SECTION DESCRIPTION BASED ON SECTION INTRINSIC TIMES FOR MEMBER MODELING*

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ABSTRACT :
For Beam-column models in the nonlinear analysis of structures concerned with earthquake ground motions, the description of nonlinear distribution along the member length and the section behavior under reversal compound forces are two important aspects to ensure the reliability of the member model. This paper introduces a new way for section description, which is based on the section intrinsic times and could be used for either homogeneous members or composite members. According to the irreversible thermodynamics, the establishment of the energy conservation equation in the partial length of member and the section constitutive equation with integral style is introduced in detail. For the sake of simplicity, a practice model with the synthetic section-time is proposed and the incremental equations convenient to numerical methods of member nonlinear analysis are introduced. The mathematical equations of the model are uniform for various loading situations, and the computer programming work at the section level could be greatly simplified when it is put into use in member nonlinear analysis. The method for determining the model parameters is discussed and some improvement suggestions are proposed for further researches at the end of the paper.

KEYWORDS: members, model studies, cross sections, nonlinear analysis, structural analysis, thermodynamics

1. INTRODUCTION

Beam-column models are widely used in the nonlinear analysis of structures concerned with earthquake ground motions. Considering the phenomenon that plasticity often happens and spreads at member ends, models with concentrated plastic-hinges are widely used in the elastic-plastic analysis of R/C frame structures due to the convenience in numerical calculation.

The single-component model (Giberson1969) describes the member plasticity through two hinges concentrated at member ends. When the plastic-hinge is concerned with the section character under compound forces, the model could be used to the biaxial dynamic analysis of frame structures (Nigam 1970; Takezawa and Aoyama 1976). For the model with finite length hinges, the member is divided into three parts and the plastic length of the two end-parts changes with the moment distribution along the member (Meyer, Roufaiel, and Arzoumanidis 1983; Roufaiel and Meyer 1987). Darvall and Mendis (1985) also defined a softening length at member ends to undergo softening analysis for plane frames. With the development of Finite Element Method (FEM), member models based on FEM gradually become popular in structure nonlinear analysis. Zeris & Marhin (1988) proposed an idea of interpolation of section force in a kind of mixed algorithm. Owing to the continued work

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(Zeris and Marhin 1991-1; Zeris and Marhin 1991-2; Spacone, Ciampi, Filippou 1996), the flexibility-based model has been gradually put into practice in structure dynamic analysis.

For member modeling methods based on section description, the description of section hysteretic behavior is the key to ensure the reliability of the model. For R/C plane frames, the relationship between moment and rotation at member end is the mainly concerned aspect in early studies. Various multiple-lines-composite models have been put forward to describe the strength and rigid features of the section under reversal forces (Clough, Benuska, and Wilson 1965; Takeda, Sozen, Nielsen 1970; Roufaiel and Mayer 1987; Park and Ang 1985). Improvements of the studies are focused on the description of shear and bond-slip effects that cause the rigid deterioration in the aspect of hysteretic characteristics.

As for the section description under compound forces, Nigam (1970) developed a section model based on the analogical theory of the plastic mechanics on describing stress-strain constitutive relationship. Further researches were made by Takizawa and Aoyama (1976) to take into account the degrading trilinear stiffness model in the hardening rules. For reinforced concrete columns, Lai, Will, and Otani (1984) proposed a special model with parallel end-springs, and the further developments of the model are focused on less spring number and on more rational description for reinforced concrete features (Lai and Will 1986; Saiidi, Ghusn, and Jiang 1989; Jiang and Saiidi 1990). In the study of member FEM, the fiber-section model was put forward to describe the section characteristics under biaxial bending and axial force (Zeris and Marhin 1988 & 1991; Neuenhofer and Fillippou 1997; Spacone, Ciampi, Filippou 1996). The compound feature of the section is automatically formed by the integral of fiber stress on the cross section. Combined with the flexibility-based iterative method, the fiber-section model could rationally control the total hysteretic history of the member without too much numerical calculation quantities.

Thus, it could be concluded that if a reasonable section model could be offered, the member model could be established by many methods, such as the single component method, the classical FEM method or the flexibility-based iterative method. A section model should be convenient to numerical calculations and capable of describing the section restoring force character under compound forces correctly. This paper introduces a new way for section description, which is based on the idea of section intrinsic variables. The theoretical establishment is introduced in detail and some application issues are discussed.

2. ENERGY CONSERVATION EQUATION IN PARTIAL ZONE OF MEMBER

Consider the deformation history of a member from the viewpoint of irreversible thermodynamics. Take the micro length of member, instead of the micro cube, as the subregion of the member. Correspondingly, member deformation and resistance at the section instead of strain and stress are taken as primitive state variables of the irreversible system. According to the energy conservation equation of the whole member,

$$\int_L \left(\rho\dot{\boldsymbol{u}}^T\ddot{\boldsymbol{u}} + \rho\dot{\boldsymbol{\theta}}^T\boldsymbol{G}\ddot{\boldsymbol{\theta}}\right)dL + \int_L \dot{e}dL = \int_L \boldsymbol{F}^T\dot{\boldsymbol{v}}dL + \int_L \left(\rho\ddot{\boldsymbol{u}}^T\dot{\boldsymbol{u}} + \rho\ddot{\boldsymbol{\theta}}^T\boldsymbol{G}\dot{\boldsymbol{\theta}}\right)dL + \int_L \dot{q}^{(e)}dL \tag{2.1}$$

where L =member length; ρ =mass per unit length; $\mathbf{u}$ = displacement vector of the section centroid; $\boldsymbol{\theta}$ = section rotation vector; $\mathbf{G}$ is a 3×3 diagonal matrix with the element being the polar moment of inertia of the section and the two moments of inertia along the two principle axes of the section; e =energy per unit length; $\mathbf{F}$ = section resistance vector with 6 components: axial resistance, two shear resistances along centroidal principle axes of the section, moment resistance of torsion, and two moment resistances of bending in centroidal principle planes; $\mathbf{v}$ = member deformation vector with 6 components: axial strain, two shear strains along centroidal principle axes of the section, torsion curvature, and two bending curvatures in centroidal principle planes; and $q^{(e)}$ = total heat flux per unit length conducted from circumstance to member. Simplification of Eq.2.1 gives:

$$\dot{e} = \mathbf{F}^T \dot{\mathbf{v}} + \dot{q}^{(e)} \quad \text{or} \quad de = \mathbf{F}^T d\mathbf{v} + dq^{(e)} \tag{2.2}$$

where "d" denotes the increment of variables.

3. ESTABLISHMENT OF THE SECTION MODEL BASED ON SECTION INTRINSIC TIME

According to the irreversible thermodynamics containing internal variables, select M groups of independent internal variables, $\mathbf{g}_1$, $\mathbf{g}_2,\ldots,\mathbf{g}_\mathrm{M}$. Each $\mathbf{g}_m$ (m =1,2,…,M) has 6 components corresponding to the 6 aspects of member deformation. From Eq.2.2, the relationship between the state variables and generalized forces could be derived by the similar way adopted in the material constitutive study (Valanis 1971). We now just write out the results without detailed derivation:

$$\mathbf{F} = \left.\frac{\partial \psi}{\partial \mathbf{v}}\right|_{\vartheta, \mathbf{g}_1, \mathbf{g}_2, \ldots \mathbf{g}_\mathrm{M}} \; ; \; \eta = -\left.\frac{\partial \psi}{\partial \vartheta}\right|_{\mathbf{v}, \mathbf{g}_1, \mathbf{g}_2, \ldots \mathbf{g}_\mathrm{M}} \; ; \; \mathbf{Q}_m = -\left.\frac{\partial \psi}{\partial \mathbf{g}_m}\right|_{\mathbf{v}, \vartheta} \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.1}$$

$$\mathbf{Q}_m^T \dot{\mathbf{g}}_m \geq 0 \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.2}$$

where $\psi = \psi(\mathbf{v}, \vartheta, \mathbf{g}_m)$ = Helmholtz free energy; η = entropy; $\mathbf{Q}_m$ (m =1,2,…,M) = M group of generalized friction forces corresponding to $\mathbf{g}_m$ (m =1,2,…,M) ; and ϑ has the significance of empirical measure of temperature. Define 6 section intrinsic times, z_i (i =1,2,…,6), to represent the measurement of irreversible extent of each aspect of the member deformations. Assume that each $\mathbf{g}_{mi}$ (i =1,2,…,6) is only the function of z_i with the same subscript, i. For the situation of small deformation and small change of temperature, assume that

$$\mathbf{Q}_m = \mathbf{s}_m \dot{\mathbf{g}}_m \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.3}$$

where

$$\dot{\mathbf{g}}_m = \left[\dot{g}_{m1}\ \dot{g}_{m2} \cdots \dot{g}_{m6}\right]^T = \left[\frac{dg_{m1}}{dz_1}\ \frac{dg_{m2}}{dz_2} \cdots \frac{dg_{m6}}{dz_6}\right]^T \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.4}$$

and $\mathbf{s}_m$ (m =1,2,…,M) are 6×6 real matrices. Substituting Eq.3.3 in Eq.3.2 gives

$$\dot{\mathbf{g}}_m{}^T \mathbf{s}_m{}^T \dot{\mathbf{g}}_m \geq 0 \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.5}$$

This demonstrates that $\mathbf{s}_m$ (m =1,2,…,M) are positive-definite matrices.

Expand $\psi = \psi(\mathbf{v}, \vartheta, \mathbf{g}_m)$ according to the Taylor Series at the reference state, ignore the item higher than two orders, and suppose that all the variables are relative to the reference state, we obtain

$$\psi = \frac{1}{2}\mathbf{v}^T\mathbf{a}\mathbf{v} + \sum_{m=1}^{\mathbf{M}}\mathbf{v}^T\mathbf{b}_m\mathbf{g}_m + \frac{1}{2}\sum_{m=1}^{\mathrm{M}}\sum_{n=1}^{\mathrm{M}}\mathbf{g}_m{}^T\mathbf{c}_{mn}\mathbf{g}_n + \mathbf{v}^T\mathbf{d}\,\vartheta + \sum_{m=1}^{\mathbf{M}}\mathbf{g}_m{}^T\mathbf{e}_m\vartheta + \frac{1}{2}f_0\vartheta^2 \tag{3.6}$$

where $\mathbf{a}$ is a 6×6, symmetrical, real, and constant matrix; $\mathbf{b}_m$ (m=1,2,…,M) are 6×6, real and constant matrices; $\mathbf{c}_{mn}$ (m, n=1,2,…,M) are 6×6, real and constant matrices with symmetrical nature of $c_{mnij} = c_{nmji}$ (i, j=1,2,…,6); $\mathbf{d}$ and $\mathbf{e}_m$ (m=1,2,…,M) are real and constant vectors with 6 components; f_0 is a real constant. Making partial derivations of Eq.3.6 and combining Eq.3.1a, Eq.3.1c and Eq.3.3, we obtain that

$$\mathbf{F} = \mathbf{a}\mathbf{v} + \sum_{m=1}^{\mathbf{M}}\mathbf{b}_m\mathbf{g}_m + \mathbf{d}\,\vartheta \tag{3.7}$$

$$\mathbf{s}_m\dot{\mathbf{g}}_m + \mathbf{b}_m{}^T\mathbf{v} + \sum_{n=1}^{\mathrm{M}}\mathbf{c}_{mn}\mathbf{g}_n + \mathbf{e}_m\vartheta = 0 \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.8}$$

Considering the independence between $\mathbf{g}_m$ (m=1,2,…,M) for different m, Eq.3.8 can be written as

$$\mathbf{s}_m\dot{\mathbf{g}}_m + \mathbf{b}_m{}^T\mathbf{v} + \mathbf{C}_m\mathbf{g}_m + \mathbf{e}_m\vartheta = 0 \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.9}$$

where $\mathbf{C}_m$ (m=1,2,…,M) are 6×6, symmetrical, real, and constant matrices. Considering the positive-definite nature $\mathbf{s}_m$ and the symmetrical nature of $\mathbf{C}_m$, there must be an unsingular matrix $\mathbf{P}_m$ (6×6) (m=1,2,…,M) making

$$\mathbf{P}_m{}^T\mathbf{s}_m\mathbf{P}_m = \mathbf{I},\ \ \mathbf{P}_m{}^T\mathbf{C}_m\mathbf{P}_m = \boldsymbol{\lambda}_m = \begin{bmatrix} \lambda_{m1} & & 0 \\ & \ddots & \\ 0 & & \lambda_{m6} \end{bmatrix} \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.10}$$

where $\mathbf{I}$ = 6×6 unit diagonal matrix; λ_{mi} (i=1,2,…,6) = six real roots of $\left|\boldsymbol{\lambda}_m\mathbf{S}_m - \mathbf{C}_m\right|$. Using Eq.3.10, Eq.3.9 could be solved as following explicit expressions:

$$\mathbf{g}_m = -\mathbf{P}_m\int_0^{\mathbf{z}}\boldsymbol{\Lambda}_m(\mathbf{z}-\mathbf{z}')\left(\mathbf{P}_m{}^T\mathbf{b}_m{}^T\mathbf{v} + \mathbf{P}_m{}^T\mathbf{e}_m\vartheta\right)d\mathbf{z}' \quad (m=1,2,\ldots,\mathrm{M}) \tag{3.11}$$

where $\int_0^{\mathbf{z}}\Gamma(\mathbf{z},\mathbf{z}')d\mathbf{z}'$ denotes $\left[\int_0^{z_1}\Gamma(z_1,z_1')dz_1' \ \ \int_0^{z_2}\Gamma(z_2,z_2')dz_2' \ \ldots \ \int_0^{z_6}\Gamma(z_6,z_6')dz_6'\right]^T$ and $\boldsymbol{\Lambda}_m(\ )$ denotes the diagonal matrix with the diagonal elements being $e^{-\lambda_{m1}(\ )}$, $e^{-\lambda_{m2}(\ )}$,…, $e^{-\lambda_{m6}(\ )}$. Substitute Eq.3.11 in Eq.3.7 gives

$$\mathbf{F} = \mathbf{a}\mathbf{v} - \sum_{m=1}^{\mathbf{M}}\mathbf{A}_m\int_0^{\mathbf{z}}\boldsymbol{\Lambda}_m(\mathbf{z}-\mathbf{z}')\left(\mathbf{A}_m{}^T\mathbf{v}\right)d\mathbf{z}' + \mathbf{d}\,\vartheta - \sum_{m=1}^{\mathbf{M}}\mathbf{A}_m\int_0^{\mathbf{z}}\boldsymbol{\Lambda}_m(\mathbf{z}-\mathbf{z}')\left(\mathbf{P}_m{}^T\mathbf{e}_m\vartheta\right)d\mathbf{z}' \tag{3.12}$$

where $\mathbf{A}_m = \mathbf{b}_m\mathbf{P}_m$ (m=1,2,…,M) are 6×6, real, and constant matrices.

For isothermal conditions, Eq.3.12 becomes

$$\mathbf{F} = \mathbf{a}\mathbf{v} - \sum_{m=1}^{\mathbf{M}}\mathbf{A}_m\int_0^{\mathbf{z}}\boldsymbol{\Lambda}_m(\mathbf{z}-\mathbf{z}')\left(\mathbf{A}_m{}^T\mathbf{v}\right)d\mathbf{z}' \tag{3.13}$$

This is the section model based on section intrinsic times, z_i (i=1,2,…,6), which could be used in member modeling. The irreversible deformation feature of the section is represented by z_i (i=1,2,…,6), and the nonlinear compound feature between different aspects of the member deformation is represented by the non-diagonal elements of $\mathbf{A}_m$ (m=1,2,…,M). Note that whether the member material is homogenous or not is not mentioned during the establishment of Eq.3.13. The section model is applicable to homogeneous members or composite members.

4. PRACTICAL SECTION MODEL AND MEMBER NUMERICAL CALCULATION

For the sake of simplicity, let $z_1 = z_2 = ... = z_6 = z$ and assume that A_{mij} =0 when $i \neq j$, Eq.3.13 is simplified as:

$$F_i = a_{ij} v_j - \sum_{m=1}^{M} \int_0^z M_{mi} e^{-\lambda_{\alpha i}(z-z')} v_i(z') dz' \quad (i=1,2,\ldots,6) \tag{4.1}$$

z is called the synthetic section-time. In the case of small deformations, define the plastic deformation vector as

$$dv_i^p = dv_i - \frac{k_i}{k_{0i}} dF_i \quad (0 < k_i < 1) \quad (i=1,2,\ldots,6) \tag{4.2}$$

where k_{0i} = elastic stiffness of each dimension of the section. Using the similar method taken in the study of Enodochronic Theory of Plasticity (Valanis 1980; Wu and Yang 1983), Eq.4.1 could be written as

$$F_i(z) = \int_0^z \mu_i(z-z') \frac{dv_i^p}{dz'} dz', \quad \mu_i(z) = \frac{k_{0i}}{1-k_i} e^{-\left(\frac{\rho_{0i}}{1-k_i}\right)z} + k_{1i} e^{-\rho_{1i} z} \quad (i=1,2,\ldots,6) \tag{4.3}$$

where dz could be defined as:

$$dz = e^{-\beta z} g\left(\kappa_1 dv_1^p, \kappa_2 dv_2^p, ..., \kappa_6 dv_6^p\right) \tag{4.4}$$

and k_{0i}, k_i, ρ_{0i}, ρ_{1i}, κ_i (i=1,2,…,6) and β are constant parameters of the section model. The incremental form of Eq.4.3 is

$$dF_i = D_i dv_i + E_i \rho_i(z) dz \quad (i=1,2,\ldots,6) \tag{4.4a}$$

$$\rho_i(z) = \int_0^z \frac{\partial \mu_i}{\partial z}(z-z') \frac{dv_i^p}{dz'} dz'; \quad D_i = E_i \mu_i(0); \quad E_i = \frac{1}{1 + \frac{k_i}{k_{0i}} \mu_i(0)}; \quad \mu_i(0) = \frac{k_{0i}}{1-k_i} + k_{1i} \tag{4.4b,c,d,e}$$

where D_i (i=1,2,…,6) is the original stiffness of dimension i of the section. If it is assumed that dimension i is elastic, $E_i \rho_i(z) dz$ in Eq.4.4a is zero and D_i is the elastic stiffness of the dimension. For the dimensions concerned with plasticity, D_i should be obtained through Eq.4.4c, Eq.4.4d, and Eq.4.4e.

Introducing $r_i(z)=k_{i1}\int_0^z e^{-\rho_{i1}(z-z')}\dfrac{dv_i^p}{dz'}dz'$ in Eq.4.4a as aiding variables, a group of differential equations could be obtained:

$$\begin{cases}\rho_{01}(F_1-r_1)dz+dF_1+(k_1-1)dr_1-k_{01}dv_1=0\\ k_{01}\rho_{11}r_1dz+k_1k_{11}dF_1+k_0dr_1-k_0k_{11}dv_1=0\\ \ldots\ldots\\ \rho_{06}(F_6-r_6)dz+dF_6+(k_6-1)dr_6-k_{06}dv_6=0\\ k_{06}\rho_{16}r_6dz+k_6k_{16}dF_6+k_{06}dr_6-k_{06}k_{16}dv_6=0\\ dze^{\beta z}=g\left[\kappa_1\left(dv_1-\dfrac{k_1}{k_{01}}dF_1\right),\kappa_2\left(dv_2-\dfrac{k_2}{k_{02}}dF_2\right),\ldots,\kappa_6\left(dv_6-\dfrac{k_6}{k_{06}}dF_6\right)\right]\end{cases} \tag{4.5}$$

Eq.4.5 could be used to solve $d\mathbf{F}$ and dz from known $d\mathbf{v}$ and the nonlinear relationship between $\mathbf{F}$ and $\mathbf{v}$ could be obtained. Rewrite Eq.4.4a as

$$d\mathbf{F}=\mathbf{D}d\mathbf{v}+d\mathbf{F}^p \tag{4.6}$$

where $\mathbf{D}$ is the diagonal matrix with elements being D_i (i=1,2,…,6) and $\mathbf{F}^p$ is called the plastic resistance vector of the section. Eq.4.5 and Eq.4.6 could be used in the various member models mentioned in the introduction and obtain corresponding iterative methods.

5. SOME DISCUSSTIONS

Eq.4.5 and Eq.4.6 is the section model based on the synthetic section-time. The mathematic equation is uniform for monotonic loading or reversal loading and for one-dimensional deformation or compound forces case. When it is applied to member nonlinear analysis, the computer programming work and the numerical calculation procedure under the section level could be greatly simplified. Because the section model is based on the relationship between section resistance and deformation rather than on the relationship between stress and strain, the member model could be used for homogeneous members or composite members.

$k_i, k_{0i}, k_{1i}, \rho_{0i}, \rho_{1i}, \kappa_i$ (i=1,2,…,6) and β in Eq.4.5 are section parameters. According to the principle that the section model should satisfy each of the single deformation condition, $k_i, k_{0i}, k_{1i}, \rho_{0i}, \rho_{1i}$ and β could be determined by the real F_i-v_i relationship under every single deformation condition. For the single deformation case of dimension i, only keep the two equations for the dimension in Eq.4.5 and take the last equation as $dz=e^{\beta z}\left|dv_i^p\right|$. It should be noticed that $k_i, k_{0i}, k_{1i}, \rho_{0i}, \rho_{1i}$ and β, in the equations are as a whole to determine the F_i-v_i theoretical curve. Except that k_{0i} is the elastic stiffness of the dimension, other parameters haven't specific meaning for the F_i-v_i curve. The principle for determining them is to ensure the theoretical curve well fitting the real one. An adaptable fitting principle should be taken and corresponding computer program method should be used in the work.

When the section is under compound forces, the definition of dz is very important for the compound force description of the section. The definition way of dz could refer to the function of the real section yielding surface. For example, in the case of biaxial bending, dz could be defined as $dz = e^{-\beta z}\sqrt{\left(\kappa_1 dv_1^p\right)^2 + \left(\kappa_2 dv_2^p\right)^2}$ and further analysis is focused on the determination of κ_1 and κ_2. However, for the more complex compound cases, same order of $\kappa_i dv_i^p$ in Eq.4.4 seems to be inadaptable and the different units between different deformations should be taken into account in the definition of dz. A systematic study is still going on for commonly used members with various shapes and materials.

On the other hand, the function style of the integral kernel in Eq.4.3 is determined by the assumed relationship between $\mathbf{Q}_m$ and $\dot{\mathbf{g}}_m$. In this paper, Eq.3.3 gives rise to the exponential kernel of Eq.3.13 and makes the hysteric curve being diamond style. Different kernel functions should also be attempted for different deformation aspect of the member.

6. CONCLUSION

A new way for section modeling is proposed in the paper, which is similar to the Endochronic Theory of Plasticity. However, there are difference between them. The original equation of the section model has integral style and the irreversible features are described by the section intrinsic time. When it is introduced into a beam-column model, the computer programming work under the section level could be greatly simplified.

For the endochronic model, the main difficulty in the utilization is the definition of intrinsic time and the determination of the model parameters. In order to make the model suitable for complex hysteretic feature, multiple section intrinsic times and different kernel functions should be taken into account to improve the reliability of the model in engineering practice.

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