeta}}{2}\cos(\gamma)-\frac{\zeta-\bar{\zeta}}{2}\sin(\gamma)\right]} \tag{5.5}$$

And finally in mapping plane (η plane):

$$\phi^{in} = e^{i\alpha\left[\frac{w(\eta)+\overline{w(\eta)}}{2}\cos(\gamma)+\frac{w(\eta)-\overline{w(\eta)}}{2i}\sin(\gamma)\right]} \tag{5.6}$$

For the case of SV wave, the incident potential function is written as:

$$\psi^{in} = e^{i\beta\left[\frac{w(\eta)+\overline{w(\eta)}}{2}\cos(\gamma)+\frac{w(\eta)-\overline{w(\eta)}}{2i}\sin(\gamma)\right]} \tag{5.7}$$

($\psi_0 = 1$). The mapping function for a crack with length "2a" is:

$$w(\eta) = \frac{a}{2}\left(\eta + \eta^{-1}\right) = \frac{a}{2}\left(e^{i\varphi} + e^{-i\varphi}\right) \tag{5.8}$$

This function is the mapping function of an ellipse with semi-major axis "a" and semi-minor axis "$b = 0$". Multiplying both sides of Eqn. 5.3 by $e^{-is\varphi}$ and integrating from $-\pi$ to π, an algebraic equation is derived. Solving this equation, unknown coefficients a_n, b_n are calculated. Knowing these coefficients, the J-integrals and consequently dynamic stress intensity factors are calculated. J-integrals are defined as:

$$J_1 = \int U dy - \int\left[T_x\left(\frac{\partial u}{\partial x}\right) + T_y\left(\frac{\partial v}{\partial x}\right)\right] ds \tag{5.9}$$

$$J_2 = \int U dx - \int\left[T_x\left(\frac{\partial u}{\partial y}\right) + T_y\left(\frac{\partial v}{\partial y}\right)\right] ds \tag{5.10}$$

In these relations U is the density of elastic strain energy. For two-dimensional case U is written as:

$$U = \left(\frac{1}{4\mu}\right)\left[\left(\frac{\kappa+1}{4}\right)\left(\sigma_x + \sigma_y\right)^2 - 2\left(\sigma_x\sigma_y - \tau_{xy}^2\right)\right] \tag{5.11}$$

In which:

$$\begin{cases} \kappa = 3 - 4\nu & \text{plane strain} \\ \kappa = \dfrac{3-\nu}{1+\nu} & \text{plane stress} \end{cases} \tag{5.12}$$

ν is the Poisson ratio and T_x, T_y are the components of traction vector:

$$T_x = \sigma_x n_x + \tau_{xy} n_y \qquad T_y = \tau_{xy} n_x + \sigma_y n_y \tag{5.13}$$

Where n_x and n_y are the components of normal vector of the surface. Also "s" is the path of the integration. Since the J-integrals are the path independence, every path surrounds the crack (the unit circle in mapping plane) can be chosen. For calculating the J-integrals in the mapping plane, all of parameters such as σ_x, σ_y, τ_{xy}, $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$ are evaluated in this domain based on calculated potential functions. After it, using equations below, stress intensity factors are achieved:

$$J_1 = \left(\frac{\kappa+1}{8\mu}\right)\left(K_I^2 + K_{II}^2\right) \tag{5.14}$$

$$J_2 = \left(-\frac{\kappa+1}{4\mu}\right)\left(K_I \cdot K_{II}\right) \tag{5.15}$$

6. NUMERICAL RESULTS

Figure 1 represents the crack with length of “2a” in an infinite medium subjected to the harmonic wave. The crack surface is free stress and Poisson ratio of medium is considered equal to 0.25. The wave number K is related to wave frequency as follows:

$$\begin{aligned} \omega &= K\left[\frac{\lambda+2\mu}{\rho}\right]^{1/2} && \text{P incident wave} \\ \omega &= K\left(\frac{\mu}{\rho}\right)^{1/2} && \text{SV incident wave} \end{aligned} \tag{6.1}$$

The wave number, K is equal to α and β in the case of P and SV incident waves respectively. Also for a circle with a radius “a” the mapping function is defined as:

$$w(\eta) = ae^{i\varphi} \tag{6.2}$$

Figure 2 shows the values of dynamic stress intensity factor for the opening mode (K_I) versus dimensionless wave number (αa) and various angle of P incident wave. These values were normalized by $\phi_0 \mu \beta^2 \sqrt{\pi a}$. In Figure 3 these values are given for sliding mode (K_{II}) and P incident wave. Figure 4 represents the values of dynamic stress intensity factor for the opening mode (K_I) versus dimensionless wave number (βa) and various angle of SV incident wave. These values were normalized by $\psi_0 \mu \beta^2 \sqrt{\pi a}$. Also Figure 5 is concern to the values of dynamic stress intensity factors sliding mode (K_{II}) and SV incident wave. An excellent agreement with the results of Chirino and Abascal [7] based on the boundary element method has been observed.

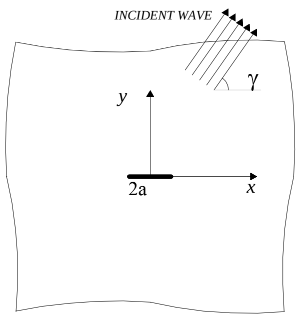

Figure 1 Crack in an infinite medium subjected to the harmonic wave

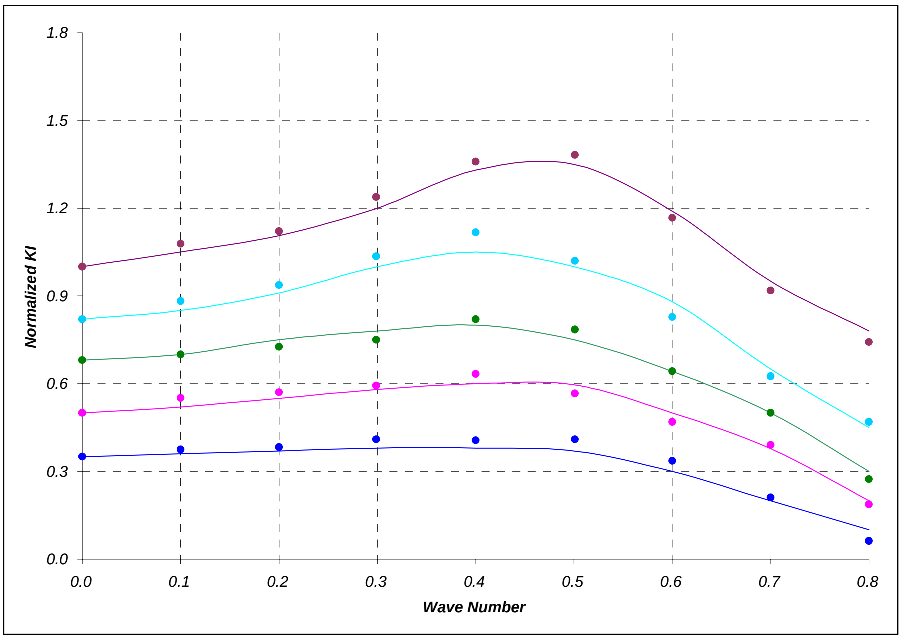

Figure 2 Values of normalized K_I versus dimensionless wave number for P wave
Solid line: present study, point line: Chirino and Abascal [7]

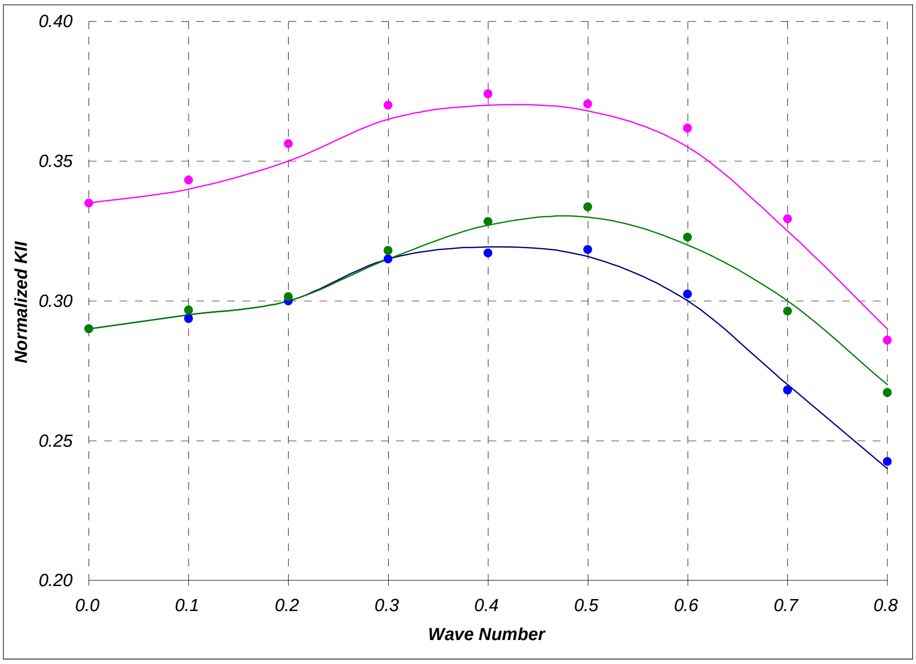

Figure 3 Values of normalized K_{II} versus dimensionless wave number for P wave

Solid line: present study, point line: Chirino and Abascal [7]

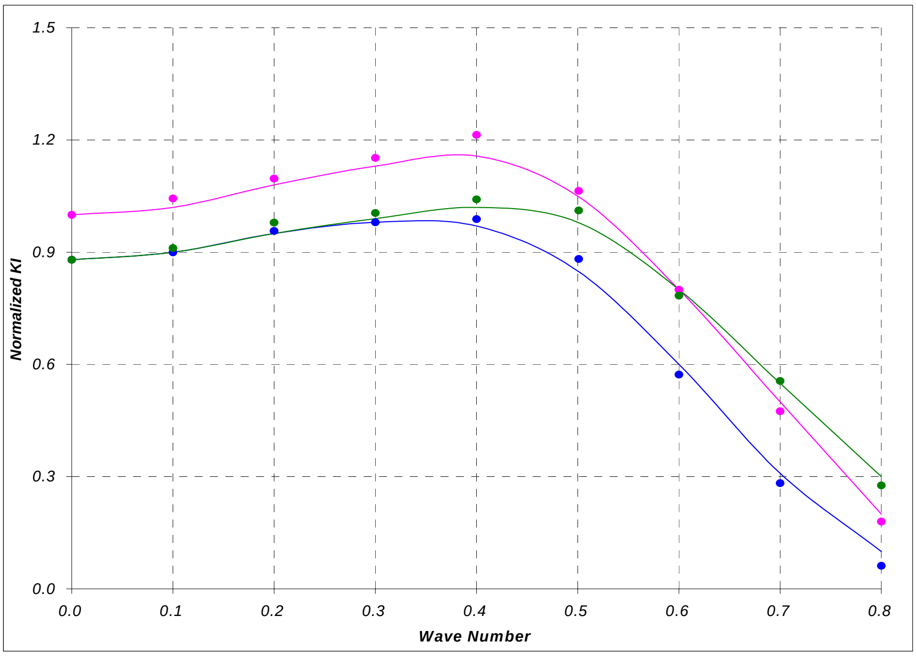

Figure 4 Values of normalized K_I versus dimensionless wave number for SV wave
Solid line: present study, point line: Chirino and Abascal [7]

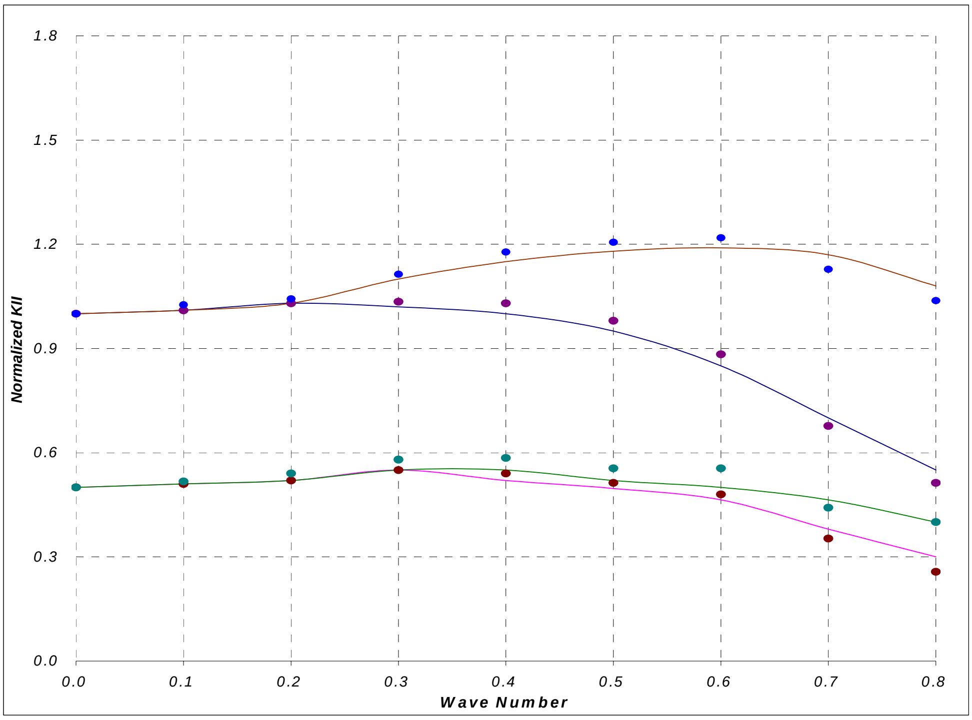

Figure 5 Values of normalized K_{II} versus dimensionless wave number for SV wave
Solid line: present study, point line: Chirino and Abascal [7]

7. CONCLUSIONS

In this paper an analytical method for calculating dynamic stress intensity factor in the mixed mode, using complex functions theory was presented. This method includes the solution of two-dimensional wave equations by means of complex functions theory. The solution is obtained in series of the Hankel functions with unknown coefficients, calculated from boundary conditions of the problem. Using this method, every dynamic problem of theory of elasticity can be solved by considering conventional mapping functions and applying the appropriate boundary conditions. Moreover, it was shown that using conformal mapping technique, significant simplification in calculation process is achieved which is mainly due to the fact that the calculations are performed on a unit circle in contrast to a crack with complicated geometry. This simplification also includes the J-integrals calculation. The well-known problem of an infinite medium containing a crack was solved by this method and the values of dynamic stress intensity factors were achieved. This method represents exact solution for dynamic problems of the crack in compare with numerical methods (e.g. boundary elements method). Also the required computer calculations by this method are significantly lower.

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