DAMAGE IDENTIFICATION OF SIMPLY-SUPPORTED BEAM USING MODAL DATA

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ABSTRACT :

This article presents a method of estimating the location and severity of damage in simply supported beams, based on experimental measurements of their fundamental vibration modes. In this context, damage is defined as a change in the stiffness of a section. Before this methodology can be applied, it is necessary to determine two modal frequencies of the system along with their respective shapes. The mass matrix of the system must also be known. Numerical simulations of a real bridge are performed to study the effectiveness of this approach and the influence of measurement errors and damage severity on its accuracy. A simply supported, wide-flanged steel beam is also tested under a sequence of progressively damaged conditions. The results obtained are satisfactory in both numerical and experimental tests; the method succeeds in precisely locating and quantifying stiffness changes in the system.

KEYWORDS: experimental testing, structural systems, evaluation and retrofit, structural response, modal analysis

1. INTRODUCTION

The structural stiffness can vary because of degradation over time: building modifications, damage, overloads, or seismic effects, for instance. This structural damage is not always visible because of either the minimal level of damage or the difficulty of accessing the structural elements such as those of bridge beams. It is therefore necessary to use a structural evaluation methodology that can detect, locate, and estimate the existing damage.

Bridges are indispensable to modern society; the deterioration or partial collapse of a single structure can have a drastic impact on the economic and social activity of a region or city (**Sheng et al.**). For this reason, the analysis of serviceable conditions and their vulnerabilities are commonly spoken of in the scientific literature (**Kwan**). Throughout its lifetime, a bridge suffers damage due to the strain caused by continuous traffic, the weight of vehicles, impacts, corrosion, and moderate earthquakes. It is necessary to remark that some structures built a few decades ago now appear vulnerable in the light of new standards (**Zhang, Frýba**).

The focus of this study is on those bridges relying on simply supported beams, which are commonly used for pedestrians, cars, and trains. National Bridge Inventory statistics show that one-third of all steel bridges are simply supported (**Nielson et al.**) This paper suggests a method to estimate the stiffness of simply supported beams, which can later be applied to the study of bridges.

There are specialized algorithms that allow one to locate and quantify damage in cantilevers, shear buildings, simply supported beams and others (**Garcés et al., Ricles, Yuan)**. If the topology of the stiffness matrix is known, one can greatly reduce the number of modes and measurement points required for accurate structural estimation. The method presented in this paper falls into this category.
The algorithm requires prior knowledge of only two vertical vibration modes and their respective frequencies. There is no need to measure rotational coordinates, which are generally difficult to obtain. This last fact means that the algorithm can be used to make prompt decisions about this type of structure, for example whether or not repair is feasible. Thus, the method is widely applicable yet requires little in the way of experimental

measurement—indeed, it drastically reduces the cost of testing.

A following paper will present the mathematics behind this identification methodology. Here, this method is applied to locate the stiffness changes in a numerically simulated bridge and a real steel beam. The proposed method successfully locates and quantifies damage in both structures with high accuracy.

2. ESTIMATION OF FLEXURE DAMAGE IDENTIFICATION IN SIMPLY-SUPPORTED BEAMS

Figure 1 shows the structural model used, with flexural behavior, vertical deformations, and globally consistent masses. Its dynamic parameters can be obtained from:

$$(\lambda_i^{-1} - F.M)\phi_i = 0 \qquad (2.1)$$

with: F= flexibility matrix, M= mass matrix , $\lambda_i=\omega_i^2$, ω_i = ith modal frequency ϕ_i= ith eigenvector

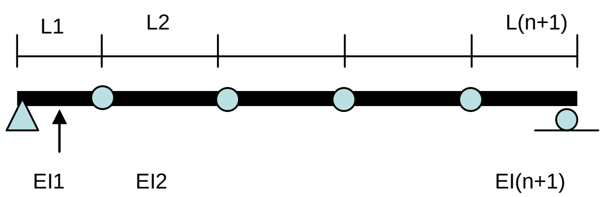

Figure 1 Simply-supported beam

Considering two eigenvalues corresponding to natural frequencies ω_a and ω_b and mode shapes ϕ_a and ϕ_b, Equation (2.2) becomes:

$$\begin{aligned}
&(f_{11}m_{11}+\ldots+f_{1N}m_{N1})\varphi_a^1+\ldots\ldots\ldots+(f_{11}m_{1N}+\ldots+f_{1N}m_{NN})\varphi_a^N=\frac{1}{\omega_a^2}\varphi_a^1\\
&(f_{11}m_{11}+\ldots+f_{1N}m_{N1})\varphi_b^1+\ldots\ldots\ldots+(f_{11}m_{1N}+\ldots+f_{1N}m_{NN})\varphi_b^N=\frac{1}{\omega_b^2}\varphi_b^1\\
&\qquad\vdots\\
&(f_{N1}m_{11}+\ldots+f_{NN}m_{N1})\varphi_a^1+\ldots\ldots\ldots+(f_{N1}m_{1N}+\ldots+f_{NN}m_{NN})\varphi_a^N=\frac{1}{\omega_a^2}\varphi_a^i\\
&(f_{N1}m_{11}+\ldots+f_{NN}m_{N1})\varphi_b^1+\ldots\ldots\ldots+(f_{N1}m_{1N}+\ldots+f_{NN}m_{NN})\varphi_b^N=\frac{1}{\omega_b^2}\varphi_b^N
\end{aligned} \qquad (2.2)$$

Each flexibility value can be evaluated as follows:

$$F(i,j) = a(j)\tau_{ab}(i) - \tau_{bc}(i,j)$$

$$a(j) = \sum_{l=j+1}^{n+1} l_l \Big/ \sum_{l=1}^{n+1} l_l \qquad j = 1,\ldots,n \qquad (2.3)$$

$$\tau_{ab}(i)=\left[\sum_{l=2}^{i}\frac{l_l}{EI_l}\frac{\sum_{k=i+1}^{n+1}l_l}{\sum_{k=1}^{n+1}l_l}\left(\frac{l_l}{3}+\sum_{k=1}^{l}l_k\right)\right]+\frac{l_1^3}{3(EI)_1}\frac{\sum_{k=i+1}^{n+1}l_l}{\sum_{k=1}^{n+1}l_l}\qquad i=1,\dots,n$$

$$\tau_{bc}(i,j)=\sum_{l=j+1}^{n}\frac{l_l}{(EI)_l}\frac{\sum_{k=1}^{i}l_k}{\sum_{k=1}^{n+1}l_k}\left[\frac{l_l}{2}\left(\sum_{k=j+1}^{l-1}l_k+\frac{l_l}{3}\right)+\sum_{k=l+1}^{n+1}l_k\left(\sum_{k=j+1}^{l-1}l_k+\frac{l_l}{2}\right)\right]+\left[\frac{l_{n+1}^2}{(EI)_{n+1}}\frac{\sum_{k=1}^{i}l_k}{\sum_{k=1}^{n+1}l_k}\left(\sum_{k=j+1}^{n}l_k+\frac{l_{n+1}}{3}\right)\right]\qquad \begin{matrix}i=1,\dots,n\\ j=1,\dots,n\end{matrix}$$

With: l_i= length section "i", E_i= elasticity modulus of the section "I", I_i= Inertia modulus of the section "I", n= number of coordinates of measure in the beam.

The goal of this procedure is the evaluation of the stiffness coefficients $(EI)_i$, for each section "i", with i = 1 to n+1. Considering equations (2.2) and (2.3) we come to:

$$A=\begin{bmatrix}a_{1,1}^a & a_{1,2}^a & \cdots & \cdots & a_{1,n+1}^a\\ \vdots & \vdots & \cdots & \cdots & \vdots\\ \vdots & & & & \vdots\\ a_{n,1}^a & a_{n,2}^a & & & a_{n,n+1}^a\\ a_{n+1,1}^b & a_{n+1,2}^b & & & a_{n+1,n+1}^b\end{bmatrix}\begin{bmatrix}\frac{1}{(EI)_1}\\ \\ \frac{1}{(EI)_n}\\ \frac{1}{(EI)_{n+1}}\end{bmatrix}=\begin{bmatrix}\frac{1}{\omega_a^2}\phi_a^1\\ \\ \frac{1}{\omega_a^2}\phi_a^n\\ \frac{1}{\omega_b^2}\phi_b^1\end{bmatrix}\tag{2.4}$$

The coefficients involved in Equation (2.4) are:

$$A(i,j)=\left(l_j\left(\sum_{l=i+1}^{n+1}l_l\Big/\sum_{l=1}^{n+1}l_l\right)\right)\left\{\left[\left(\left(\sum_{k=1}^{j-1}l_k\right)\left(\frac{l_j}{2}+\sum_{k=1}^{j-1}l_k\right)\gamma\right)-\left(\sum_{l=1}^{j-1}l_l\sum_{l=1}^{j-1}\left(\frac{l_j}{2}+\sum_{k=1}^{j-1}l_k\right)+\frac{l_j}{2}\left(\frac{2l_j}{3}+\sum_{k=1}^{j-1}l_k\right)\right)\alpha\right]\right\}$$

$i=2..,n \qquad j=2..,i$

$$A(i,j)=\left(l_j\left(\sum_{l=1}^{i}l_l\Big/\sum_{l=1}^{n+1}l_l\right)\sum_{l=j+1}^{n+1}l_l\right)\left\{\left(\left(\frac{l_j}{2}+\sum_{l=1}^{j-1}l_l\right)\gamma\right)-\left[\sum_{l=1}^{j-1}\left(\frac{l_j}{2}+\sum_{k=l+1}^{j-1}l_k\right)\alpha\right]\right\}$$

$i=1..,N-1 \qquad j=i+1..,n$

$$A(i,j)=\left(\frac{l_j^2}{2}\left(\sum_{l=1}^{i}l_l\Big/\sum_{l=1}^{n+1}l_l\right)\right)\left\{\left(\left(\frac{l_j}{3}+\sum_{l=1}^{j-1}l_l\right)\gamma\right)-\left[\sum_{l=1}^{j-1}\left(\frac{l_j}{3}+\sum_{k=l+1}^{j-1}l_k\right)\alpha\right]\right\}$$

$i=1..,n \qquad j=i+1..,n+1$

$$A(i,j)=\left(\frac{l_j^2}{2}\left(\sum_{l=i+1}^{n+1}l_l\Big/\sum_{l=1}^{n+1}l_l\right)\right)\left(\frac{l_j}{3}+\sum_{l=1}^{j-1}l_l\right)\gamma\tag{2.5}$$

$i=1..,n \qquad j=1..,i$

$$\alpha=\sum_{k=1}^{n}m_{lk}\phi_a^k\qquad \gamma=\sum_{l=1}^{n}\sum_{k=1}^{n}\left(\sum_{u=k+1}^{n+1}l_u\Big/\sum_{u=1}^{n+1}l_u\right)m_{lk}\phi_a^k$$

With: l_i= length section “i”, m_{lk}= the lk mass coefficient, ϕ_a^k = modal coordinate k of modal shape “a”, and ω_a = modal frequency of mode a.

Equation (2.4) defines a system of (n+1) equations with (n+1) unknowns, because (n+1) unknown coefficients, (EI) are considered. This fact imposes the requirement of two mode shapes and their corresponding frequencies to produce (n+1) equations.

The system (2.5) can be rewritten as follows:

$$[A](x) = (c) \tag{2.6}$$

Where (x) is the vector of unknown coefficients EI_i of each section of the beam.

3. NUMERICAL STUDY

To demonstrate the effectiveness of our damage identification method in simply supported beams, we begin by conducting a numerical study using a finite element model of a real bridge. The dynamic parameters of the undamaged model are compared to a scenario where various sections of the deck have been damaged. The effect of noise on the measurement of modal shapes and natural frequencies is also studied.

3.1 Numerical Model

Figure 2 shows the geometric characteristics of a bridge model from Nielson et al., which is also used in this study. The bridge is composed of three simply supported spans. The lateral spans are 12.2 m long, and the central span is 24.4 m. long. The deck is supported on eight steel girders spaced 1.83 m apart; the deck is thus 15 m wide (Figure 2).

The natural frequencies and mode shapes of the bridge were calculated by performing a finite element analysis with **SAP2000**. Following Nelson et al., the deck sections are composed of a material equivalent to homogeneous steel, with an elastic modulus of 200 GPa (**Nelson et al.**). Table 1 shows the elastic proprieties of the middle span and the two end spans (**Nelson et al.**).

This study is limited to measurements of the middle span. In the simulation the middle span is divided into eight sections, defining seven coordinates of measurement for the modal shapes.

Table No 1 Elastic properties of deck sections (**Frýba et al.**)

Span	A (m^{2})	I_z(m^4)	I_y(m^4)	Weight (kN/m)
End	0.51	0.03	9.78	39.00
Center	0.68	0.11	13.00	52.00

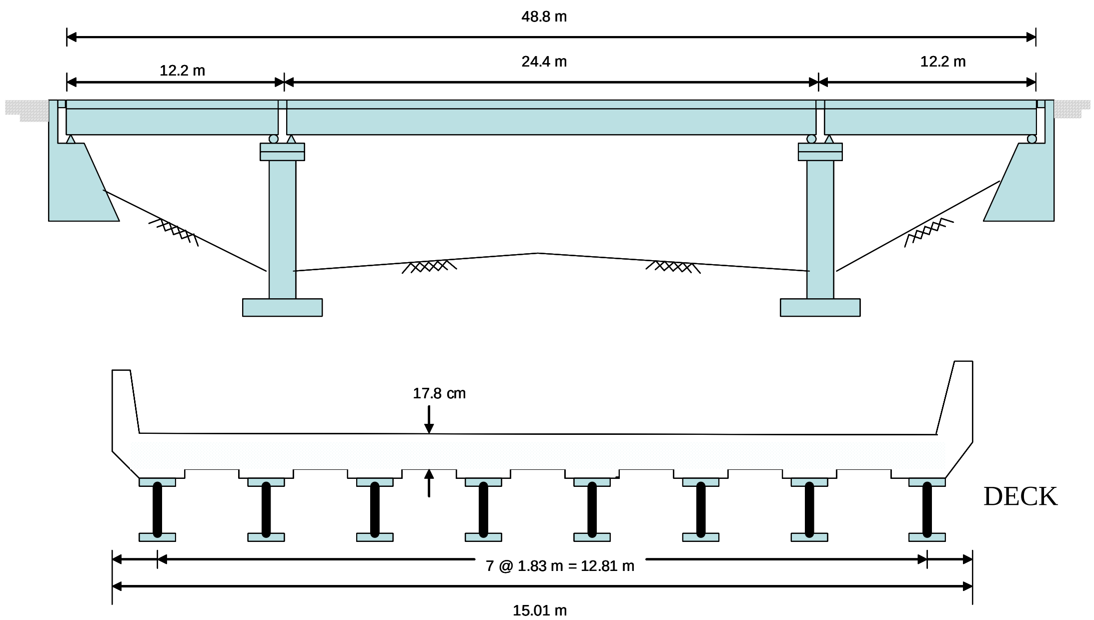

Figure 2 Geometrical characteristics of the bridge **(Nielson et al.)**

3.2 Study cases

Two cases were examined: a) the initial (undamaged) structure, b) damage in four sections (see Table 2).

Table No 2 Stiffness for case b

Section	1	2	3	4	5	6	7	8
EI_0/EI_f	0.90	1.0	0.70	0.70	0.80	1.0	1.0	1.0

3.3 Effects of errors in dynamics measurements

Uncertainties in modal data can affect the quality of damage estimation. To address this issue, a normally distributed random perturbation is added in the values of frequencies and mode shapes calculated by the finite element model. The noise was simulated by generating a random number between 0 and 1. Three cases were used to study the effect of measurement noise on damage identification:
a: The frequency is corrupted and the mode shape is uncorrupted, b: The frequency is uncorrupted and the mode shape is corrupted, c: Both frequency and mode shape are corrupted at the same signal-to-noise level.
Two values of the noise level were considered: 5% and 10%.

3.3.1 Results

Table 3 shows the results of our damage estimation procedure. The location of the damage is identified with precision, as are the changes in stiffness. The damage identification is affected by noise in the measurement, but is usually quite good. The quality of damage identification does not depend on whether measurement errors reside in the frequencies or modal shapes.

4. EXPERIMENTAL ASSESSMENT

4.1 Model Tested

To verify the effectiveness of this damage estimation method, we also tested the dynamic modes of a physical model. The dynamical parameters of the model are determined from the experimental data.

We have tested the method on a wide-flanged steel I-beam (IPN 80), with an 80 mm deep web and a 42 mm wide flange. The beam is 4100 mm long, with 4 meters suspended between the two outermost supports. This part of the beam is divided into 5 sections of 800 mm. The outer supports are elastomeric bearings (Figure 3).

In order to acquire lower values of the beam's modal frequencies and facilitate the process of obtaining a variety of modal shapes, the beam was supported in the flanges. The geometric and mechanical properties of the model are: $I_{xx} = 5.69$ cm^4, $I_{yy} = 74.9$ cm^4, $A = 7.66$ cm^2, Total weight (model and accelerometers) = 22.84 kg, Elastic Modulus = 209.5 GPa.

Table 3 Relative Errors

	Noise Level					
	Case a		Case b		Case c	
Section	5%	10%	5%	10%	5%	10%
1	1.12	1.01	1.39	1.99	3.73	1.03
2	0.50	0.66	0.29	0.03	0.10	0.56
3	0.15	0.70	0.59	1.08	1.55	1.78
4	0.09	0.97	0.13	0.56	1.21	0.58
5	0.00	0.87	0.65	1.15	1.51	0.41
6	0.15	0.60	0.15	0.20	0.61	3.15
7	0.16	0.45	1.01	1.58	2.16	3.48
8	0.38	0.62	0.89	0.74	0.31	8.63

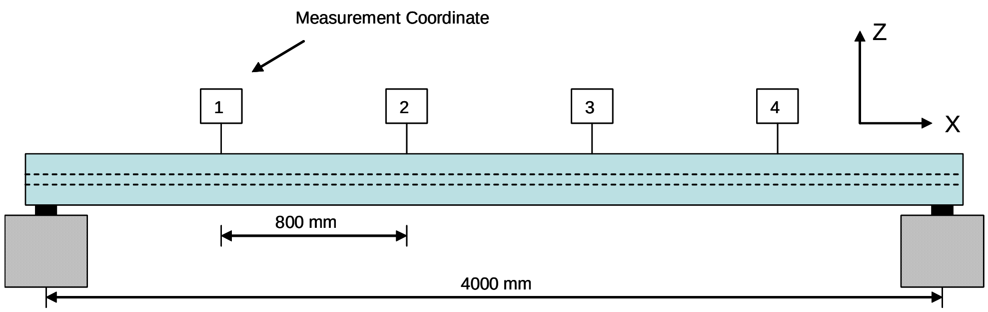

Figure 3 Tested Model

4.2 System identification

Free vibrations were induced by applying the appropriate initial displacements or velocities to each measurement coordinate. Vibration responses were registered with unidirectional accelerometers Kinemetrics FBA-11 and processed with an Altus K2 Kinemetrics signal processing device. Measurements were taken for a frequency range of 0-50 Hz. and the dynamic response was captured by 4 accelerometers.

The dynamic properties of the model were then assessed using SADEX, a software system developed to process structural dynamic signals in experimental tests (**García et al.**) Once the natural frequency of the vibration was estimated, SADEX determined the corresponding mode shape.

5. STUDY CASES AND ESTIMATION RESULTS

After measuring the natural vibration modes, damage was introduced into the beam in several stages. To reduce the stiffness of a beam segment, cracks were made in the flanges. These cracks were in the center of the affected section. Four cases were established:

Case a: Initial undamaged beam, Case b: Beam with damage in the 2nd section, Case c: Beam with damage in the 2nd and 3rd sections, Case d: Beam with damage in the 2nd, 3rd and 4th sections

A variety of free vibration tests were performed for each of the cases described above. In this manner we were able to compare several records, and to choose those which provided the most information on the structure. The modal frequencies and mode shapes determined for each case are reported in Table 4 and Figure 4 respectively. Given the beam mass and this experimental information, a linear system is formed (2.5) from the expressions described in (2.6). Solving system (2.5), we obtain the stiffness changes for every case relative to the initial structure (case a).

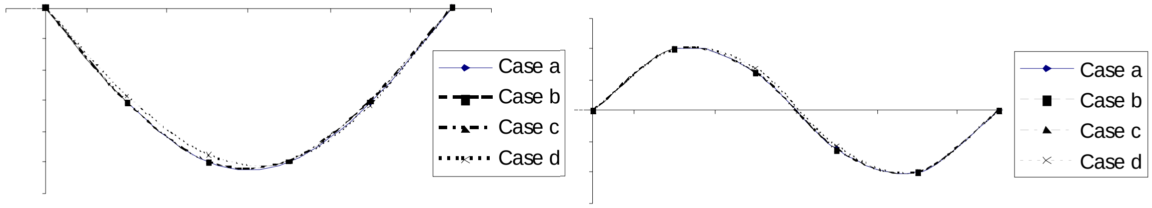

Figure 4 Experimental mode shapes

Table 4 Natural frequency from the free vibration test

	ω1 (rad/s)	ω2 (rad/s)	ω3 (rad/s)
Undamaged	25.50	97.60	---
Case b	25.12	95.87	---
Case c	24.93	95.68	219.55
Case d	23.01	87.63	196.93

Table 5 shows the final damage estimations for cases b, c, and d. In each case, the damaged and undamaged sections are identified correctly. In the undamaged sections, the maximum error in the stiffness estimation is 2%. The methodology precisely estimated the degree of damage in other sections as well, in terms of a change in stiffness relative to case (a).

Table 5 Stiffness changes estimation

	Case b Damage in the 2nd section	Case c Damage in the 2nd and 3rd section	Case d Damage in the 2nd, 3rd and 4th section
Section	EI_f/ EI_o	EI_f/ EI_o	EI_f/ EI_o
1	0.99	1.01	1.02
2	0.92	0.93	0.93
3	0.99	0.89	0.90
4	0.99	0.99	0.63
5	0.99	0.99	1.00

6. CONCLUSION

This paper proposed a damage identification procedure for simply supported beams. The methodology requires that the mass matrix and two of the beam's natural vertical vibration modes (shape and frequency) are known beforehand.

The damage identification procedure was illustrated with a numerical example: a finite element model of a real bridge. Damage to the model was successfully estimated with low relative error. It has been shown that the method also behaves satisfactorily under noisy conditions. The quality of damage identification does not depend on whether measurement errors reside in the frequencies or modal shapes.

The accuracy of our damage identification method is also unaffected by the severity of damage. Whether the stiffness of a section is greatly reduced or multiple sections are affected, the method accurately estimates the location and magnitude of stiffness changes.

A real steel beam was progressively damaged and subjected to the same method. In all three cases, the estimation method performed just as well as in the numerical models.

This approach can be applied not only to simply supported beams but also to simply supported girder bridges. It has two important advantages: only a small number of natural vibration modes need to be known beforehand, and stiffness changes can be accurately estimated using a small number of dynamical tests. Furthermore, this method requires only measurements along a single axis; rotational coordinates and refined FE models are unnecessary.

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