SIMULATE REVERSED NON-LINEAR HYSTERETIC RELATIONSHIPS BY FINITE STATE MACHINE

Deyin JIN and Yuji ITO

FORUM8 Co., Ltd. (http://www.forum8.co.jp)
2-1-1 Nakameguro GT Tower 15F, Kami-Meguro Meguro-ku Tokyo, 153-0051, JAPAN
Email: kin@forum8.co.jp

ABSTRACT:
This paper presents the application of Finite State Machine (FSM) theory to the programming of nonlinear hysteretic model simulation software for both known and newly created rules. Reversed nonlinear constitutive simulation of materials and components is crucial for the analysis of structures under strong earthquake forces. However, complicated reversed internal paths, which not only depend on material properties, but also on load history, often confuse rule creators and scholars. In this paper, we first summary the hysteretic models and their properties and introduce the FSM theory conceptually. Then we explain how it is applied to reversed and diverse hysteresis routes including state definitions and procedures. Two application examples, the Bilinear Model and the Mander Model, are explained with state definition and state diagram. The successful application in UC-win/FRAME(3D) is described too. Finally, several characteristics of the application of FSM to nonlinear simulation are summarized.

KEYWORDS: Hysteretic Relationships, Finite State Machine, Nonlinear Simulation, Earthquake Responses, Reversed Routes, Constitutive Laws

1. INTRODUCTION

The simulation of structural responses to earthquakes has developed from static analysis to dynamic analysis and from linear analysis to nonlinear analysis. Particularly following a series of disasters, such as the Northridge Earthquake (1994) and the Kobe Earthquake (1995), both material nonlinear analysis and dynamic behavior are accounted for in the design standards for safety checks under the concept of performance-based design [1]. The nonlinearity of structural responses to earthquakes mainly originates from materials has been considered to the extent that materials loss their resistance ability against the forces, For example, drastic loss in resistance for a steel bar includes breaking in tension and buckling in compression. Under reversed cyclic loads, nonlinear responses present a complex hysteric characteristic route which is a key part to any seismic response simulation.

In the structural analysis, the nonlinearity of concrete and steel and the composition of RC sections are crucial concerns. Up until now, many kinds of constitutive rules, for example, the Mander Model [2] for concrete, the Menegotto-Pinto Model [3] for steel bars and the Takeda Model [4] for P-δ relationship of whole RC sections, have been developed in attempt to reflect material or component hysteretic properties during strong earthquake activity. Researchers put great time and energy into establishing new models and modifying current ones in order to more precisely describe material properties. Nonetheless, due to complicated issues such as internal routes and interactions between composite materials, researchers have not yet found an efficient method to describe each constitutive law and create quick simulations on the computer.

FSM [5], [6] is a theory that describes state contents and transitions that are widely applied to computer programming. For example, a computer is a basic state machine. Each instruction is input to the present state and then the whole machine transits into a new state. During computer executing, its state changes from one state to another state. Comparing to nonlinear hysteric relationship, it can also be modeled into a series of states and links between states, the nonlinear proceeding can be taken as state changes. Thus we can apply the FSM theory to constitutive law and use the state set to present any constitutive law.

In this paper the application of FSM theory to nonlinear hysteresis simulation is presented. First the nonlinear hysteresis relationships and their characteristics are summarized in the micro material level and macro section level. Then the FSM theory and mathematical expression are introduced generally. Next, the simulation based on FSM e theory is explained in detail through demonstrating how to define the states and state transitions to describe the hysteretic routes for a constitutive model under reversed loads, such as dynamic earthquake loads. As the application examples, the Bilinear Model and the Mander Model are employed to generate state set and state diagram. The application of FSM in UC-win/FRAME(3D)[7] is mentioned too. Finally, conclusions regarding of the application of FSM theory to nonlinear simulation are summarized.

2. REVERSED HYSTERETIC MODELS

According to up-to-date seismic design concepts, such as performance-based design [1], structures attacked by a strong earthquake are allowed to enter nonlinear fields and retain ductility in order to absorb the earthquake-induced vibration energy as much as possible, and avoid collapse. When structures enter the nonlinear state prior to collapse, the ductile phase and reversed nonlinear calculations will determine the structural response. Therefore, the establishment and accurate integration of a constitutive model can affect the accuracy of the response prediction.

The structural ductile development, plastic phase, can be depicted in stress-strain relationship in material level. And many kinds of constitutive law for the different materials such as concrete and reinforcements([2], [3]) have been proposed. For reversed forcing action, those constitutive laws not only explain the monotonically change trend but also give unloading or reloading paths.

Although stress-strain constitutive relationships can model nonlinear behaviors of structural responses more accurately from the micro material point, the integrated parameters (such as M-φ, P-δ) for a whole section are often used to describe the nonlinear characteristics of structures in the early constitutive models([4], [8],[9]). This is due to the limitations of modeling methods and computer capacities. Presently, because some models have been fully developed and priorly accepted, they are still used broadly by the researchers and engineers in their structural computations. For example, the Takeda Model, which describes the force-displacement nonlinear hysteretic relationship in a RC section, is stated clearly in the seismic code of bridge design [10].

As for composite sections such as RC and PRC, the constitutive relationship between force and displacement is not only determined by the material properties of the concrete and steel, but also by the arrangement of reinforcement and section's shape. Particularly, for unsymmetrical, irregular-shaped sections, the reversed constitutive law becomes more difficult to determine. In earlier models, its constitutive relationship was simply taken as a bilinear line that represented the degradation of section stiffness after the reinforcement yield. After further observation, tri-linear or tetra-linear models are proposed to account for the cracking of concrete and after-peak responses. With the unloading and reloading processes resulting from the vibrating reversed earthquake acting, following to the damage state, the rule of degrading stiffness and point-directed are defined like the historical maximum (minimum) point or origin point. The slope of decline route can be straight line or curves.

Fig. 1 shows three constitutive models of concrete, reinforcement, and RC section. By observing those constitutive models or others, we can generalize the following properties about their line moving and shapes.

- Skeleton curve (piece lines or functional curves) under monotonically increasing load;
- Unloading and reloading paths;

- Directional difference (concrete tension or compression, unsymmetrical sections);
- Boundary limit (deflect in some meanings);
- Historical state dependence.

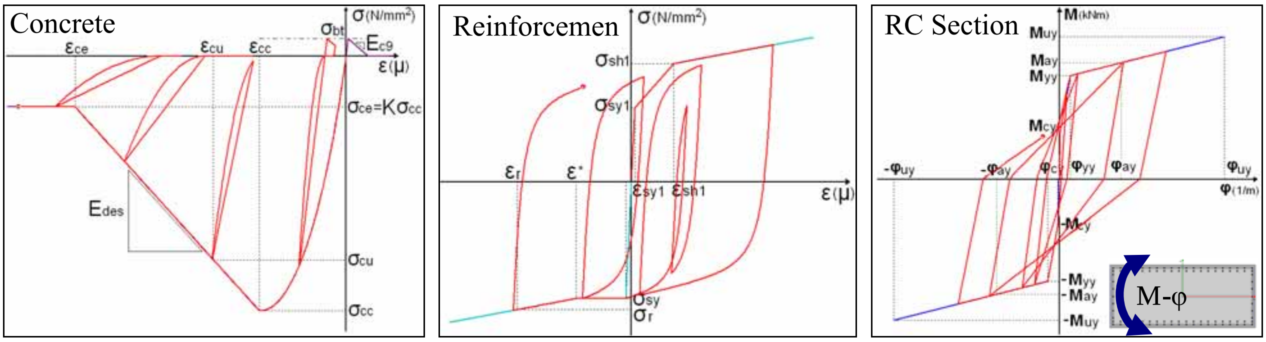

Fig. 1 Constitutive Law Samples of Concrete, Reinforcement, and RC Sections

Furthermore, each model can be abstracted into a series of continuous lines (curves), called state later, and the linkage between those states. These lines are not only directional but also boundary limited, and connected at start points and finish points. Thus we can apply states to describe them and use a directional state network to present a constitutive model.

3. FINITE STATE MACHINE (FSM) THEORY

Finite State Machine theory deals with the transition and behavior between states which can be applied to any specific object or abstract object. The specific object can be cited as a device that stores the status of something at a given time and can operate on input to change the status and/or cause an action or output to take place for any given change. For example, a computer is basically a state machine, with each instruction acting as input, causing the present state to transition into a new state. The FSM model has been widely applied to fields such as electrical engineering, computer science, etc.

In mathematics, the FSM can be expressed as

$$S \text{ x } \Sigma \rightarrow S \qquad (3\text{-}1)$$

With five parameters (Σ, S, S_0, δ, F), in which,

Σ is the input alphabet (a finite non empty set of symbols),

S is a finite non empty set of states,

S_0 is an initial state, an element of S. In a Nondeterministic finite state machine, S_0 is a set of initial states,

δ is the state transition function,

F is the set of final states, a (possible empty) subset of S.

Fig. 2 shows one abstract state element graph of a state with multi-state input and multi-state output. In the practical application, the number of input routes or output routes of a state may vary with specific conditions and even a state may have no output. Finally, the state set like those composes a system or a functional model.

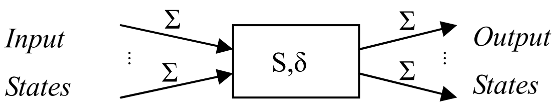

Fig. 2 State transition diagram in FSM

4. FSM APPLICTIONS TO SIMULATE THE CONSTITUTIVE MODELS

4.1. Geometrical Characteristics of Constitutive Laws

According to above description about constitutive law, we can generalize the following properties about the paths and the moving in the constitutive laws. Here we assume the horizontal coordinate as x and vertical as y for a more general constitutive curve graph. Some pre-works as following have been familiarized or finished.

a. Constitutive model is composed of a series of continuous pieces or lines of a curve,
b. Jumping from one piece line to another piece line usually results from the moving direction change or over defined limit,
c. The initial condition (x_0, y_0) must be defined,
d. Calculate y from input x or Δy from Δx based on the last step results and the current path,
e. The curve can stop at any point.

Based on the above generalized contents, any hysteretic model can be considered as a series of states, each of which is composed of the defined lines or curves, boundaries, reversed actions, etc. The connection between states is established by the inner definition in the each state which may be due to the moving direction change or to the defined limit crossing. For example during the loading, if unloading is applied, the unload line diverges from loading with a different slope. If the reloading is applied again, the moving path will become a new line route. The line state transition depends on the state definition and experienced routes. In term of the concept of FSM, we can apply FSM to describe the constitutive law and finish simulation through the state definition.

4.2. Procedures of FSM Application

In ordinary FSM, states can have multi-input or multi condition-based output and the connections are multi-linear. But for the constitutive law corresponding to the summarized properties above, its FSM model has its own properties such as no end state. The procedures to obtain a FSM constitutive model have the following 4 steps.

Step 1. Separate the law into the finite states,
Step 2. Define the details of each state including state name, line functions, boundary state, etc.,
Step 3. Give linkage between states,
Step 4. Generate the state diagram.

When actions or input is performed on the model, the responses or output can be expressed as:

$$y = [x, \Delta x, S_{model}] \tag{4-1}$$

In which,

Δx : the increase of x,
S_{model} : a constitutive model.

While a specific constitutive model is used, it can be presented as a summation of states including an initial state.

$$S_{model} = \bigcup S_j \tag{4-2}$$

In which,

S_j : state elements,
$\bigcup$: state assembly.

In further a state element can be defined as follow and its diagram is also depicted.

$$Sj = \{xi, yi, \Delta x, fj(x), Direction, Limit, So, Sr\} \tag{4-3}$$

In which,

x_i, y_i : the current point,

Δx : the increment of x from x_i,
$f_j(x)$: the moving function within the state,
Direction (+ or –) : plus or negative moving,
Limit : the state limit for x moving,
So : the following state over Limit,
Sr : the state when the x reverses directionally.

Any state has an entering path, but its departing state may limit to itself regardless of proceeding or reversing.

4.3. Application to Bilinear Model

As an example, a symmetrical bilinear constitutive model is displayed here in Fig. 3. The rules for a symmetrical bilinear model are defined in the following.

(1) the initial path is linear (elastic) with a slope of K1 before yielding
(2) the path is along the liner (yielding) with a slope of K2 after yielding
(3) In the yielding state, if the load is reversed, the unloaded with a slope of K1 from tuning point until to cross to other directional yield line
(4) When crossing with the yielding, the path will go along the yielding line
(5) In (4) state, if the proceed is reversed, repeat (3).

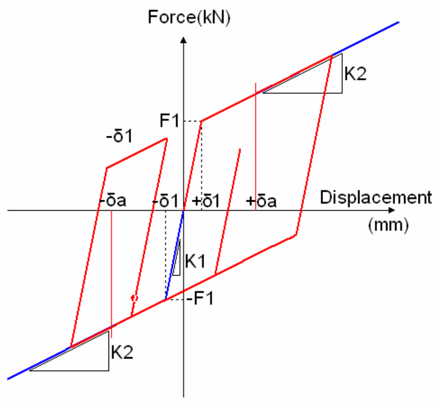

Fig. 3 The Bilinear Constitutive Law

Its FSM state set for Eqn.4.2, 4.3 can be expressed as below.

$S_{Bilinear}$ = Sp1 U Sn1 U Sp2 U Sn2 ;
Sp1 = $\{x_i, y_i, \Delta x, fp1(x), +, y1, Sp2, Sn1\}$;
Sn1 = $\{x_i, y_i, \Delta x, fn1(x), -, -y1, Sn2, Sp1\}$;
Sp2 = $\{x_i, y_i, \Delta x, fp2(x), +, \infty, Sp2, Sn1\}$;
Sn2 = $\{x_i, y_i, \Delta x, fn2(x), -, -\infty, Sn2, Sp1\}$.

In which,

$\Delta x = x - x_i$;
$fp1(x) = y_i + K1 \cdot (x - x_i)$;
$fn1(x) = y_i + K1 \cdot (x - x_i)$;
$fp2(x) = y1 + K2 \cdot (x - x1)$;
$fn2(x) = -y1 + K2 \cdot (x + x1)$.

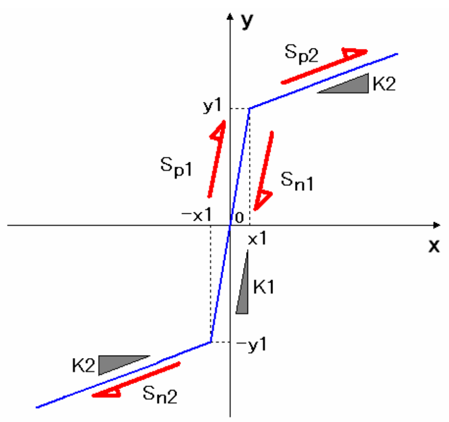

Fig. 4 The Bilinear Hysteretic States

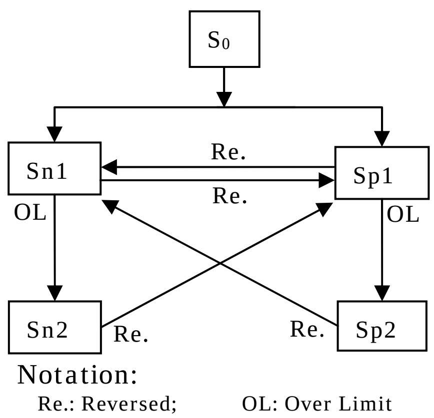

Fig. 5 The State Diagram for the Bilinear Model

Here K1 is the initial stiffness; K2 is the second stiffness; (x1, y1) is the yield point; (x_i, y_i) is the current point. The parameters of the states are shown in Fig. 4 and its FSM diagram is shown in the Fig. 5.

A numerical simulation case is demonstrated in Ref. [12] and the route and state experience can be checked more concretely.

4.4. Application to The Mander Model

The Mander model was proposed for stress-strain relationship of confined concrete by reinforcement [2] as shown in Fig. 6 and later modified by many researchers such as Sakai [11]. It can simulate the stress-strain physical trend of confined concrete under reversed uni-axial forces. By adjusting parameters, the Mander model not only use for reinforcement confinement but also for FRP confinement. The Mander Model mainly is composed of compressive curve and unloading/reloading curves and their function formula is defined as below.

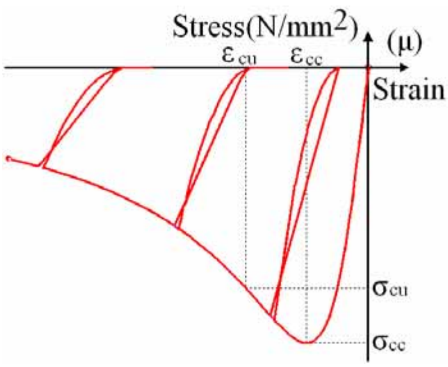

Fig. 6 The Mander Model

- Compressive Curve Function:

$$\sigma = (\sigma_{cc}\frac{\varepsilon}{\varepsilon_{cc}} r) \Bigg/ \left(r - 1 + \left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^r \right); \quad r = \frac{E_{c0}}{E_{c0} - E_S}; E_S = \frac{\sigma_{cc}}{\varepsilon_{cc}} \tag{4.4}$$

  Where ε_{cc} and σ_{cc} are peak strain and stress and E_{c0} is the initial stiffness of confined concrete.

- Unloading Curve Function:

$$\sigma = \sigma_{ul}\left(\frac{\varepsilon - \varepsilon_{pl}}{\varepsilon_{ul} - \varepsilon_{pl}}\right)^2 \tag{4.5}$$

  Where ε_{ul} and σ_{ul} are strain and stress of unloading point, ε_{pl} is the plastic strain

- Reloading Curve Function:

$$\sigma = \begin{cases} 2.5\sigma_{ul}\left(\dfrac{\varepsilon - \varepsilon_{pl}}{\varepsilon_{ul} - \varepsilon_{pl}}\right)^2 & \textit{for 0 stress start} \\ E_{ul}(\varepsilon - \varepsilon_{ul}) + \sigma_{ul} & \textit{for nonzero stress start} \end{cases} \tag{4.6}$$

  Where E_{ul} is an average stiffness [11].

The Mander Model's FSM state set for Eqn.4.2, 4.3 can be expressed as below.

S_{Mander} = Sp0 U Sn0 U Snl U Spu U Snr ;
Sp0 = {x_i, y_i, Δx, fp0(x), +, ∞, Sp0, Sn0};
Sn0 = {x_i, y_i, Δx, fn0(x), –, ε_{pl} , Snr, Sp0};
Snl = {x_i, y_i, Δx, fnl(x), –, –∞, Sn1, Spu};
Spu = {x_i, y_i, Δx, fpu(x), +, ε_{pl} , Sp0, Snr};
Snr = {x_i, y_i, Δx, fnr(x), –, ε_{ul} , Snl, Spu}.

In which,
Δx = x - xi ;

$fp0(x) = 0$;
$fn0(x) = 0$;
$fn1(x) = Es \cdot r \cdot x / [\, r - 1 - (x/\varepsilon_{cc})^r \,]$;
$fpu(x) = \sigma_{ul} \cdot [(x - \varepsilon_{pl})/(\varepsilon_{ul} - \varepsilon_{pl})]^2$;
$fnr(x) = 2.5\,\sigma_{ul} \cdot [(x - \varepsilon_{pl})/(\varepsilon_{ul} - \varepsilon_{pl})]^2$ or $E_{ul}(x - \varepsilon_{ul}) + \sigma_{ul}$;

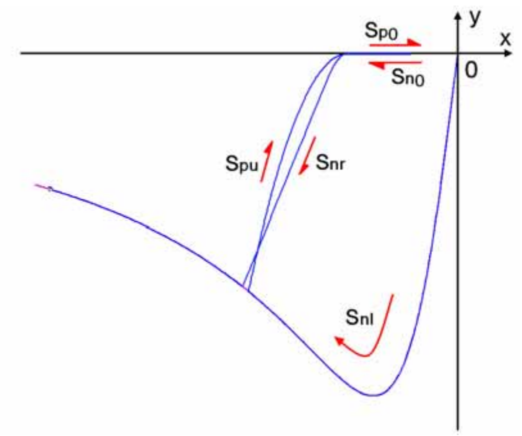

Fig. 7 The Mander Model States

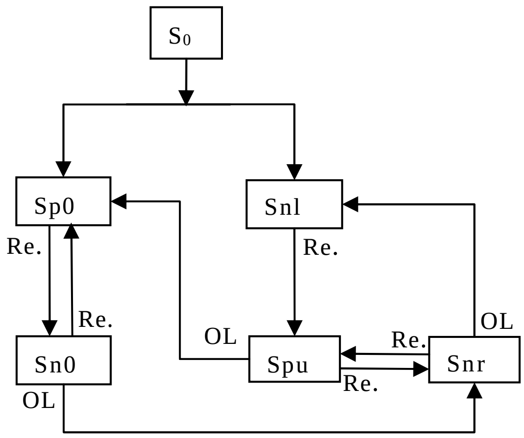

Fig. 8 The Mander Model State Diagram

4.5. Application in UC-win/FRAME(3D)

The above two examples are rather simple application case. For more complicated models such as the Takeda Model [4], the FSM application shows a greater advantage over traditional description and programming methods.

UC-win/FRAME (3D) is a nonlinear dynamic analysis software program for spatial frames [7]. The structure nonlinear properties are taken into consideration by defining material constitutive law, sectional properties, and spring element nonlinear relationships. The FSM has been successfully applied to the reversed relationship definition and nonlinear computation. FSM use not only brought efficiency to programming but also avoided errors that arise from a lack of understanding of the various constitutive relationships. The hysteretic models are applied to a structure's dynamic nonlinear analysis, especially for bridge designs with a strong earthquake input.

Fig. 9 shows the two constitutive relationships of the Takeda Model [4] for RC plastic hinges and the Hardening Model for rubber bearings using in bridge dynamic analysis.

5. CONCLUSIONS

The application of FSM to hysteretic rule simulation was discussed through an explanation of procedures and two example case studies of the Bilinear Model and the Mander Model. Besides giving us a different view on the hysteretic rules extensively used in the dynamic nonlinear computation, FSM application can also provide a deeper understanding of path and route changes in a mathematical sense. At the same time, the application of FSM can contribute to program simulation efficiency. The created constitutive laws can be reused and conveniently modified.

In summary, we find the following advantages when using FSM to the reversed relationship definition.

1. Avoid program errors;

2. Easy to reuse code;
3. Easy to create a new model;
4. Help to deeply understand on the rules;
5. Help to generalize different scale problems by an abstract FSM graph.

For nonlinear analysis, there are a variety of hysteresis models that have been or will be created to model the mechanical properties of components or materials under reversed earthquake action. By representing the hysteresis model through the states and the linkages of FSM, we can quickly realize program simulations. By now we only applied it to uni-axial or uni-directional hysteresis relationship, but for the multi-axial or multi-direction coupled hysteresis relationship, the further research is needed.

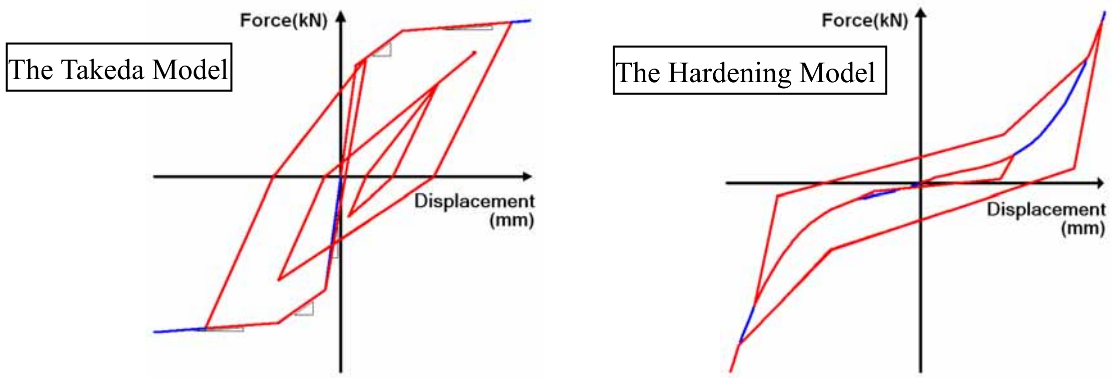

Fig. 9 Constitutive Models Provided by UC-win/FRAME(3D)

REFERENCES

[1] BERTERO, V. V. (1997). Performance based seismic engineering: A critical review of proposed guidelines, *Workshop on Seismic Design Methodologies for the Next Generation of Codes*, Slovenia,
[2] Mander, J.B., Priestley, M.J.N. and Park, R. (1988). Theoretical Stress-Strain Model for Confined Concrete, ASCE *Journal of Structural Engineering*, **Vol. 114**, No.8, pp.1804-26
[3] Menegotto, M. and Pinto, P.E. (1973). Method of Analysis for Cyclically Loaded RC Plane Frames Including Changes in Geometry and Non-Elastic Behavior of Elements under Combined Normal Force and Bending, *Proc. IABSE Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well defined Repeated Loads*, pp15-22
[4] Takeda, T., Sozen, M. A. and Nielsen, N. N. (1970). Reinforced Concrete Response to Simulated Earthquakes, *Journal of Structural Division, Proc. of the ASCE,* **Vol. 96**, No.ST12, pp.2557-2573
[5] Wagner, F. (2006). Modeling Software with Finite State Machines: A Practical Approach, *Auerbach Publications,* ISBN 0-8493-8086-3
[6] Gill, A. (1962). Introduction to the Theory of Finite-State Machines, McGraw-Hill
[7] http://www.forum8.co.jp/english/uc-win/frame3d-e.htm
[8] Clough, R. W. and Johnston, S.B. (1966). Effect of Stiffness Degradation on Earthquake Ductility Requirements, *Proc. 2nd Japan National Conference on Earthquake Engineering*, pp. 227-232
[9] Otani, S. (1981). Hysteresis Models of Reinforced Concrete for Earthquake Response Analysis, *Journal of Faculty of Engineering, University of Tokyo,* **Vol. XXXVI,** No. 2, pp. 407 – 441
[10] Japan Road Association (2002). Specification for Highway Bridges, Part V: Seismic Design, Maruzen
[11] Sakai, J. and Kawashima, K (2006.1) Unloading and Reloading Stress-Strain Model for Confined Concrete, *Journal of Structural Engineering*, ASCE, **Vol. 132**, No. 1, pp. 112-122
[12] Jin, D. and Ito, Y. (2008.10). The Application of Finite State Machine Theory to the Simulation of Reversed Non-linear Hysteretic Relationships, *The 12th International Conference on Computing in Civil and Building Engineering(ICCCBE-XII) and 2008 International Conference on Information Technology in Construction*, Tsinghua University, Beijing