PROPOSAL AND APPLICATION OF STRESS-STRAIN MODEL FOR CONCRETE CONFINED BY STEEL TUBES

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ABSTRACT :

Confinement of concrete by transverse reinforcement has been one of the important research fields in concrete researches. Traditionally, spirals or hoops have been generally adopted to confine concrete. For high-strength concrete (HSC), however, confinement by conventional spirals or hoops needs a large quantity of steel, which might cause problem concerning with concrete placing. Therefore, instead of the conventional spirals or hoops, a new and effective confinement method is desirable for HSC in particular.

Confinement by steel tubes is an alternative method that can meet this need due to its following advantages: 1) to provide strong confinement to HSC easily without causing problem of concrete placing, 2) to confine the whole section of a concrete member so that one can expect enhancement not only of ductility but also of ultimate load-carrying capacity, and 3) to work as form for columns. To promote the use of confinement by steel tubes in particularly HSC structures, a stress-strain model for concrete confined by steel tube needs to be developed, and its applicability to the assessment of flexural behavior to be verified.

Objective of this paper are to propose a stress-strain model for normal- and high-strength concrete confined by steel tubes, and to verify its applicability and validity to the assessment of flexural behavior of the concrete members confined by steel tubes. The proposed model is simple and comprehensive, and can cover a wide range of material (concrete and steel) strengths and different confinement configurations (square steel tubes and circular steel tubes).

KEYWORDS: Confinement by steel tube, High-strength concrete, Stress-strain curve, Confined concrete, Strength enhancement ratio

1. INTRODUCTIONS

High-strength concrete (HSC), due to its high load-carrying capacity and durability, has recently gained increasing use in construction industry. To promote the use of HSC in building structures located on earthquake -prone regions, prevention of inherent brittle failure mode of the HSC becomes an important issue.

Tomii et al have proposed a so-called super transversely confinement (STC) method to prevent concrete short columns from brittle shear failure [Tomii et al., 1989]. In the STC method, steel tubes are utilized in lieu of traditional spirals or hoops to confine concrete in short columns. Confinement by steel tubes has several advantages over the conventional confinement by spirals or hoops in that the steel tubes can provide strong confinement to HSC easily and can confine the whole section of a concrete member so that one can expect enhancement not only of ductility but also of ultimate load-carrying capacity.

To effectively utilize the first advantage of steel tube, Sun et al. have applied the confinement by steel tubes to confine HSC concrete in low-story columns of high-rise building structures and experimentally verified effectiveness of the confinement by steel tubes in enhancing seismic performance of HSC columns and frames under high axial compression [Sun et al., 1998, 2001]. Figure 1 exhibits concept of confinement by steel tubes in the columns. As shown in Figure 1, unlike the steel tube in concrete-filled steel tubular columns, the steel tube in the STC method is used to provide lateral confining pressure to concrete rather than to provide a direct

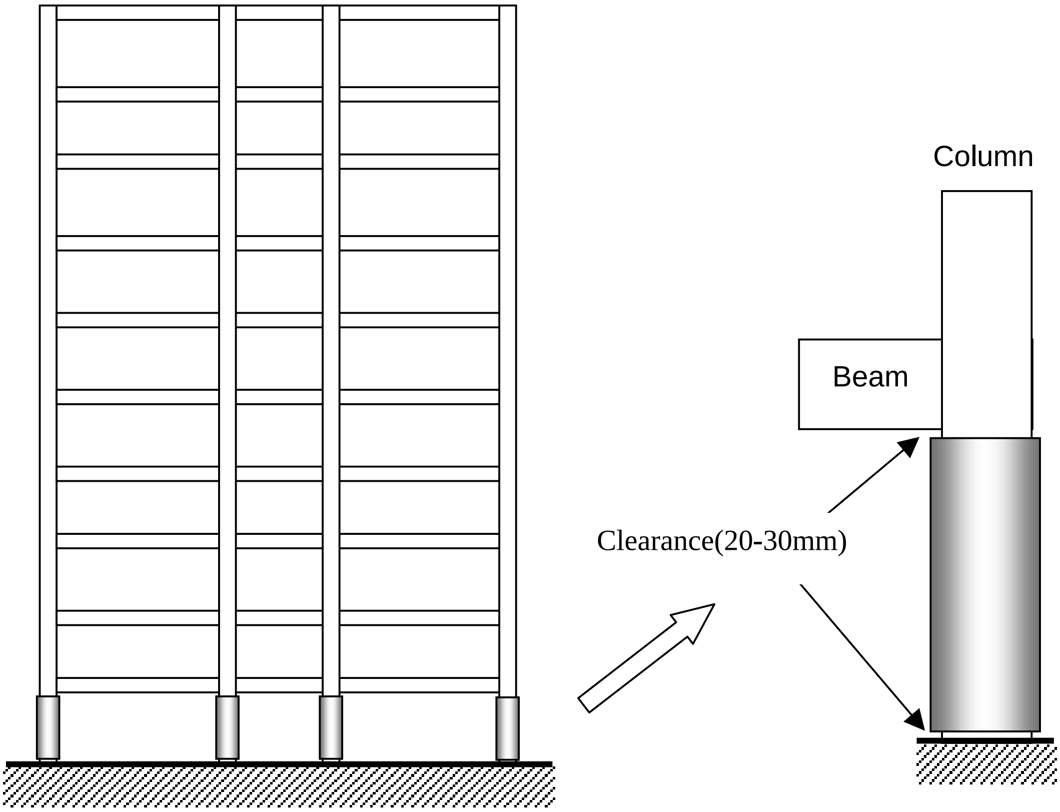


Figure 1 Concept of confinement by steel tubes in concrete columns

resistance to the axial stresses in concrete members induced by gravity and moment. The clearances are provided at both ends of the steel tubes to meet this requirement.

To promote the use of confinement by steel tubes in HSC concrete structures, understanding and evaluation of confinement effect by steel tubes are indispensable. This paper is to present a comprehensive stress-strain model for the concrete confined by steel tubes. The proposed model can take confinement effect by steel tube into consideration in a very simple form, an can be applied to concrete confined either by square steel tube or by circular steel tube.

2. COMPLETE STRESS-STRAIN MODEL FOR CONCRETE CONFINED BY STEEL TUBES

2.1 Confinement effect

Confinement by steel tubes generally has two types of configuration according to shape of concrete columns. One is square or rectilinear confinement and the other is circular confinement as shown in Figure 2. In the case of confinement by square steel tube, to enhance confinement effectiveness of the perimeter steel plate, cross inner stiffeners are sometimes wielded.

As to the evaluation of confinement by traditional spirals and hoops, numerous studies have been conducted since the pioneer study by Richart [Richart et al., 1928]. For the evaluation of confinement effect by circular spirals or hoops, while many models have been proposed until now, it is unquestionable that the model proposed by Richart et al has still been the simplest and most reasonable one. In the case of confinement effect by square or rectilinear transverse reinforcement, however, the evaluation is still in controversy, since there are many factors influencing confinement effectiveness. To solve the problem concerning with the evaluation of confinement effect by rectilinear transverse reinforcements, the author and his colleagues have proposed a new evaluation model for rectilinear confinement by introducing a so-called effective lateral confining factor that covers a wide range of confinement configurations, material strengths, and steel amount [Sun et al., 1996]. It

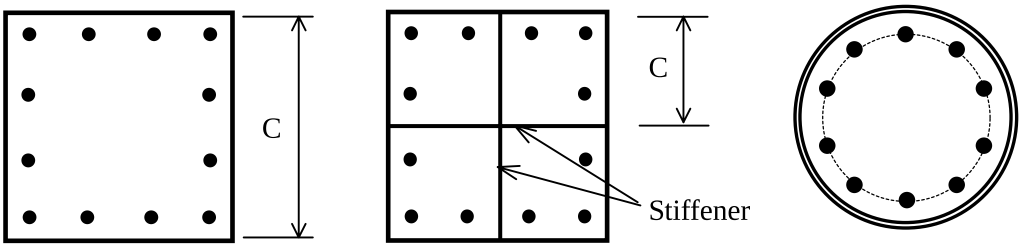


Figure 2 Typical configurations of confinement by steel tubes

has also been verified that the proposed model can evaluate confinement effect by rectilinear hoops very well.

Since the steel tubes can be considered as the limit state of traditional hoops with zero spacing, in this paper, the evaluation models proposed by Sun [Sun et al., 1996] and by Richart [Richart et al., 1928] will be applied to assess confinement effect by square steel tubes and by circular steel tubes, respectively. Following these two models, the strength of concrete confined by square steel tube and of the concrete confined by circular steel tube can be calculated by Eqn. 2.1 and Eqn. 2.2, respectively.

$$f_{cc}' = Kf_p = \left(1+11.5\frac{\rho_t f_{yt}}{f_p}\left(\frac{t}{C}\right)\right)f_p = \left(1+46\frac{(B/t-1)}{(B/t-2)^2}\left(\frac{t}{C}\right)\frac{f_{yt}}{f_p}\right)f_p \tag{2.1}$$

$$f_{cc}' = Kf_p = \left(1+2.05\frac{\rho_t f_{yt}}{f_p}\right)f_p = \left(1+8.2\frac{(B/t-1)}{(B/t-2)^2}\frac{f_{yt}}{f_p}\right)f_p \tag{2.2}$$

where f_{cc}' is the confined concrete strength, f_p is the unconfined concrete strength, ρ_t, f_{ty}, t, and B are the volumetric ratio, the yield strength, the thickness, and the outside width of square tube or the diameter of circular tube, respectively, the C is the unsupported length of perimeter plate of the square tube as defined in Figure 2, and K is the strength enhancement ratio. Figure 3 shows relationships between the strength enhancement ratio K and the wall thickness of square (without inner stiffener) and circular steel tubes. One can see from Figure 3 that confinement effectiveness of circular steel tube is much higher than that of square steel tube. For a column of 1000 mm in diameter, for example, confining the column with 6mm thick circular steel tube having f_{yt} = 300 N/mm^2 could bring 53% strength gain to the concrete with f_p = 30N/mm^2 as compared with the only 2% strength gain provided by the confinement of square steel tube having the same thickness, which

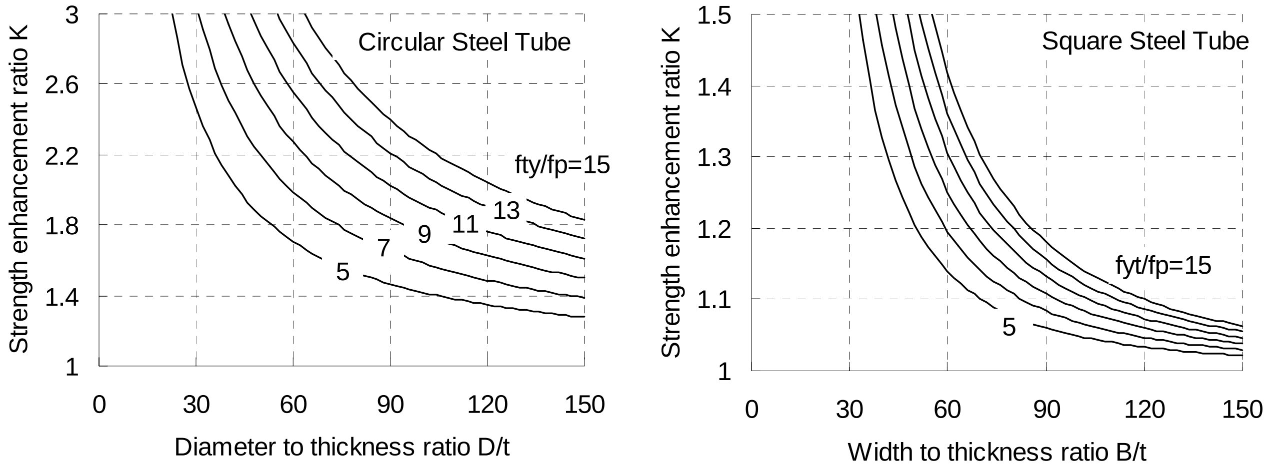


Figure 3 Examples of strength enhancement ratios of confined concrete

is the main reason for strengthening of the perimeter plate with inner stiffener as shown in Figure 2.

2.2 Stress-strain relationship

The stress-strain relationship proposed by Sun et al for the concrete confined by conventional hoops or spirals is directly applied to predict the axial behavior of concrete confined by steel tubes, and is written as follow:

$$f_c = Kf_p \frac{aX + (b-1)X^2}{1 + (a-2)X + bX^2} \tag{2.3}$$

where K is the strength enhancement ratio of confined concrete (see Eqns. 2.1 and 2.2), f_p is the unconfined concrete strength and X is the axial strain normalized by the strain at the peak. In order to predict the stress-strain relationship of confined concrete, it is necessary to determine values of three parameters in Eqn. 2.3. These parameters are: 1) strain at the peak ε_{co}, 2) parameter a that controls shape of the ascending branch, and 3) parameter b mainly governing the descending portion of the curve. Based on concentrically compressive test results of forty-eight stub square concrete columns made of concrete having cylinder strength $f_c^{'}$ = 20.6N/mm^2 – 67.7N/mm^2, and confined by normal strength square steel tubes with width-to-thickness ratio B/t = 31 – 107, the following expressions had been derived to predict these parameters [Sun et al., 1996] .

$$\frac{\varepsilon_{co}}{\varepsilon_o} = \begin{cases} 1 + 4.7(K-1), & K \le 1.5 \\ 3.35 + 20(\mathrm{K} - 1.5), & K > 1.5 \end{cases} \tag{2.4}$$

$$a = \frac{E_c}{E_{\sec}} = \frac{E_c \varepsilon_{co}}{Kf_p}, \quad b = 1.5 - 0.017 f_p + 2.4\sqrt{\frac{(K-1)f_p}{23}} \tag{2.5}$$

$$\varepsilon_o = 0.94(f_p)^{1/4} 10^{-3}, \quad E_c = \left(0.69 + 0.332\sqrt{f_p}\right) \times 10^4 \ (in\ MPa) \tag{2.6}$$

As can be seen from Eqn. 2.4 through Eqn. 2.6, the proposed model is a two-parameter model. Only if the unconfined concrete strength f_p and the strength enhancement ratio K, an index measuring the confinement level of steel tube, are known, one can completely determine the stress-strain curve for the confined concrete.

When using Eqn 2.1 through Eqn. 2.6 to predict the axial behavior of confined concrete, definition of the unconfined concrete strength f_p needs to be clarified. According to the research by Neville [Neville et al., 1966], it is reasonable to take standard cylinder strength $f_c^{'}$ as f_p for the concrete confined by square steel tube, while f_p =0.85 $f_c^{'}$ is suitable for the concrete confined by circular steel tube.

2.3 Comparisons with experimental results

To verify validity of the proposed stress-strain model, the previous test results are compared with the calculated stress-strain curves in Figure 4 and Figure 5 for squarely confined concrete and circularly confined concrete, respectively.

The tests shown in Figure 4 were conducted by Sakino [Sakino and Sun, 1994], while the tests shown in Figure 5 for the concrete confined by circular steel tubes were conducted by the author [Sun et al., 1999]. As apparent from Figure 4, the stress-strain model proposed by Sun et al. can predict axial behavior of the concrete confined by square steel tube up to large deformation very well. Some discrepancy between the measured and the theoretical result is observed for the HSC confined by thick steel tube with B/t ratio of 31. This discrepancy is mainly due to that the steel tube didn’t yield when the confined concrete reached its peak stress while it is assumed the steel tube will yield in Eqn 2.1. However, the discrepancy between the stress-strain curves converges as the strain increases, which implies that the proposed model is a good and reasonable model for the concrete confined by square steel tubes. For the concrete confined by circular steel tubes, comparison shown in Figure 5 indicates that Eqn. 2.3 can predict axial behavior of circularly confined concrete very well either.

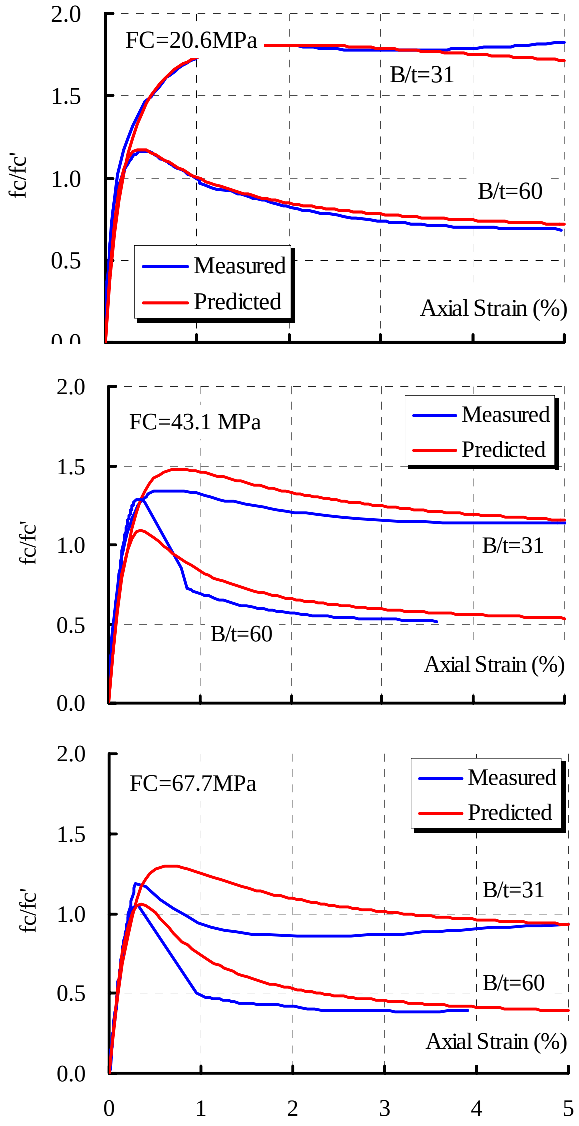


Figure 4 Comparison of stress-strain curves of squarely confined concrete

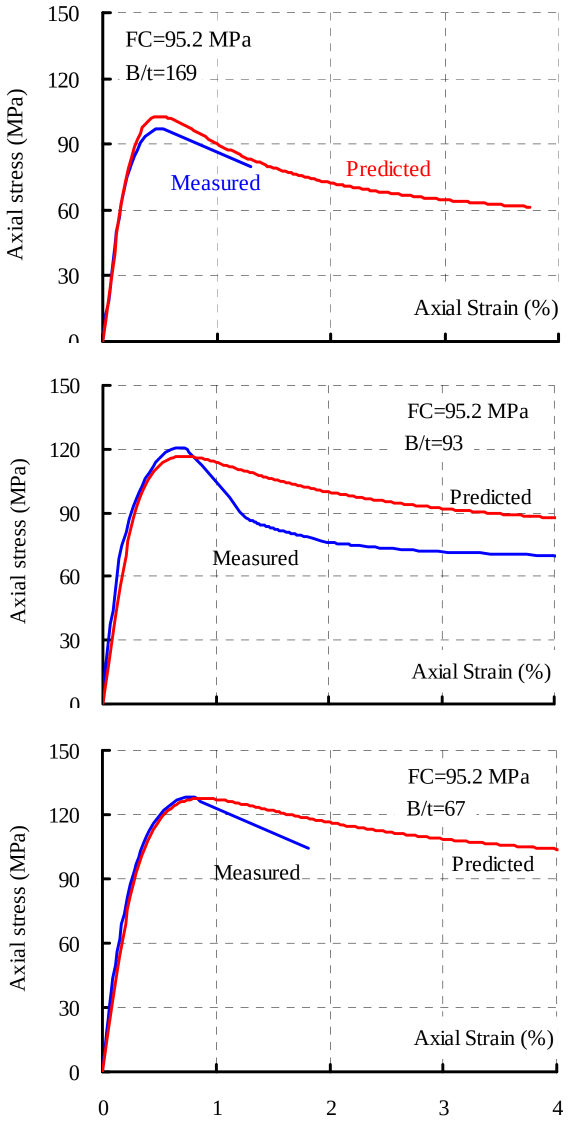
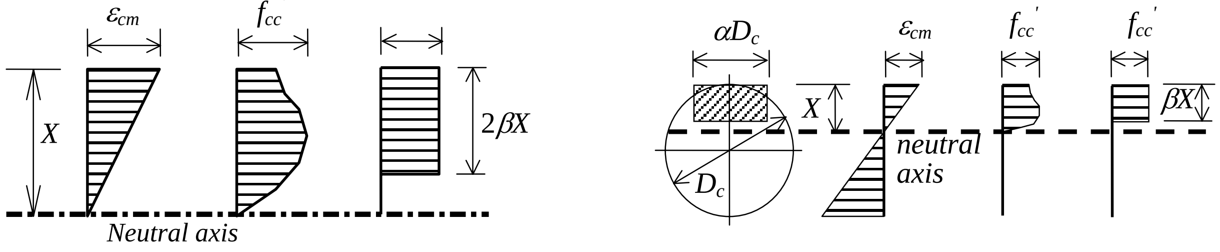


Figure 5 Comparison of stress-strain curves of Circularly confined concrete

3. APPLICATIONS OF THE CONFINEMENT MODEL

3.1. Equivalent rectangular stress block

The stress-strain model of concrete is basically used to predict flexural property such as moment-curvature



(a) strain profile (b) actual stress distribution (c) equivalent stress block

(a) square column section

(a) concrete section (b) strain profile (c) actual stress (d) stress block

(b) circular column section

Figure 6 Concepts of equivalent rectangular stress block relationship for concrete members. However, for structural engineers, information at the peak point in a moment-curvature curve is more desirable rather than the complete moment-curvature curve. Information at the peak includes ultimate flexural strength as well as ultimate curvature. The interaction diagrams between the axial load N, the ultimate moment M_m, and the ultimate curvature ϕ_m (shorten to N-M-ϕ interaction diagram hereafter) are usually adopted to calculate the ultimate flexural strength and deformation for reinforced concrete columns. For a given column, the N-M-ϕ interaction diagram can be obtained by computing the moment-curvature relations for various levels of axial load. This method, however, is very tedious, since it involves an iterative procedure to find the depth of the neutral axis for the internal forces to balance the external applied load. Therefore, simplification of the calculation procedure is desirable.

To simplify the calculation procedure, equivalent rectangular stress blocks have been developed by the author and his colleague [Sun and Sakino, 1998, 2001] to replace the actual stress state of the compressed concrete in square and circular column sections. Figure 6 shows concept of the stress blocks. As shown in Figure 6, for column section confined by square steel tube, the equivalent stress block has depth of $2\beta X$ with an uniformly distributed stress $\alpha f_{cc}{}' / (2\beta)$, while for column section confined by circular steel tubes, the stress block has depth of βX and width of αD_c with uniformly distributed stress $f_{cc}{}'$, where D_c is the diameter of concrete section.

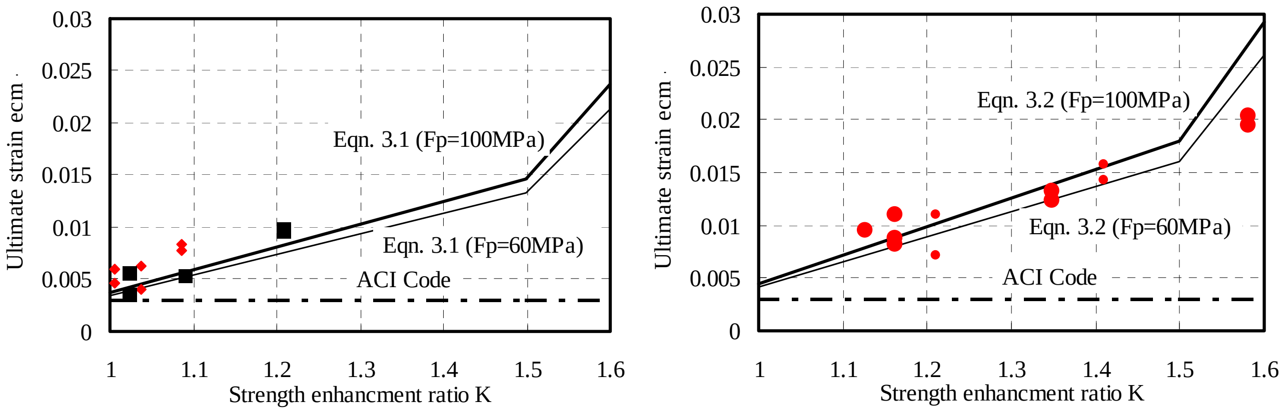
After intensively investigating the flexural behavior of confined concrete columns, the following equation can be derived to evaluate the ultimate strain ε_{cm} for the confined concrete in compressed region:

$$\frac{\varepsilon_{cm}}{\varepsilon_{co}} = 1.375 + 0.108K - \frac{0.1}{K^4}\frac{f_p}{42} \quad \text{for square section} \tag{3.1}$$

$$\frac{\varepsilon_{cm}}{\varepsilon_{co}} = 1.465 + 0.315K - 0.168\frac{f_p}{42} \quad \text{for circular section} \tag{3.2}$$

where K is the strength enhancement ratio of confined concrete as defined by Eqn. 2.1 or 2.2, and ε_{co} is the strain at the peak stress of confined concrete (see Eqn. 2.4). Figure 7 shows comparison of Eqns. 3.1 and 3.2 with the previous test results [Ikenono et al., 1999, Sun et al., 2001]. One can see from Figure 7 that Eqns. 3.1 and 3.2 can predict the ultimate strain of normal- and high-strength concrete confined by square or circular steel tubes very well.

Furthermore, Sun and Sakino [1996] have also developed the following expressions to evaluate the parameters defining the stress blocks:



(a) squarely confined concrete [Ikenono et al. 1999] (b) circularly confined concrete [Sun, 2001]

Figure 7 Comparisons of the ultimate strain of confined concrete

$$\alpha = 0.724 + 0.107K - \frac{0.037}{K - 0.007}\frac{f_p}{42},\ \beta = 0.383 + 0.046K - \frac{0.019}{K + 0.687}\frac{f_p}{42} \quad \text{for square section} \qquad (3.3)$$

$$\alpha\beta = A(K, X_n) - B(K, X_n)\frac{f_p}{42}, \qquad \frac{\beta}{2} = C(K, X_n) - D(K, X_n)\frac{f_p}{42} \quad \text{for circular section} \qquad (3.4)$$

where Xn is the depth of neutral axis normalized by diameter of column section. As obvious from Eqns. 3.3 and 3.4, the stress block can be completely determined by concrete strength and strength enhancement ratio for square column, which implies that calculation of ultimate capacities can be simplified easily. On the other hand, parameters of the stress block for circular column depend upon the depth of neutral axis due to shape of section, which implies complexity in calculation of ultimate capacities by the stress block.

To further simplify the calculation of ultimate flexural capacities of the circularly confined concrete members, after intensive analysis of the properties of the *N-M* interaction diagrams, author has developed a very simple equation to define the ultimate *N-M* interaction diagrams in form of [Sun, 2007]:

$$m = \begin{cases} m_0\left[1 - \left(\dfrac{n - n_0}{n_0 + r}\right)^2\right], & n \le n_0 \\ m_0\left[1 - \left(\dfrac{n - n_0}{n_0 - K - r}\right)^2\right], & n > n_0 \end{cases} \qquad (3.5)$$

where

$$n = \frac{N}{A_g f_p}, \qquad m = \frac{M}{A_g D_c f_p}, \qquad r = \frac{p_g f_{ys}}{f_p} \qquad (3.6)$$

$$n_0 = \frac{1}{\pi}\left(0.1K^2 + 1.3K - 2.2K^{-1} f_p \times 10^{-3}\right)$$
$$m_0 = \frac{1}{\pi}\left[0.31K + (0.61K - 0.85) f_p \times 10^{-3} + r\frac{D_s}{D_c}\right] \qquad (3.7)$$

where N is the applied axial load, M is the ultimate moment capacity corresponding to the N, A_g is the gross sectional area of the column, D_c is the diameter of the column section, and D_s is the distance between the centroids of longitudinal steels located at the extreme compressive and tensile side.

3.2.Verification of the proposed methods

Verification of the proposed stress block for squarely confined concrete members and the design equation for circularly confined concrete members made of normal-strength concrete have been reported by the author and his colleagues [Sun et al., 1996; Sun, 2007], respectively. Therefore, in this paper, the test results by Ikenono et al [1999] and Sun et al [2001] will be used to check applicability of the proposed methods to the confined concrete members made of high-strength concrete.

Figure 8 shows comparison between the measured and the theoretical ultimate moments for confined concrete columns made of high-strength concrete. It is obvious from Figure 8 that the proposed methods (stress block for square column and design equation for circular column) can predict the ultimate flexural strength of high-strength concrete members confined by square or circular steel tubes very well.

4. CONCLUSIONS

To provide structural engineers a useful and powerful tool for evaluating the confinement effect by steel tubes on ultimate properties of normal- and high-strength concrete members, a comprehensive stress-strain model for

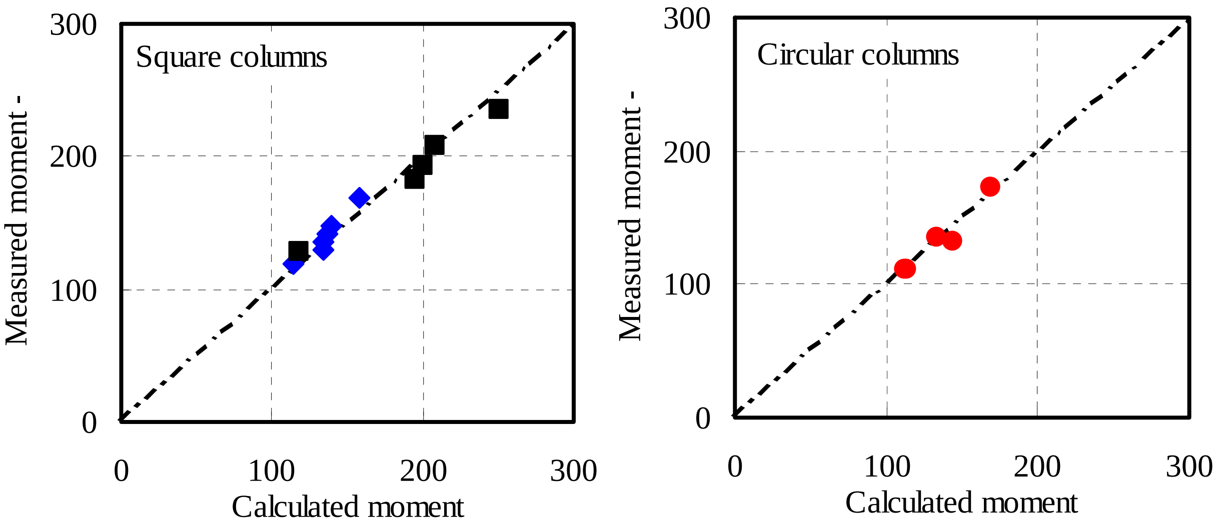


Figure 8 Comparison of the measured and theoretical moments of high-strength concrete members

confined concrete has been proposed. Comparisons with the previous tests have indicated that confinement effect by the steel tubes on concrete strength and deformation capacity can be accurately evaluated by using the simple equations included in the proposed model. The proposed stress-strain model predicts the stress-strain relationships of normal- and high-strength concrete confined by square or circular steel tubes very well up to large strain. It has also been shown that the flexural strength based on the proposed stress block or design equation (see Eqn. 3.5) agreed fairly well with the previously test results, which implies that the confinement model could provide structural engineering a powerful and reliable tool to conduct reasonable design for concrete members confined by steel tubes.

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