THE APPLICATION OF INCLINOMETER IN NATURAL CHARACTERISTICS TESTING OF BEAME BRIDGES

Chen Jianli Qu Shuying Hou Xingmin

Chen Jianli, Graduate student, Dept. of Structural Engineering, Yaitai University, Yaitai. China
Email: cjl_155@163.com

ABSTRACT :

Natural characteristics (natural frequency, damping ratio, vibration mode) is the important content in bridge detection. Expatiates on the theories of mode testing methods proposing recently years in this paper, including modal synthesis method, complex modal method, empirical mode decomposition (EMD) and random decrement technique (RDT) method, mode strain energy method, curvature model method, strain modal method etc. Analysis the testing method and analytic method of simply-supported beam bridge natural characteristics. Academic and examinational preparations were offered for the application of inclinometer in natural characteristics testing of beam bridges.

KEYWORDS: beam bridge; modal analysis; dynamic characteristics; inclinometer

1 INTRODUCTION

With the development of vibrating and experimental modal analysis technology, a new approach for bridge safety is monitoring, using testing data to correct dynamic model has developed fully. Vibration modal analysis technology is based on vibration testing, data acquisition, signal analyzing and processing, the structure's physical parameters, such as mass, stiffness, damping, and so on, can be restroestimated from the dynamic characteristics of structure. If there are damages in the structure, physical parameters will change, and modal parameter will change too. Thus, damage can be detected from structure modal parameters. QY inclinometer can be used for static and dynamic real-time dip angle measurement of bridges, modal parameters can be obtained form numerical calculation, thereby providing a new approach for natural characteristic measurement of bridges.

2 MODE TESTING METHOD

The classical definition (Fu Zhifang, 1995) of modal analysis is: physical coordinate are transformed into modal coordinate in time-invariant system of linear differential equations which are made to decouple, becoming one group of modal coordinate and modal parameters describing independent equations, so as to obtain modal parameters. The ultimate objective for modal analysis is to identify structure's modal parameters, providing basis for vibration characteristic analysis, vibration fault diagnosis and prediction, as well as dynamic characteristics of structure's optimization and design of structure.

The condition of some modal testing methods' theory and engineering application was explained as follows.

2.1 Mode synthesis method

Mode synthesis method is a very effective method of calculating dynamic parameters of the large complex structures. Whose basic thought (Wang Wenliang, etc, 1985) is: the complex structure was divided into some substructures by its characteristics, obtain the branch modal of each substructure using finite element method or experimental mode analysis method, and then physical coordinate of each substructure was transformed into modal coordinate; then all substructures' modal coordinate are simply integrated the whole structure's modal coordinate; finally do the second coordinate transformation using interface join condition of each substructure, obtain a set of independent generalized coordinate which is composed of each substructure's modal coordinate and can describe movement of the whole system. The essence of which is that one high order eigenvalue problem was transformed into some lower-order eigenvalue problem.

For general dynamic analysis problem, the dynamic equation after the modal synthesized of every substructure

$$[M]^*\{\ddot{q}\}+[C]^*\{\dot{q}\}+[K]^*\{q\}=[R]^* \tag{2.1}$$

In the formula, $[M]^*$, $[K]^*$, $[C]^*$ and $[R]^*$ are modal mass, modal stiffness, modal damping and exciting force of the structure respectively after modal synthesized. The order of the equation equals to the total number of the selected retention mode subtracting the joint degree of freedom. The above analysis can be generalized in the structure with more than two substructures.

Mode synthesis theory has widely used in aerospace and various large engineering fields. The fixed-interface method (Ju Jianmin, etc, 1999) and the free-interface method (Wang Ailun, etc, 2002) are currently enjoying wide application in modal synthesis methods. They have showed predominant efficiency and precision in solving dynamic analysis of the complex structures, but they also have many faults. Another more efficient Computing Method, namely the mixed-interface mode synthesis method (Gao Hongfen, etc, 2004) emerged in order to improve these deficiencies. Compared with other methods, this method is an easy way with low computational complexity and high precision.

2.2 Complex modal method

When damping is not proportional damping, and doesn't meet damping orthogonality condition, normal modal method can't make damping matrix diagonalized, but complex modal method solves this problem very well. It provides a decoupling way of the motion equations in state space for the generic viscous damping linear multiple degrees of freedom structures which doesn't satisfy damping orthogonality condition.

The motion equation of non-proportional linear damped multiple degrees of freedom structure is:

$$[\tilde{m}]\{\ddot{x}\}+[\tilde{c}]\{\dot{x}\}+[\tilde{k}]\{x\}=\{\tilde{f}(t)\} \tag{2.2}$$

In the above formula, damping matrix $[\tilde{c}]$ doesn't meet the condition for diagonalization, deriving the auxiliary equation $[\tilde{m}]\{\dot{x}\}-[\tilde{m}]\{\dot{x}\}=0$, the two formulas are combined, getting the first order differential equation:

$$[M]\{\dot{y}\}+[K]\{y\}=\{F(t)\}=\begin{Bmatrix}\{0\}\\ \{f(t)\}\end{Bmatrix} \tag{2.3}$$

In the formula, $[M]=\begin{bmatrix}[0] & [\tilde{m}]\\ [\tilde{m}] & [\tilde{c}]\end{bmatrix}$, $[K]=\begin{bmatrix}-[\tilde{m}] & [0]\\ [0] & [\tilde{k}]\end{bmatrix}$.

Equation (2.2) and equation (2.3) describe the motion of the same system, so they have the public eigenvalue. Impulse response function matrix $[h(t)]$ and frequency function matrix $[H(\omega)]$ could be got by complex modal transformation.

$$[h(t)]=[u]diag[h_i(t)]diag[m_i^*]^{-1}[v]^T=\sum_{i=1}^{2n}\frac{\{u_i\}\{v_i\}^T}{m_i^*} \tag{2.4}$$

$$[H(\omega)]=[u]diag[H_i(\omega)]diag[m_i^*]^{-1}[v]^T=\sum_{i=1}^{2n}\frac{\{u_i\}\{v_i\}^T}{m_i^*(j\omega-p_i)} \tag{2.5}$$

In the formula, $[v]$ and $[u]$ are the left and right modal matrixes; $\{v_i\}$ and $\{u_i\}$ are the left and right eigevectors; p_i is the eigevector of the system; m_i^* is the modal mass after weighted orthogonal treatment.

Complex modal method is a powerful tool for calculation of structure dynamics, it can analysis the dynamic response of general structure whose damping matrix doesn't meet normal modal condition. It can solve the structure's natural vibration characteristics, and can do time-domain analysis for earthquake response of the structure too. The analysis of reliability constraints of wind-induced optimization design (Li Tun, etc, 2007) of the structure could use this method. This method also can be used for earthquake response analysis and optimization design (Li Chuangdi, etc, 2003) of the structure. Using complex method is proper when analyzing such structure response of considering foundation interaction.

2.3 Empirical mode decomposition (EMD) combined with the random decrement technique (RDT) method

Empirical mode decomposition (EMD) was based on time scale characteristic of the number for decomposition. It is suitable for processing non-stationary and nonlinear data, and it is widely used in the field of bridge engineering. For environment random-excitation, the structure's modal response after EMD decomposition actually composites of two parts: free vibration response and forced vibration response aroused by external load. Using random decrement technique gets the free vibration response of the corresponding mode, and then using the parametric recognition method of single-mode system obtains frequency and damping the structure.

EMD was established on the basis of three assumptions (Huang N E, etc, 1999). Then, the original data sequence $x(t)$ can be expressed as the sum of a group of IMF component and a residue

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{2.6}$$

RDT means that: the data processing method of free vibration response was obtained from one or more stationary random response sample functions of linear structure's vibration. Zhou Chuanrong (1989) has elaborated on its principle in detail

The free vibration response of a linear single degree of freedom structure whose initial displacement is A and initial velocity is 0 is as follows

$$X(t) = E[x(t)] = AD(t) + E[\dot{x}(t_i)]V(t) + \int_0^t h(t-\tau)E[f(\tau)]d\tau = AD(t) \tag{2.7}$$

In the formula, $D(t)$ is a free vibration response of which the initial displacement is 1 and the initial velocity is 0; $V(t)$ is a free vibration response of which the initial displacement is 0 and the initial velocity is 1; $f(t)$ is the stationary random excitation whose mean value is 0; $x(t)$ and $\dot{x}(t)$ are initial displacement and initial velocity respectively of the structure; $E[\dot{x}(t)]$ and $E[f(\tau)]$ are mathematical expectation. RDT is also suitable for linear structure with multiple degrees of freedom.

The free vibration of a certain measuring point was extracted by RDT, then solve it and obtain i order natural frequency and i order modal ratio

$$\omega_{ni} = \sqrt{a_i^2 + b_i^2}\ , \quad \xi_i = -\frac{a_i}{a_i^2 + b_i^2} \tag{2.8}$$

EMD can decompose the non-stationary signal into the quasi-stationary IMF component, and then RDT can be used for extracting the free vibration response of each component, finally modal parameters can be obtained by parameter recognition theory and optimal estimation theory comprehensively. Therefore, EMD combined with RDT method provides a new way for modal parameters recognition of the large bridges.

2.4 Mode strain energy method

Mode strain energy method is on the basis of structure damage level evaluation which is deduced by first order perturbation theory. Unit modal energy is used as damage location of the structure.
Using the change of element modal energy as the location indicator factor can diagnose damage position of structure.

$$MSECR_{ij} = \frac{\left| MSE_{ij}^{d} - MSE_{ij} \right|}{MSE_{ij}} \tag{2.9}$$

In the formula, $MSECR_{ij}$ is the modal energy change ratio of the jth element about the i order mode. MSE_{ij} and MSE_{ij}^{d} are the element modal energy before and after damage of the structure.

Multiple modal vibrations can be used simultaneously for diagnosing the structure's damage position in order to decrease the effect of random noise of the test modal vibration

$$MSECR_{j} = \frac{1}{m}\sum_{i=1}^{m}\frac{MSECR_{ij}}{MSECR_{i\max}} \tag{2.10}$$

Tang Tianguo (2005) has used Modal Strain energy method for damage testing recognition of fractures and cracks, the results show this method has highly precision. Wang Genhui (2006) has researched on damage location of the bridges, showing that this method can recognize the damage position of structure accurately form the analysis and discussion of the calculation results and measurement results in different damage condition. This method is simple and effective, and convenient for practical application. Because the noise level of testing modal vibration is higher, this method still need to further study for the condition of the lower partial stiffness loss.

2.5 Curvature model method

Curvature mode is the typical dynamic characteristics of the bending structure, which could be used to damage diagnosis for beam and bridge structures. For a beam structure, using the difference method in finite element analysis can obtain

$$\upsilon_{i} = \frac{y_{i+1} - 2y_{i} + y_{i-1}}{h^{2}} \tag{2.11}$$

In the formula, υ_i and y_i are a certain order curvature modal vibration and displacement modal vibration of i point respectively. h is the space between the measuring points.

The bending distortion is corresponding to strain, and ε could denote as follows

$$\varepsilon = -\frac{h'}{\upsilon} = -h' \cdot \frac{y_{i+1} - 2y_{i} + y_{i-1}}{h^{2}} \tag{2.12}$$

In the formula, h' is the distance between the point on the beam and the natural layer. The formula show that curvature mode of beam relates to strain mode directly. For the uniform section beam, the measured strain can reflect the change of curvature directly; for the variable section beam, the measured strain also can denote the curvature change after simple mathematical processing.

The damage position can be determined by detecting a certain order curvature mode change in normal state

$$MSC(i) = \upsilon_{ci} - \upsilon_{di} \tag{2.13}$$

In the formula, υ_{ci} and υ_{di} are the curvature modes before and after damage respectively.

For multiple order modes, the mean value of each order curvature mode difference can be used to indicate the occurrence of injury, showing as follows

$$MSC(i) = \frac{1}{n}\sum_{r=1}^{n}\Delta\upsilon_{r}' \tag{2.14}$$

In the formula, $\Delta\upsilon'_r$ is the curvature mode difference before and after damage of the rth order mode of the i point; $MSC(i)$ is the curvature change value of the i point, the place having larger variation may be the damage position.

Using curvature modal curve can indicate and locate the damage of the structure effectively, and can indicate the structure's damage degree simultaneously. This method has some advantages with high rate of identification and doesn't need modal signal before damage of the structure; it can attempt to be used in practical engineering (Li Debao, etc, 2002). For bridges, theory calculation shows that just using curvature modal curve to indicate damage is very difficult, but the difference between curvature modes can indicate the existing and position of the damage accurately. And for damage with different degree, the difference curvature between modes takes on different peak value. The more the damage is large, the more the peak value is significant, so it shows that curvature mode can indicate the damage degree of structure.

2.6 Strain modal method

Strain is the first order derivation of displacement, strain distribution state is called strain mode corresponding displacement mode. The Strain model reflects the natural characteristics of structure. From dynamic mechanics theory, the displacement response of continuum beam structure lists as

$$\{u\} = [\phi_r][Y_r][\phi_r]^T\{F\} \tag{2.15}$$

In the formula, ϕ_r is displacement modal vibration; F is exciting force; $Y_r = (-\omega^2 m_r + j\omega c_r + k_r)^{-1}$

Based on the bending theory of beam, strain is the first order derivation of displacement, so strain response ε_x is

$$\varepsilon_x = \psi_r^\varepsilon Y_r[\Phi_r]^T F \tag{2.16}$$

In the formula, $\psi_r^\varepsilon = \dfrac{\partial\phi_r}{\partial x}$ is called strain mode.

The displacement vector of three directions in 3-d geometric space is $[u \quad v \quad \omega]^T$, according to elastic mechanics principle, when just considering positive strain, its strain is

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{Bmatrix} = \begin{Bmatrix} \psi_{rx}^\varepsilon \\ \psi_{ry}^\varepsilon \\ \psi_{rz}^\varepsilon \end{Bmatrix} [Y_r][\phi_u \cdot \phi_v \cdot \phi_\omega]^T \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} \tag{2.17}$$

Writing in matrix form is

$$\{\varepsilon\} = [\psi_r^\varepsilon][Y_r]\phi^T\{F\} \tag{2.18}$$

Strain mode can be obtained by experimental measurement directly, also can obtained by bending displacement mode indirectly, namely on the foundation of displacement mode measurement, using differential calculation obtains strain mode. The bridge structure whether or not existing damage and the position of damage can be judged by contrasting the change of strain mode before and after damage of structure.

Strain modal analysis method is very effective for damage diagnosis and monitoring of the structure, especially for the structure having lower natural frequency. This method still needs deeply and comprehensively research in theory and experiment so as to apply for the practical engineering. Gu Peiying (2006) has studied the principle of strain mode and strain modal parameter recognition in theory, and discussed the recognition method of strain modal parameters. Du Siyi (2003) researched damage recognition of rigid bridge form numerical simulating calculation using strain modal method. This method can be used for bridge damage online monitoring and discover problem immediately, which has important significance for avoiding the occurrence of bridge accident and ensuring the safety operation of bridge.

2.7 Comparison of the methods

These methods all use the analysis of structure dynamic parameter's variation (including modal parameter and structure parameter) to detect the safety of structure, and discover the existing damage of structure, then take safety actions to avoid the occurrence of accidents.

Mode synthesis method is applicable to calculating the dynamic parameter of large-scale complicated structure; complex modal method is used in the general viscous damping linear structure with multiple degrees of freedom whose damping matrix doesn't satisfy damping orthogonality, providing a new way for motion equations decoupling; EMD can decompose non-stationary signal into the quasi-stationary IMF components, and combine with RDT to extract the free vibration response's every component, then modal parameters can be obtained by using parameter recognition theory and optimal estimation theory comprehensively; Mode strain method uses the rate of change of element modal strain energy as damage location factor of structure, this method is simple and effective, and it is convenient for practical application; The curvature mode difference can indicate the existing and position of damage accurately; Strain mode can be obtained by experimental measurement directly, and is very effective for damage diagnosis and monitoring of the structure.

3 NATURAL CHARACTERISTIC OF BEAM BRIDGES

Dynamic characteristic of structure is the basic feature of structure vibration system, and is the necessary parameter for vibration analysis. In bridge vibration analysis, the natural frequency and the damping (dynamic characteristics) must be determined first of all. Damping characteristic only can be determined by test.

3.1 The natural characteristic test method of beam bridges

3.1.1 Natural frequency test

Exciting method can be used for creating free vibration of bridges, record damped vibration waveform of structure by actual measurement, as shown in Figure 1.

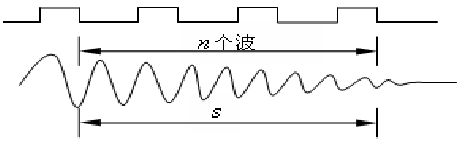

Figure 1 Solving natural frequency with attenuation curve

According to vibration waveform on the record, natural frequency f_0 can be calculated directly using timescale symbol

$$f_0 = \frac{Ln}{t_1 S} \tag{2.1}$$

In the formula, L is the space between the two timescale (mm); n is wave number; S is the distance of n wavelengths (mm); t_1 is time interval of timescale.

Structure will generate continuous and periodicity forced vibration when using vibration generator. Structure will create resonance phenomenon, and its amplitude reaches maximum value when the vibration frequency of vibration generator is the same as the natural frequency of structure.

3.1.2 Damping ratio test

Damping characteristic of bridge is generally denoted by log decrement. In practical engineering, m waveforms are usually measured to obtain damping decrement:

$$\delta = \frac{1}{m} \ln \frac{A_i}{A_{i+m}} \tag{2.2}$$

In the formula, A_i and A_{i+m} are amplitude values of the wave, can be measured directly on attenuation curve.

3.2 Analytic method of beam bridge natural characteristic

In practical engineering, bridge is usually abstracted to computation module of uniform mass. Takes constant section beam bridge with no load or uniform static load for example, it can be regarded as an elastic system with the continuous uniform mass, as shown in Figure 2.

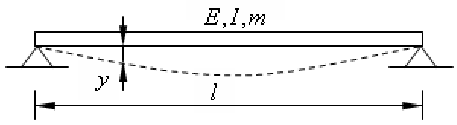

Figure 2 Simply supported beam bridge model

Its basic equation of natural vibration is

$$EIy'''' + m\ddot{y} = 0 \tag{2.3}$$

Supposing, $y(x,t) = Y(x)F(t)$, then separating variables obtains the equation as follows

$$\frac{EIy''''}{mY} = -\frac{\ddot{F}}{F} = \text{constant} = \omega^2 \tag{2.4}$$

Obtain two independent linear homogeneous simultaneous differential equations.

$$\ddot{F} + \omega^2 F = 0 \tag{2.5}$$

$$EIy'''' - m\omega^2 Y = 0 \tag{2.6}$$

Solving formula (2.5) as follows

$$F(t) = A\sin\,\omega t + A\cos\omega t = a\sin(\,\omega t + v)$$

Solving formula (2.6) as follows

$$Y(x) = A\cos kx + B\sin kx + C\cos kx + D\sin kx$$

In the formula, $k = \sqrt[4]{\frac{\omega^2 m}{EI}}$, the four integral constant A, B, C and D are determined by edge conditions.

Based on edge conditions of simply supported beam, using $Y(0) = Y''(0) = 0$ and $Y(l) = Y''(l) = 0$, obtain $A = B = C = 0$ and $kl = n\pi$, $(n = 1,2,3\cdots)$. Therefore, the mode shape which is satisfied with edge condition could write as follows

$$Y_n(x) = D_n \sin\frac{n\pi x}{l}, \quad (n = 1,2,3\cdots) \tag{2.7}$$

The natural frequency corresponding to $Y_n(x)$ is

$$\omega_n = (\frac{n\pi}{l})^2 \sqrt{\frac{EI}{m}} \quad (n = 1,2,3\cdots) \tag{2.8}$$

4 INCLINOMETER

Consisting of capacitive transducer technique and passive servo technology, inclinometer is a dip angle measurement apparatus with high sensitivity and anti-jamming capability, its output voltage is proportional to the rotating angle of the testing section of bridge. QY inclinometer (Hou Xingmin, etc, 2002) is really a kind of ultra-low frequency acceleration sensor of which the amplitude in low frequency range and phase frequency characteristic satisfying the undistorted condition of the low-pass measurement signal, and its testing results don't need the correction of software. Laying inclinometer on the testing bridge and measuring the dip angle of each testing point of bridge can calculate dynamic deflection, static deflection and dip angle value of any key point, etc. Based on this testing ability, combining with the modal analysis methods mentioned above, inclinometer can be used for the natural characteristics measurement of bridge, providing a new way for the natural characteristics measurement of structure. In practical engineering, QY inclinometer has been used for the deflection measurement of bridges (Hou Xingmin, etc, 2004) with high sensitivity, and obtained good testing results.

5 CONCLUSION

The vibration of the currently bridges need scientific analysis in order to solving the various performance changes of the bridges, which is a vexing issue and waiting to be solved in bridge engineering field at present time. In view of the current situation, this paper analyzes the concept, development and safety testing condition applying to bridges of the modal analysis technology; the theories of some modal testing methods and their application value in practical engineering are especially emphasized; the testing method and analytic method of natural characteristics of the simply-supported beam bridge are analyzed. Academic and examinational preparations were offered for the application of inclinometer in natural characteristics testing of beam bridges.

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