

## Killer Pulses of Hazardous Earthquake Motions and Transition of Natural Periods of Buildings

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### ABSTRACT :

A theoretical approach of elasto-plastic response with a constant damping factor is proposed because almost all the damping mechanisms of buildings are not viscous but frictional. The skeleton curve of proposed single degree of freedom system is derived from the condition that the dissipative energy during one cycle of the elasto-plastic system is exactly same as that of the visco-elastic system. When the natural period of the elasto-plastic system is selected as the equivalent period at the maximum response, the shapes and amplitudes of the acceleration, velocity, displacement and total dissipative energy response spectra are approximately same as that of visco-elastic response spectra. Although the proposed elasto-plastic response system is non-linear, the spectrum with equivalent period shows linear response with fluctuations in amplitudes of input motions. The transition of equivalent period of the elasto-plastic system indicates significant relation between response amplitudes and damping factors of the system. The transition rate of equivalent period is proportional to the rate of response amplitude and the power of damping factor. This mechanism gives appropriate interpretation on the observed transition of natural periods of buildings. The proposed theory suggests that the natural period of wooden buildings increases about four to five times during severely damaged earthquake motions. This result gives good elucidation that the predominant periods of one second to two second during the 1995 Hyogoken Nanbu earthquake, known as the killer pulses, damaged rather shorter period buildings.

**KEYWORDS:** Natural Period of Buildings, Response Spectrum, Elasto-Plastic Response

### 1. INTRODUCTION

Although the principal mechanism of dissipation in existing buildings is frictional phenomenon, the viscous model is used for the damping factor of the single degree of freedom system calculating the response spectrum.

The mechanisms of energy dissipation during oscillation in buildings are varied in nature but mainly: (a) damping intrinsic to the structural material, (b) damping due to friction in the structural joints and between structural and non-structural elements, (c) energy dissipated in the foundation soil, and (d) energy radiation through the foundation to ground. Lagomarsino has noted that the first two factors constitute the structural damping of the building and (a) structural damping in steel buildings can be attributed to the friction actions in the joints; (b) in reinforced concrete buildings structural damping is due both to the slips between structural and non-structural elements, and to the dissipation in the material (Sergio Lagomarsino, 1993).

If we accept the damping factor of buildings is primarily caused by friction, we have to recognize the natural periods of buildings are not constant but variable and change depending on the response amplitude (Ogawa, 2006).

## 2. ANALYTICAL METHOD

### 2.1. Restoring Force Characteristics with a Constant Damping Factor

Damping factor  $h$  of visco-elastic system is given as follows:

$$h = \frac{1}{4\pi} \cdot \frac{\Delta W}{W} \quad \dots(1)$$

where  $\Delta W$  is the dissipative energy during one cycle and  $W$  is the maximum potential energy stocked in the elastic element.  $\Delta W$  and  $W$  are given as the area of hysteresis loop and the area of triangle concerning to the maximum restoring force, respectively.

Now we adapt Equation (1) to the hysteresis damping of elasto-plastic system and define the function of the skeleton curve  $y = F(x)$  as follows:

$$y = F(x) \quad \text{for } x > 0, \quad y = -F(|x|) \quad \text{for } x < 0, \quad F(0) = 0 \quad \dots(2)$$

where  $x$  is displacement, and applying the Masing's law of the relationship among the skeleton curve and hysteresis loops, Equation (1) in integral form is given:

$$h = \frac{1}{4\pi} \cdot \frac{8 \left[ \int_0^x F(x) dx - \frac{1}{2} X \cdot F(X) \right]}{\frac{1}{2} X \cdot F(X)} \quad \dots(3)$$

where  $\pm X$  ( $X > 0$ ) is the displace amplitude of the hysteresis loop.

Here we denote the parameter  $h'$  as

$$h' = \frac{h}{1 - \pi h/2} \quad \dots(4)$$

and substituting Equation (4) into Equation (3), we have a resolvable Equation (5).

$$h' = \frac{1}{4\pi} \cdot \frac{8 \left[ \int_0^x F(x) dx - \frac{1}{2} X \cdot F(X) \right]}{\frac{1}{2} X \cdot F(X) - \left[ \int_0^x F(x) dx - \frac{1}{2} X \cdot F(X) \right]} \quad \dots(5)$$

Consequently, the function of skeleton curve is given as follows. Where  $C$  is the integral constant to provide the gradient of skeleton curve.

$$F(x) = C \cdot x^{\frac{1}{1+\pi h'}}, \quad \log[F(x)/C] = \frac{1}{1+\pi h'} \log x \quad \dots(6)$$

As shown in Equation (6), the restoring force function gives constant damping factor when the relation between the skeleton curve and displacement is linear in logarithmic scale. This suggests if the relation of the transition of natural periods according to the amplitude of oscillation of a building is linear in logarithmic scale, the restoring force function of the building is elasto-plastic and damping factor is constant.

## 2.2. Application of Mechanical Model for Restoring Force Characteristics

The mechanical model proposed by Iwan is applied for the restoring force characteristics of the single degree of freedom system (Iwan, 1966). This model is composed of springs and Coulomb sliders as shown in Figure 1. When the function of skeleton curve is a single-valued function and the gradient of tangent decreases monotonically, Iwan's model satisfies Masing's law. Because springs and sliders compose Iwan's model, elastic energy stored in the model is calculated from expansions and contractions of springs, and dissipative energy is calculated from movements of sliders.

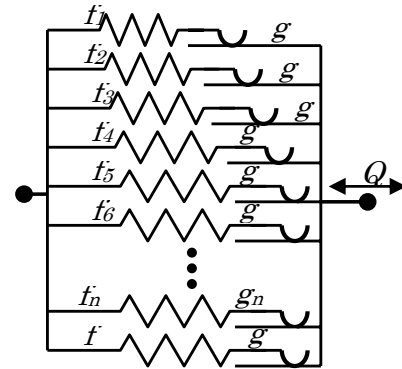


Figure 1 Iwan's Model

## 2.3. Calculation for Response of Single Degree of Freedom System

The elasto-plastic response to the incident acceleration motion  $\ddot{z}$  is given by solving the Equation (7) in time domain:

$$m\ddot{x} + Q(x) = -m\ddot{z} \quad \dots(7)$$

where  $m$  is mass and is normalized to a unit mass (1kg), and  $Q(x)$  is restoring force mentioned above. We regard the equivalent period as the natural period of the elasto-plastic response system. The equivalent period  $T_{EQ}$  is expressed as follows:

$$T_{EQ} = 2\pi \sqrt{\frac{x_{max}}{a_{max}}} = 2\pi(1/C)^{\frac{1+\pi h'}{2}} a_{max}^{\frac{\pi h'}{2}} = 2\pi(1/C)^{\frac{1}{2}} x_{max}^{\frac{\pi h'}{2(1+\pi h')}} \quad \dots(8)$$

where  $x_{max}$  and  $a_{max}$  are the absolute values of the maximum relative displacement and the maximum absolute acceleration, respectively.

Because we could not know the equivalent period of system beforehand, we define the setting period  $T_{SET}$  to calculate Equation (7) on the bases of the secant rigidity at a unit displacement (1m). As shown in Figure 2, the setting period  $T_{SET}$  is fixed to variations in the damping factors.

We also could not know the required maximum restoring force beforehand. Iwan's model shows perfect plasticity when the external force exceeds the maximum frictional force. Therefore, the maximum restoring force of skeleton curve in Equation (6) has been set as 20N initially, and if the inertial force in Equation (7) exceeds the fixed maximum restoring force during calculations, then the maximum restoring force is reset and calculates again. The skeleton curve of Equation (6) is transformed to discrete quantity divided by 1000 at initial condition. The discrete quantity translates 1000 elements composed of one pair of spring and slider (so called Jenkin's element) of Iwan's model. To keep accuracy of calculations, the maximum restoring force is reduced when the inertial force is less than 1/3 of the maximum restoring force. The acceleration increment  $\Delta\ddot{x}$  is derived from Equation (7) integrated with linear acceleration method and converged into tolerance of 0.001% or  $1.0 \times 10^{-5} \text{m/sec}^2$  by repetitive calculation.

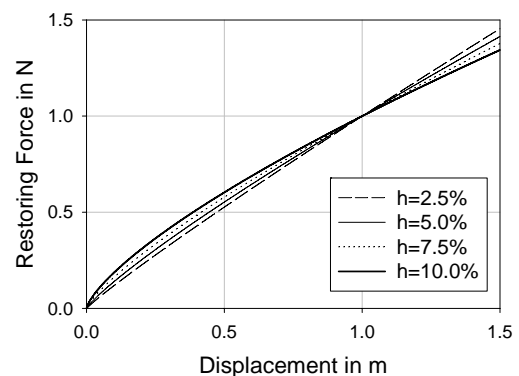


Figure 2 Typical Skelton Curve (C=1N)

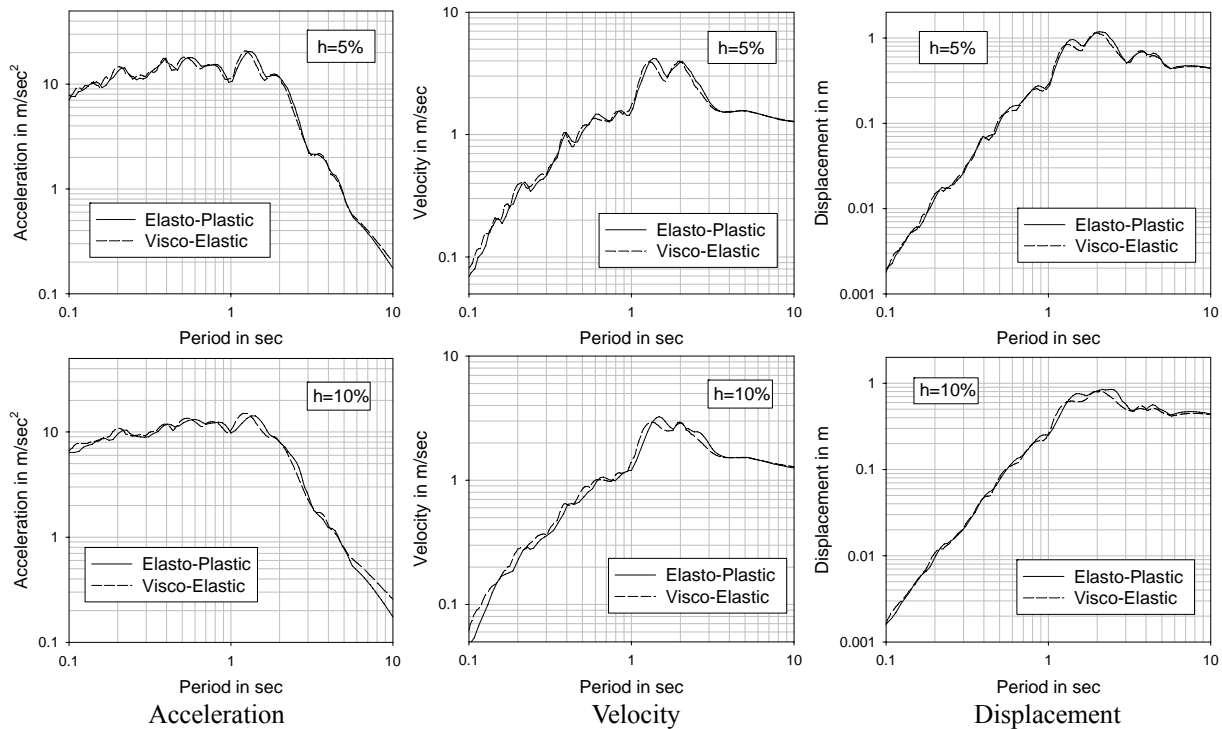


Figure 3 Comparisons of Elasto-Plastic and Visco-Elastic Spectra (1)  
 Input motion is NS component of 1995 Hyogoken Nanbu earthquake observed at Takatori station.

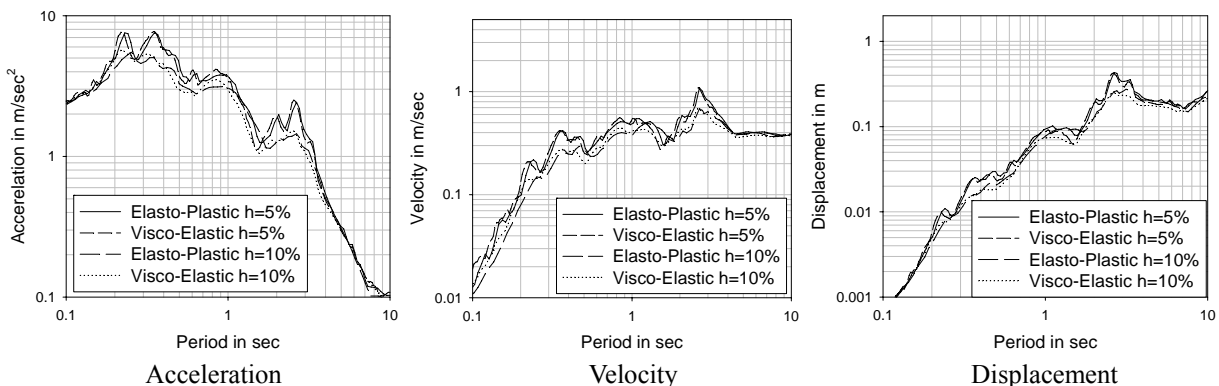


Figure 4 Comparisons of Elasto-Plastic and Visco-Elastic Spectra (2)  
 Input motion is NS component of 1968 Tokachioki earthquake observed at Hachinohe port.

### 3. Elasto-Plastic Response and Visco-Elastic Response

#### 3.1. Comparisons of Spectra

Figure 3 shows the spectra of the elasto-plastic response and visco-elastic response. Input motion is the NS component of 1995 Hyogoken Nanbu earthquake observed at Takatori station. Damping factors are 5% ( $h=5\%$ ,  $h'=5.426\%$ ) and 10% ( $h=10\%$ ,  $h'=11.864\%$ ), respectively. The natural period  $T$  is the equivalent period  $T_{EQ}$  for the elasto-plastic response and the un-damped natural period for the visco-elastic response, respectively.

As for the acceleration, velocity and displacement response spectra of 5 % damping factor, the elasto-plastic response spectra are almost same as that of visco-elastic response. In the case of 10 % damping factor, the elasto-plastic response is slightly longer than the visco-elastic response, both spectra are agree well each other.

Figure 4 shows the spectra of Hachinohe NS motion of 1968 Tokachioki earthquake. Although this

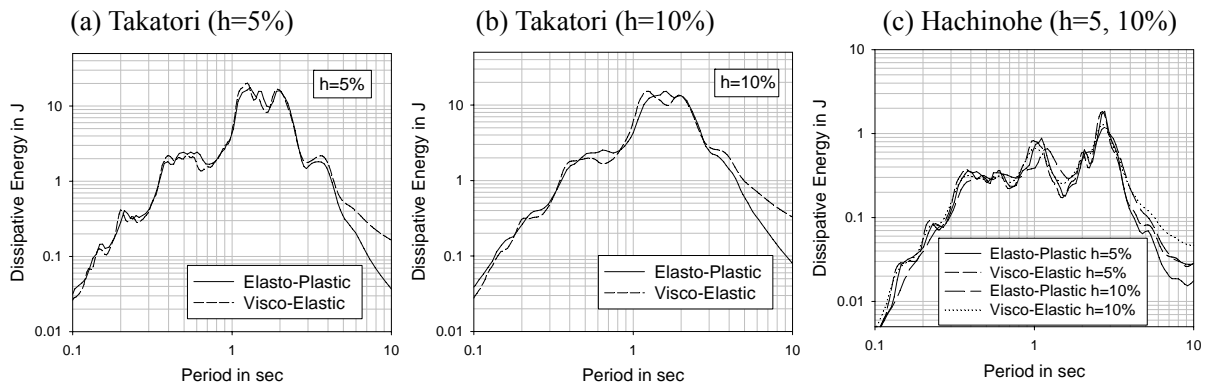


Figure 5 Comparisons of Elasto-Plastic and Visco-Elastic Energy Dissipation Spectra  
 Input motion are Takatori (same as Figure 3) and Hachinohe (same as figure 4)

motion includes many oscillations during main shock, the elasto-plastic response spectra agree well with that of visco-elastic response. Because these agreements are observed in the other earthquake motions, the elasto-plastic response spectrum is almost equivalent to that of visco-elastic spectrum.

Dissipative energy spectra are shown in Figure 5. Despite the difference of mechanisms of energy dissipation, both spectra agree up to three second.

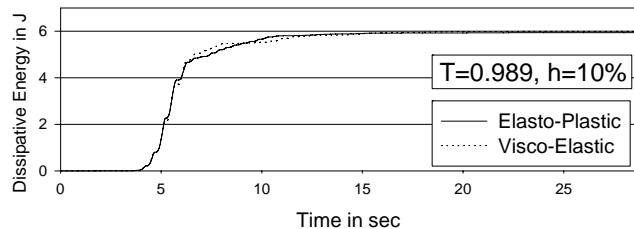
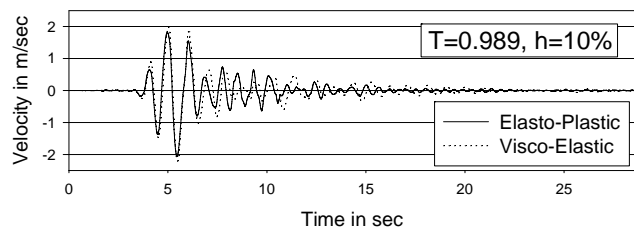
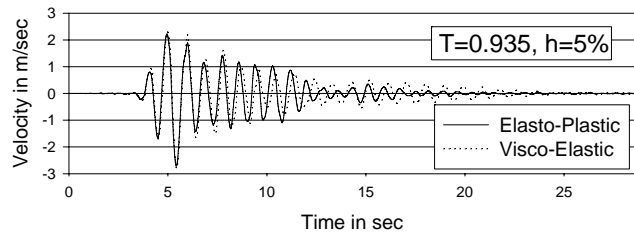


Figure 6 Time Histories of Velocity Response and Energy Dissipation (Input motion is Fukiai)

### 3.2. Comparisons of Time Histories

Time histories of the velocity responses and the energy dissipation are shown in Figure 6. Input motion is N330E component of 1995 Hyogoken Nanbu earthquake observed at Fukiai. Relations between restoring force and displacement during the main oscillation of the Fukiai responses (see Figure 6) are shown in Figure 7. The unsymmetrical shapes caused by Masing's law are seen at the reverse point of the loci of elasto-plastic response.

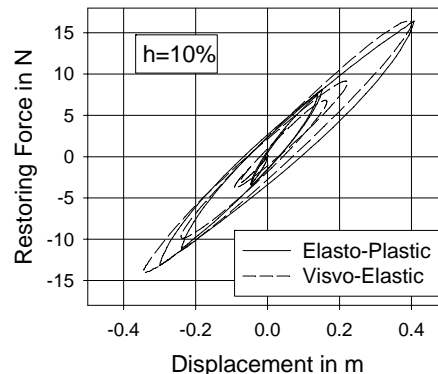
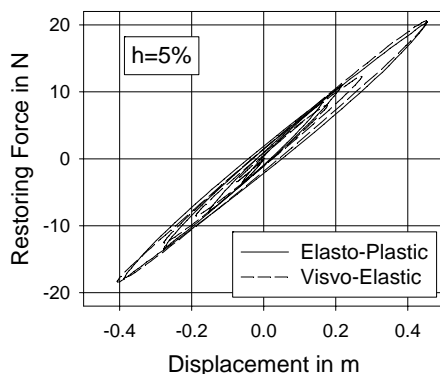


Figure 7 Relations between Restoring Force and Displacement  
 Input motion is N330E component of 1995 Hyogoken Nanbu earthquake observed at Fukiai

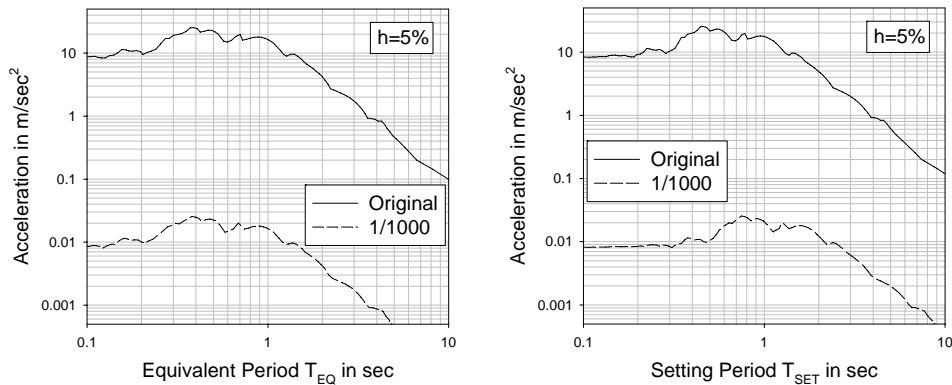


Figure 8 Comparisons of Spectra Represented with Equivalent Period  $T_{EQ}$  and Setting Period  $T_{SET}$  Kobe Marine Meteorological Observatory, 1995 Hyougoken Nanbu Earthquake

#### 4. Transition of Natural Period and Hazardous Earthquake Motions

##### 4.1. Transition of Natural Period with Proposed Elast-Plastic response spectrum

It is expected from the agreements mentioned above, proposed elasto-plastic response spectrum responds linearly to increase and reduction in amplitude of input motion. Figure 8 shows the response spectra of the original input motion and the 1/1000 reduced motion. The original motion is NS component of 1995 Hyougoken Nanbu Earthquake observed at the Kobe Marine Meteorological Observatory. When period  $T$  is selected as the equivalent period  $T_{EQ}$ , the periodical characteristics of reduced spectrum agrees with the original one. On the other hand, the spectrum represented with the setting period  $T_{SET}$ , the periodical characteristics are different each other.

The relation between the setting period of original motion  $T_{SET}$  and that of the  $(1/K)$  reduced motion  $T'_{SET}$  is given as follows.

$$T'_{SET} = (1/K)^{\frac{\pi h'}{2(1+\pi h')}} \cdot T_{SET} \quad \dots(9)$$

The single degree of freedom system with the elasto-plastic restoring force characteristics proposed here, the equivalent period  $T_{EQ}$  varies depending on the amplitude of input motion (see Figure 9). Change Equation (8), we have the ratio of equivalent period  $T_{EQ}$  as follows:

$$\frac{T_{EQ,1}}{T_{EQ,2}} = \left( \frac{A_1}{A_2} \right)^\alpha \quad \dots(10)$$

where  $A_i$  are the response amplitudes and  $T_{EQ,i}$  are the equivalent periods corresponding to the ordinal number; multiplier  $\alpha$  is  $\pi h'/2$  and  $\pi h'/2(1+\pi h')$  for acceleration amplitude and displacement amplitude, respectively. When  $A_i$  is velocity amplitude,  $\alpha$  is given by approximation in the equivalent natural period as  $\pi h'/(2+\pi h')$ .

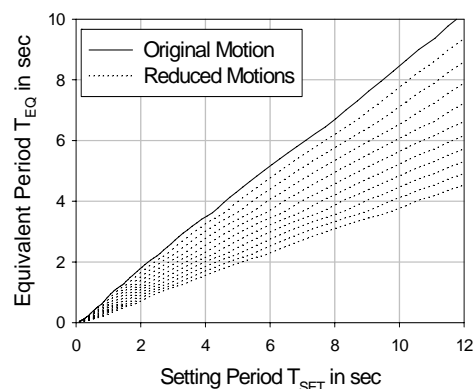


Figure 9 Relations between Equivalent Period  $T_{EQ}$  (Ordinate) and Setting Period  $T_{SET}$  (Abcissa)

Broken lines show Reduced Input Motions as 1/2, 1/4, 1/8, ..., 1/1024.

$h=8.462\%$  ( $h'=10\%$ )

NS Component of Kobe Marine Met. Obs.

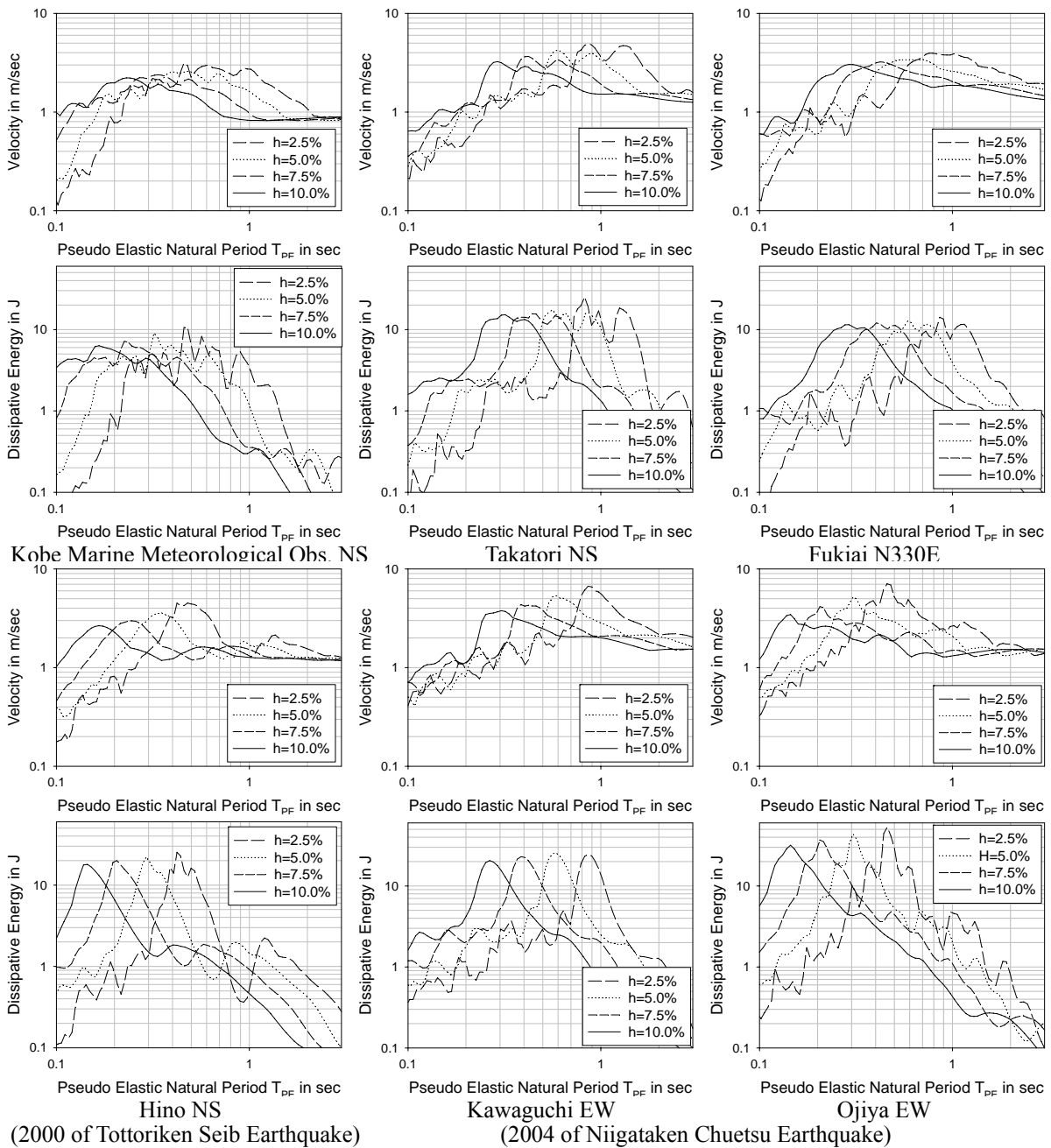


Figure 10 Elasto-Plastic Response Spectra Represented with the Pseudo Elastic Natural Period  $T_{PE}$

#### 4.2. Damage to Buildings during past earthquakes and hazardous earthquake motions

Generally, the natural periods of buildings are determined from the observation results of small amplitude oscillations. Conversely, they significantly become longer during strong shakings if the structural members do not reach yielding conditions (Ogawa, 2006). If this transition of natural periods of buildings could be explained by the elasto-plastic system proposed here, the equivalent period  $T_{EQ}$  should be translated to that of small amplitude oscillations.

Now we define the pseudo elastic natural period  $T_{PE}$  as the natural period during micro-vibration conditions. The relation between the pseudo elastic natural period  $T_{PE}$  and the equivalent period  $T_{EQ}$  is:

$$T_{PE} = T_{SET} \cdot x_0^{\frac{\pi h'}{2(1+\pi h')}} = T_{SET} \left( \frac{T_{SET}}{2\pi} \cdot v_0 \right)^{\frac{\pi h'}{2+\pi h'}} = T_{SET} \left[ \left( \frac{T_{SET}}{2\pi} \right)^2 a_0 \right]^{\frac{\pi h'}{2}} \quad \dots(11)$$

where  $x_0$ ,  $v_0$  and  $a_0$  are amplitudes of relative displacement, relative velocity and absolute acceleration, respectively. This relation is also expressed in Equation (10):  $A_1$  and  $T_{EQ1}$  are transformed as  $a_0$  (or  $x_0$ ,  $v_0$ ) and  $T_{PE}$ , respectively. We could the response spectra represented with the pseudo elastic natural period  $T_{PE}$  from the usual visco-elastic response spectrum if we accept a small difference.

Figure 10 shows the elasto-plastic response spectra of some earthquake motions represented with the pseudo elastic natural period  $T_{PE}$ . The amplitude of micro-vibration is chosen as relative velocity and supposed to be the oscillation of micro-tremor and  $1 \times 10^{-4}$  m/sec.

#### *4.2.1. Damages of RC Buildings during 1995 Hyogoken Nanbu Earthquake*

The statistical analysis was reported that more than seven-story buildings (including seven-story) except pilloti buildings suffered more than moderate damage extensively (Hayashi et. al., 2000). The natural period of these buildings is estimated to 0.378 sec and more and the damping factor of RC buildings is usually set to about 5%. In the spectra of Fukiai, the dissipative energy and the maximum velocity increase at 0.4 second. In the spectra of Takatori, these values increase around 0.5 second and this natural period corresponding to nine-story buildings.

#### *4.2.2 Damages of Wooden Buildings during Earthquakes Occurred Recently*

During recent earthquakes, severe damages against wooden buildings were occurred: in the spectra of Takatori, Fukiai and Kawaguchi of Figure 10 are typical cases. Conversely, in Hino and Ojiya, very strong ground motions were observed but damages were not so severe. Dynamic characteristics of existing wooden buildings were measured that the natural period is 0.1 second to 0.32 second, and the distribution of damping factors, the center was 9% and ranged from 4.8% to 17% (Onozuka et. al., 1998) Because the natural period of old houses are rather long and these houses are vulnerable, the range from 0.2 second to 0.32 second is focalized to the natural period. In Figure 10, the peaks of dissipative energy spectra and velocity response spectra are agree well with the damaged and not-damaged sites.

## **5. CONCLUSIONS**

A theoretical approach of elasto-plastic response with a constant damping factor is proposed and the following conclusions are obtained:

- (1) The transition rate of equivalent period is proportional to the rate of response amplitude and the power of damping factor.
- (2) The proposed theory suggests that the natural period of wooden buildings increases about four to five times during severely damaged earthquake motions. Also the natural period of RC buildings increases about two times if the structural members do not reach yielding conditions.
- (3) These results seems to give good elucidation that the predominant periods of one second to two second during the 1995 Hyogoken Nanbu earthquake, known as the killer pulses, damaged rather shorter period buildings.

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