ASYMMETRIC UPLIFT ANALYSIS OF UNANCHORED STEEL STORAGE TANKS

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ABSTRACT:

During severe earthquakes, the most important parameter affecting the response of unanchored tanks is the uplift of their base plates from the foundation. This study mainly focuses on obtaining the overturning moment-base rotation diagram of unanchored tanks. To this end, a method is introduced which is based on the uplift of tapered beams. Then after, the accuracy of the method is examined by the experimental investigation of a broad tank model. Finally, a simplified non-iterative method is introduced which can accurately predict the uplift response of unanchored tanks.

KEYWORDS: Unanchored Tanks, Asymmetric uplifting, Tapered beams

1. INTRODUCTION

Damages to tanks in destructive earthquakes may lead to environmental hazards, loss of valuable contents, and disruption of fire-fighting efforts. Inadequately designed tanks have suffered extensive damages in past earthquakes (Cooper *et al.*, 1997). Earthquake damages to unanchored tanks, due to uplifting, can take several forms including: (1) increase in the probability of tank's wall elephant-foot buckling. The weight of the tank's wall concentrates on a narrow zone of the wall, due to the decrease of the contact area between the tank's wall and its foundation. (2) damage to the piping connections that are incapable of accommodating vertical displacements, (3) rupture of the base plate-wall junction due to excessive joint stresses, and (4) uneven settlement of the foundation.

In general, only some of the existing tanks are anchored to their foundations and others rest on their foundations without any anchorage. In case of unanchored tanks, their response is highly influenced by the uplift mechanism (Rammerstorfer, 1990).

Clough *et al.* (1979) investigated the uplift response of a tall and broad tank model using the static tilt test. They concluded that, in fixed roof tanks and tanks which have a top circumferential stiffener, distortional modes contribute to the tank's wall deformation. However, the test results of Sakai and Isoe (1989) showed that these modes have negligible contribution to the tank's wall deformations of the fixed roof and top stiffened tanks. Malhotra and Veletsos (1994a, 1994b) used a finite number of infinite length prismatic beams with constant width to model the bottom plate uplift of unanchored tanks. They used the free body diagram of the tank's wall to estimate its resisting moment. Ahari *et al.* (2008) made use of tapered beams to model the base plate uplift of unanchored tanks. They used the overall equilibrium of the tank to estimate its base plate resisting moment.

The present paper discuses the establishment of the overturning moment-rotation diagram of unanchored tanks whose base plates are modeled with a finite number of tapered beams. The accuracy of the theoretical model is investigated by the experimental tilt-up test which is performed on a broad tank model. Finally, a simple non-iterative method is addressed which can be used to easily obtain the moment-rotation diagram of unanchored tanks.

2. ANALYTICAL METHOD OF UPLIFT ANALYSIS OF UNANCHORED TANKS

Ahari *et al.* (2008) presented the analytical solution for the prismatic tapered beam model (Figure 1). In this model, the contact effects and geometrical nonlinearity of the base plate are considered along with the formation of a plastic hinge at its end. Horizontal and rotational constraints, resulting from the elastic tank's wall stiffness, are applied at the end of the beam. The stiffness matrix of these constraints is determined by the condition of the axisymmetric distributed bending moments and shears on the bottom edge of the tank's wall. Out-of-round deformations of the tank's wall are not considered. The governing differential equation of the beam can be written as:

$$\frac{d^2M(x)}{dx^2}-\frac{NR}{EI_R x}\cdot M(x)=-\frac{P_R}{R}x \tag{1}$$

where $M(x)$ is the bending moment of the beam; N is the axial force of the beam; R and E are the tank's radius and module of elasticity of the bottom plate's material; L is the uplifted length of the beam; $P_R=\rho_l g H_l b$ and $I_R=bt_b{}^3/12$; b and t_b are the maximum beam's width and bottom plate's thickness; H_l is the liquid's height; and g and ρ_l are the gravitational acceleration and liquid's density, respectively.

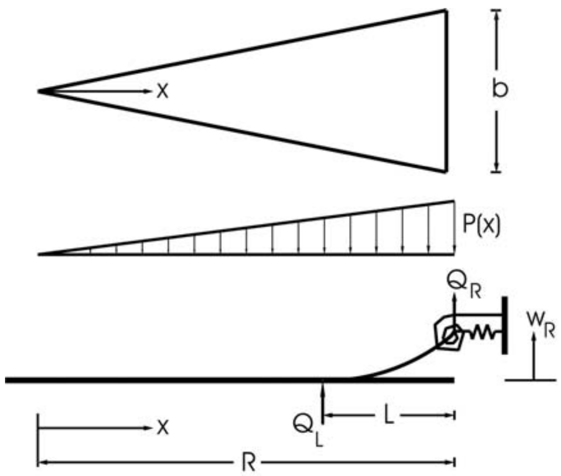


Figure 1 The tapered beam model (Ahari *et al.*, 2008)

The vertical force-uplift diagram of a tall unanchored tank is shown in Figure 2. Before point B, the beam does not experience any uplift. In part BC, the beam's material behaves linearly. After point C, a plastic hinge will form at the uplifted end of the beam. In this part, the axial force of the beam dominates the uplift behavior of the beam. It should be noted that, the tank's wall and its base plate are made of mild steel whose module of elasticity and yielding stress equal $2.1\times10^{11}\ N/m^2$ and $2400\times10^5\ N/m^2$ respectively. The specific gravity of the contained liquid is equal to 1. All other tank models in this paper have the same material properties and the specific gravity of their contained liquid is equal to 1.

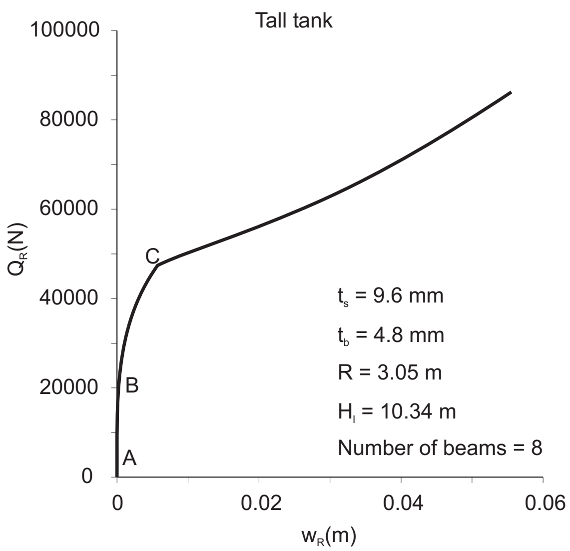


Figure 2 Vertical uplift force vs. uplift displacement of the tall tank

Based on the uplift of tapered beams, the resisting moment of the base plate is calculated from a modified form of NZSEE (1986) method. Based on Figure 3, in which the base plate is modeled with a finite number of tapered beams, with the assumption of the rigid body rotation of the tank's wall, the resisting moment of the base plate can be written as:

$$M_{ROT} = \left(\sum Q_{Ri}\right)\times R - \sum Q_{Li}\left(d_i' - R\right) + \left(\sum W_{bi}\right)\times\left(R - \bar{d}\right) \tag{2}$$

The first part in the right hand side of the equation above shows the resisting moment produced by the axial force in the side of tank's wall which is in contact with foundation. The second part indicates the moment due to reaction forces at the other side of the uplifted length of the beams. And the third stands for the moment due to the distance between the center of gravity of the contained liquid and the center of the fluid pressure on that area of the base plate which is in contact with foundation. $\sum W_{bi}$ is the weight of the liquid which is on the in-contact part of the base plate. W_{bi} can be written as:

$$W_{bi} = P\left(x = R - L_i\right)\times\frac{\left(R - L_i\right)}{3} \tag{3}$$

The d_i, d_i', d_i'' and Y_i shown in Figure 3b can be written as:

$$d_i' = d_i - L_i\cos\frac{2\pi}{n}\left(i-1\right) \tag{4a}$$

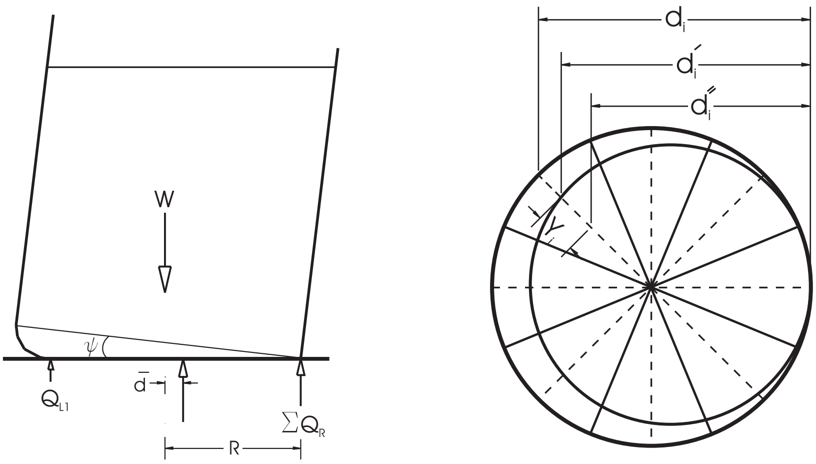
$$d_i = \left[1 + \cos\frac{2\pi}{n}\left(i-1\right)\right]R \tag{4b}$$

$$d_i'' = d_i - \left(L_i + Y_i\right)\cos\frac{2\pi}{n}\left(i-1\right) \tag{4c}$$

$$Y_i = \frac{1}{3}\left(R - L_i\right) \tag{4d}$$

The distance between the center of gravity of the contained liquid and the center of the liquid pressure on the in-contact part of the base plate is equal to:

$$\bar{d} = \frac{\sum(W_{bi} \times d_i'')}{\sum W_{bi}} \tag{4e}$$



(a) (b)

Figure 3 (a) The free body diagram of the asymmetric uplifted tank, (b) geometrical parameters of the base plate used in the estimation of resisting moment

3. COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

To investigate the accuracy of the proposed model, a broad tank model is experimentally studied under static tilt tests (Figure 4). To this end, one side of the tank's support is lifted gradually. Therefore, the difference between the hydrostatic pressures exerted on sides of tank's wall causes an overturning moment on the base of the tank's wall. With the increment of the slope of the tank's support, its base plate may uplift from its support. As shown in Figure 5, the base plate of the tank is instrumented to measure the uplift of its different points. A top ring stiffener is added to the top end of the tank. Hence, the tank's wall does not distort in test times and its in-extensional deformation modes don't contribute to the tank's wall deformation. It should be noted that the radius and height of the tank are equal to $900\,mm$ and $1000\,mm$. The thickness of its base plate and wall are equal to $1\,mm$ and $1.25\,mm$, respectively.

Figure 4 Experimental investigation of broad tank model

The comparison of the analytical solution and experimental investigation results of the base uplift of the tank under study is shown in Figure 6. The analytical and experimental results are in good agreement. It should be noted that, in these test series, the material of the base plate behaves linearly. Here, the nonlinearity of the base uplift is due to the second order effects of the axial forces of the base plate and the gradual variation of the contact area of the base plate and its support.

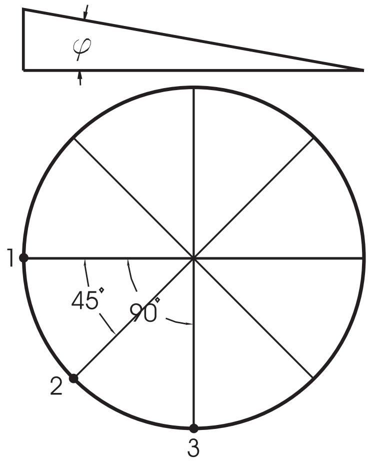


Figure 5 The arrangement of LVDTs under the base plate of the tank

If the tank's wall has inextensional deformations, the assumption of its rigid body rotation will not be correct any more. Clough (1979) concluded that, in fixed roof tanks and tanks with top stiffeners, the tank's wall experiences distortional deformations. However, Sakai (1989) concluded that the existence of top stiffeners omits the distortional deformations of the tank's wall. The good agreement between the test and analytical results indicates that with the existence of top stiffeners, the distortional modes of the tank's wall would have negligible contribution to its deformations.

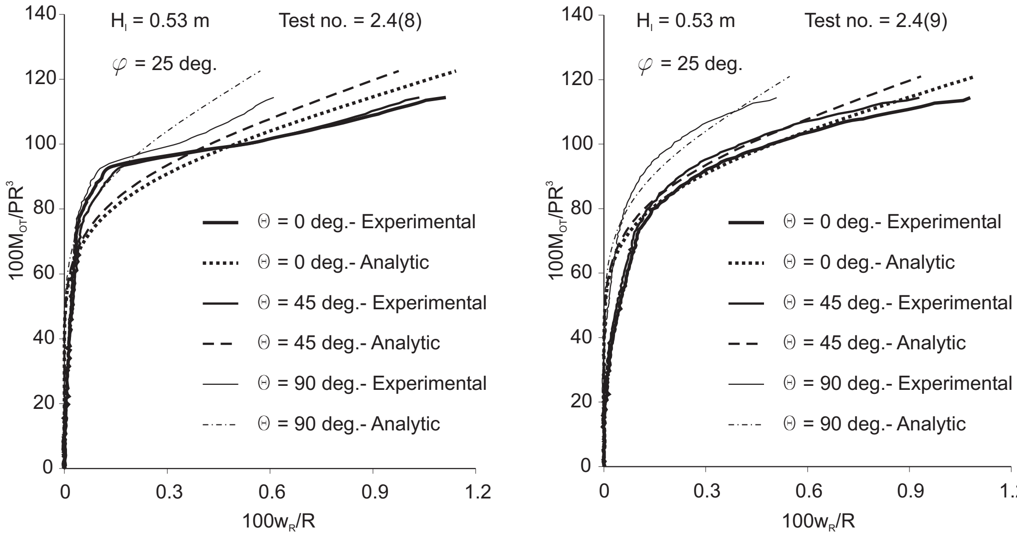


Figure 6 Comparison of the overturning moment- base rotation diagrams in analytical and experimental methods

4. SIMPLIFIED METHOD OF OBTAINING VERTICAL FORCE-UPLIFT DIAGRAM OF CONSTANT WIDTH BEAM

The method which is introduced to establish the overturning moment-rotation ($M_{OT} - \psi$) diagram of the base plate is based on the uplift analysis of a finite number of tapered beams. Also, to establish the ($M_{OT} - \psi$) diagram, constant width beams can be used instead of tapered beams. Most of the times, the difference between the tapered and constant beam results are less than 15 % .The analytical uplift analysis of the tapered or constant beam is complicated and needs a computer programming software. It is an iterative solution in which there are some numerical difficulties at small uplift amounts. In this section, a simple procedure is introduced to establish the vertical force-uplift ($Q_R - w_R$) of the constant beam which can easily be extended to tapered beams (Ahari, 2008). In this procedure, there is not any iterative solution and the ($Q_R - w_R$) diagram can be obtained easily with sufficient accuracy. The differential equation of the deformed shape of the constant beam of width b can be written as:

$$\frac{d^2 M(x)}{dx^2} - \frac{N}{EI} M(x) = -P \tag{5}$$

It should be noted that, the bending solution is obtained from Eqn. 5 when its second left hand side term is omitted. And the string solution is obtained from Eqn. 5 when its first left hand side term is omitted. The bending and string solutions are illustrated by Malhotra & Veletsos (1994a).

As evident in Figure 2, the $Q_R - w_R$ diagram of the beam can be divided to three parts: 1) Part AB in which the vertical force (Q_R) is less than the summation of the weight of the tank's wall multiplied by $b/2\pi R$ and the distributed vertical force which is exerted on the boundaries of the base plate by the wall's hydrostatic pressure multiplied by b. In this case, the beam does not experience uplifting. 2) Part BC in which point B marks the initiation of the uplift. In this part, the beam's material behaves linearly and the second order effects due to the beam's axial force are negligible. Here, the uplift response of the beam is similar to the bending solution of the Eqn. 5. 3) Part after point C. At point C, a plastic hinge is formed at the uplifted end of the beam. The geometrical nonlinearity significantly affects the uplift response of the beam. In this part, the uplift response of the beam follows the string solution of Eqn. 5. The following procedure is suggested to establish the $Q_R - w_R$ diagram:

1. The initial uplift length is calculated from Eqn. 6. The vertical force at point B is equal to $Q_0 = \frac{3}{4} q L_0$.

$$\frac{q}{24 E I_b \beta_s} L_0^3 + \frac{q}{8 D_s \beta_s^2} L_0^2 - \frac{\gamma R^2 H_l}{E t_s}\left(1 - \frac{1}{\beta_s H_l}\right) = 0 \tag{6}$$

in which q and γ are the hydrostatic pressure of base plate and liquid density, respectively. $I_b = t_b^3/12$, $D_s = E t_s^3 / 12(1-\nu^2)$ and $\beta_s = \sqrt[4]{3(1-\nu^2)} / \sqrt{R t_s}$.

2. The uplift length of point C (L_1) can be estimated by solving Eqn. 7. At least for two uplift lengths less than the uplift length of point C, the bending moment of the uplifted end of the beam should be calculated from Eqn. 8. Then after, the uplift and vertical force related to these uplift lengths can be calculated form Eqns. 9 and 10, respectively. Finally, by substituting L with L_1 and M_R with $-M_y$ in Eqns 9 and 10, the uplift and vertical force of point C can be calculated as follows:

$$k_{\theta\theta} p L^3 - 6 k_{\theta\theta} M_y L - 12 E I M_y = 0 \tag{7}$$

$$M_R = -\frac{k_{\theta\theta}}{k_{\theta\theta} + 2EI/L} \frac{pL^2}{6} \tag{8}$$

$$w_R = \frac{pL^4}{24EI} + \frac{M_R L^2}{6EI} \tag{9}$$

$$Q_R = \frac{pL}{2} - \frac{M_R}{L} \tag{10}$$

in which $k_{\theta\theta} = 2b\beta_s D_s$, $k_{\theta u} = -2b\beta_s^2 D_s$, $k_{uu} = 4b\beta_s^3 D_s$.

3. The part after point C can be replaced with a straight line. The slope of this line can be estimated from Eqn. 11.

$$0.7p\left(\frac{k_s/p}{1+0.15k_s R/AE}\right)^{\frac{1}{3}} \tag{11}$$

in which $A = t_b b$, $k_s = k_{uu} - k_{\theta u}^2 / k_{\theta\theta}$.

4.1. *Comparison of Vertical Force-Uplift Diagrams of Exact and Simplified Solutions*

The $Q_R - w_R$ diagrams of exact and simplified solutions are compared against each other in Figures 7 and 8. Figure 7 belongs to the tall tank and Figure 8, to the broad tank. The geometrical parameters of the tanks are shown in these Figures. As it is evident, the exact and simplified solutions are in good agreement with each other. There are just some discrepancies in the results of tanks with large amounts of t_s/R. Therefore, the overturning moment-base rotation of unanchored tanks can be obtained using the simplified diagram.

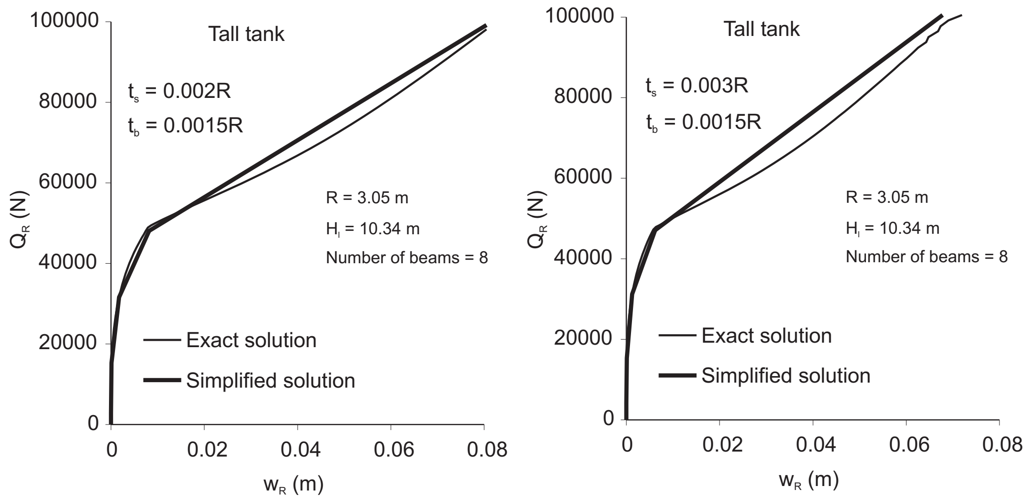


Figure 7 Comparison of vertical force- uplift diagram of exact and simplified solutions in tall tank

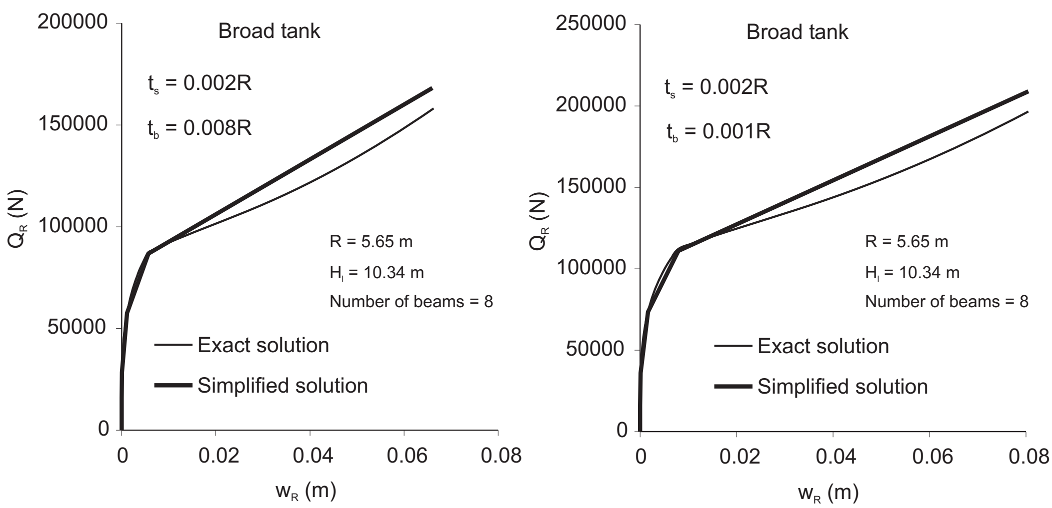


Figure 8 Comparison of vertical force- uplift diagram of exact and simplified solutions in broad tank

5. CONCLUSION

The objective of the present paper was to obtain the overturning-base rotation diagram of unanchored tanks. To this end, a modified version of NZSEE (1986) method was used. The method was based on tapered beam models. To ensure the accuracy of the method, an experimental model of a broad tank was statically tilted up. Finally, a simplified method was introduced to establish the overturning moment- base rotation diagram of such tanks. The method is able to obtain the diagram accurately without any iterative efforts. The main results of the study are as follows:

1. The theoretical method which is based on the uplift of prismatic tapered beams can provide accurate solutions for the base plate uplift of unanchored tanks which rest on rigid foundations and have fixed roofs or top circumferential stiffeners.
2. The results of the experimental investigation of the broad tank model under study show that the existence of a top circumferential stiffener causes the distortional deformation modes of the tank's wall to have negligible contributions to its deformation. Also, in the existence of top stiffeners, the tank's wall will rigidly rotate under overturning moments.

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