OBTENTION OF UNIFORM HAZARD SPECTRA TO USE ON THE PERFORMANCE BASED SEISMIC DESIGN

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ABSTRACT :

This paper presents a new methodology based on structural performance to determine uniform hazard design spectra with the same probability of exceedence of a performance level for a given design. The obtained spectra correspond to a given exceedence rate of a specific performance index characterizing the performance level for which the structures will be designed. The proposed formulation takes into account the non-linear behaviour of the structures and a given damage configuration under design conditions. This procedure involves the evaluation of the seismic hazard, necessary to define a seismic design level consistent with the performance based seismic design philosophy, using a large enough number of seismic records of several magnitudes, These records are simulated with an improved empirical Green function method. The statistics of the performance of a reference single degree of freedom system are obtained by using Monte Carlo simulation, assuming the seismic demand, the fundamental period and the strength of the structure as uncertain. With these results, the conditional probability that a structure exceeds a specific performance level is obtained. It is concluded that the proposed procedure is a significant improvement of other considered in the literature and useful research for the further development of uniform hazard spectra of use in the performance based seismic design and retrofit of structures.

KEYWORDS: Uniform Hazard Design Spectra, Uncertain Structural Properties, Performance Based Seismic Design

1. INTRODUCTION

In the practice of earthquake engineering there are many instances where essential aspects in the determination of structural performance are not properly considered due to a lack of information or to the limitations of the analytical models to handle uncertain information. As during the design of a structure, the engineer does not know with certainty the characteristics of the design seismic demands and of the physical properties of the structure, *e.g.*, properties of the concrete and/or steel, which form the structural elements and quality of the construction when it will be built, among others, it is generally accepted that in order to face these problems it is necessary to use probabilistic methods which implicitly consider the randomness of the involved variables. Up to a certain point, the uncertainties attached to the seismic demands and the structural properties could be diminished with reliable seismic information, obtained from enhanced seismic instrumentation at sites of interest and from more research results on the characteristics of potential seismic sources; more elaborate analysis and design procedures, better characterization of the properties of the materials and stricter controls on the quality of the construction; unfortunately, any of these requirements would necessarily increase the cost of the structures without an assurance that the effect of the diminished uncertainties involved in this process would be really reduced.

Current procedures, based on the performance based seismic design philosophy, have as objective that an evaluated or designed structure has a predictable performance when subjected to demands consistent with those assumed during its design. It is convenient within this design philosophy that for each considered design objective there should be a performance level, characterized by a realistic performance index, and a seismic design level, characterized by uniform hazard design spectra for a given rate of exceedence of the considered performance index, and not design spectra in terms of rates of exceedence of earthquake intensity. Unfortunately, current design codes based on this philosophy characterize seismic design levels with spectra associated to rates of exceedence of given earthquake intensities ignoring the performance that the structure would have under these design conditions. Bearing in mind this problem is that some research efforts have been dedicated to the definition of design spectra consistent with the performance based seismic design philosophy not only by considering the rate of exceedence of seismic intensity but also the probability of exceeding the performance level for which the structure is designed and the inherent uncertainties in the definition of relevant structural properties..

Examples of these efforts are works done by Bazzuro and Cornell (1994a and 1994b) and Shome (1999) within a probabilistic framework and by Avelar *et al.* (2003) who propose, in accordance with the seismic design philosophy discussed in documents such as Vision 2000 (SEAOC, 1995), the use of uniform hazard spectra for given rates of exceedence of a performance index. However, all these formulations do not consider the uncertainties in the structural properties relevant to the definition of performance under seismic action as the performances considered are those of structural models with deterministic properties. To overcome this limitation, this paper presents a consistent formulation of a method to introduce the randomness of all important variables in the definition of uniform hazard spectra characterizing the seismic design levels used in the performance based seismic design of structures.

2. PROCEDURE FOR THE DETERMINATION OF UNIFORM HAZARD DESIGN SPECTRA

It is an accepted fact that the most important source of uncertainty in the performance estimation of new and existing buildings is the evaluation of the characteristics of future seismic events at the site of interest (Kircher *et al.*, 1997). The effect of this acceptance is evident when looking at most current seismic codes throughout the world where the prescribed elastic design spectra are defined in terms of spectral accelerations for an assumed exceedence rate of earthquake intensity. It is not difficult to show, however, that there are other uncertainties involved in the determination of the performance of reinforced concrete and steel structures subjected to seismic demands, among these are, for example, those involved in the determination of the stiffness and strength of the structural elements due to the contribution of steel reinforcement which in general, tends to increase the stiffness when the strength is increased, something that is not considered in current design practice. Due to this and other ignored facts is that in practice is common to find a large dispersion in the performance of structures designed with spectra obtained with methods which are not fully explained and/or understood by practising engineers specially when these performances are compared with those obtained by different researchers using more elaborate methods involving non-linear analysis and simulated structural properties proposed, and, above all with the nominal periods considered during the design process, leading with this to sub or over estimate the seismic demand for which the structure should be designed.

As previously discussed, it is important to consider in the definition of the design spectra the randomness of the seismic demand to which the structure is subjected under design conditions. However it should be pointed out that there are some structural characteristics, which also strongly influence the characteristics of design spectra, whose uncertainties are not explicitly considered. Current design codes consider other sources of uncertainty such as those associated to the structural properties, by using somehow empirical modification factors aimed to match the performance of code designed structures to those calculated from non-linear analyses and/or observations after real events. These factors are defined, in most cases, from analytical calibrations and expert opinions based on analyses of structural damage data after destructive earthquakes. However, in spite of the simplicity of this approach, it has the disadvantage of not formally considering the randomness of the structural characteristics, something that does not guarantee that the performance expected to occur during design conditions would take place.

In this paper, to obtain uniform hazard spectra which take into account, besides the uncertainties in the seismic demand and also those in the relevant properties of the structure, a formulation based on the concepts originally proposed by Esteva (1976) is used.

$$E\left(\nu_f\right) = \int_0^\infty f_R\left(r\right) \int_0^\infty -\frac{\partial \nu_y\left(u\right)}{\partial u} P_F\left(r \middle| u\right) du\, dr \tag{2.1}$$

where the first integral in this equation is a multiple integral involving all the uncertainties involved in the structural properties through $f_R(r)$, $\nu_y(u)$ is the rate of exceedence of a seismic event of characteristic u, necessary information to obtain the expectation of the rate of exceedence of a structural performance index, $E(\nu_f)$.

Considering that the problem is to quantify the rate of exceedence of a given performance level, *i.e.*, the expected number of times per unit time that the performance of a structure exceeds this level when subjected to earthquakes of different magnitudes, generated at different sources defining the seismic hazard of the site, the equation which defines the expected rate of exceedence must be established as the sum for all possible seismic sources and magnitudes considered of the product of the derivative of the rate of exceedence of magnitudes of the i^{th} source multiplied by the conditional probability that the structure exceeds a chosen performance level, given that an earthquake of magnitude M generated at a distance L occurs, may be expressed, from eq. 2.1, as

$$\eta\left(R,T\right) = \sum_{i=1}^{N} \int_{M_0}^{M_U} -\frac{d\lambda_i\left(M\right)}{dM} P_{r,i}\left(R_d > R \middle| T, R, M, L_i\right) dM \tag{2.2}$$

where R is the considered seismic performance level for the structure under study, $\lambda_i(M)$ represents the local seismicity of the i^{th} source and $P_{r,i}(R_d > R\,|T, R, M, L_i)$ is the probability that the performance R_d exceeds the considered performance level, given that an earthquake of magnitude M, generated at an i^{th} seismic source located at a distance L_i from the site, occurs.

Considering that the seismic hazard of the site is defined by only one seismic source, as is the case of sites in Mexico City and a life safety performance level, and that the index defining the considered performance level is a displacement ductility, μ_D, eq 2.2 may be simplified to

$$\eta\left(R,T\right) = \int_{M_0}^{M_U} -\frac{d\lambda\left(M\right)}{dM} P_r\left(\mu > \mu_D \middle| T, R, M\right) dM \tag{2.3}$$

Since the probability that a structure has a ductility larger than μ_D is equal to the probability that it has a strength smaller than that required to reach this ductility, eq. 2.3 may be expressed as

$$\eta\left(R,T\right) = \int_{M_0}^{M_U} -\frac{d\lambda\left(M\right)}{dM} P_r\left(R_d > R \middle| T, R, M\right) dM \tag{2.4}$$

where R_d is the strength that must be provided to the structure to reach the chosen performance level.

To obtain the conditional probability of exceeding a given performance level, the above equation may be evaluated using either a stochastic approach, *e.g.* Flores and Ayala (2005), or, as it is done in this paper, the Monte Carlo Method. Once the rate of exceedence of magnitudes, $\lambda(M)$, and the conditional probability that the structure exceeds the performance level for which it was designed are obtained, the integral over all the magnitudes in eq. 2.4 corresponding to the local seismicity of the site for which the desired spectra, may be calculated. The results of this calculation may be represented by a surface formed by curves of period *vs*. strength. The characteristic of each of these curves is that they have a uniform rate of exceedence of the selected performance level.

2.1. Seismic simulation

For the evaluation of eq. 2.4 using the Monte Carlo method it is necessary to have a large number of real earthquake records at the site of interest, something that, unfortunately, never happens. To circumvent this problem, in this research, synthetic records simulated with an improved empirical Green function method which produces more realistic simulations are used. In this method real earthquake records of small magnitude recorded at the site of interest are used as empirical Green functions; the probability distribution function, pdf, of the rupture times of the N cells, in which the source is divided, is approximated by considering that the Fourier spectrum of the seed and of the simulated signals are defined by two instead of only one corner frequency as is normally done, (Niño, 2008). Moreover, the sum scheme used to construct a simulated record is divided in two stages, (Kohrs-Sansorny *et al.*, 2006), an approximation which leads to sufficiently independent records associated to a larger number of rupture processes.

2.2. Uniform hazard spectra for structures with uncertain vibration period

To include the uncertainty in the vibration period of the structure in the definition of the rate of exceedence of a given performance level, it is assumed that there is a conditional probability of the performance which depends not only on the seismic magnitude but also on the structural period. Several researchers, *e.g.*, Quiroz *et al.*, (2004), show, based on existing field and experimental data. that the pdf which best represents the dispersion of the data on nominal *vs.* real structural period is the log-normal pdf, *i.e.*,

$$P(T) = \Phi\left(\frac{1}{\zeta_{T\mu}} \ln \frac{T}{T_\mu}\right) \tag{2.5}$$

where T_μ is the nominal period and $\zeta_{T\mu}$ is the standard deviation of the natural logarithm of T.
Thus, the equation defining the rate of exceedence of a given performance level, considering the uncertainty of the real vibration period is evaluated from

$$\gamma(R, T_\mu) = \int_0^\infty \eta(R, T) p_T(T|T_\mu) dT \tag{2.6}$$

where η (R, T) is the rate of exceedence of the performance level for each considered period and p_T $(T|T_\mu)$ is the pdf of the period T given that the structure was designed with a nominal period T_μ.

2.3. Uniform hazard spectra for structures with uncertain lateral strength

Besides the variations between the nominal period of vibration and the period the structure has once it is built, there are also differences between the nominal and the real strengths of the structure. The observed dispersions are due to factors such as the standardization of the steel reinforcement in structural elements, uncertain material properties for the concrete and the reinforcement steel, among others. Due to the importance of these uncertainties in the determination of structural performance, it is necessary to consider in the definition of a design spectrum the real strength as a random variable. To do this, a formulation similar to that used for the structural period was considered, i.e.,

$$\beta(R_\mu, T) = \int_0^\infty \eta(R, T) p_r(R|R_\mu) dR \tag{2.7}$$

where p_r $(R|R_\mu)$ is the pdf of the strength given that the structure was designed with a nominal strength R_μ.

Saito *et al.*, (1997) have used the log-normal pdf to characterize the dispersion of strengths, *i.e*,

$$P(R) = \Phi\left(\frac{1}{\zeta_{R\mu}} \ln \frac{R}{R_\mu}\right) \tag{2.8}$$

where R_μ is the nominal strength and $\zeta_{R\mu}$ is the standard deviation of the natural logarithm of R.

3. OBTAINED UNIFORM HAZARD DESIGN SPECTRA

3.1 Probability of exceedence of a given performance level

Using the formulation described in the previous section, uniform hazard spectra for the SCT site located in the lakebed zone of Mexico City are obtained using as index for the life protection performance level a ductility value of 4.

The evaluation of the seismic hazard defined by a rate of exceedence of the seismic magnitude, initial step in the determination of uniform hazard spectra, has been previously presented by Avelar *et al*. (2003) and Niño (2003) for the SCT site considering that for this site, only one source, the Guerrero Gap, generates earthquakes of magnitudes of significance for the seismic hazard.

The Monte Carlo simulations of the strength spectra used in this work were obtained for the considered performance index by Niño, (2008), using synthetic records of earthquakes of magnitudes ranging from 7.2 to 8.2 with increments of 0.1, Fig. 3.1. For each magnitude considered, 1000 records were simulated, *i.e.*, a total of 11000 simulated records with 11 different magnitudes. The 11000 strength spectra were calculated using a post-yielding to initial stiffness ratio of 0.24 and a 5% damping ratio, parameters consistent with an acceptable damage distribution in reinforced concrete buildings under design conditions in accordance with the current design philosophy of weak beams – strong columns.

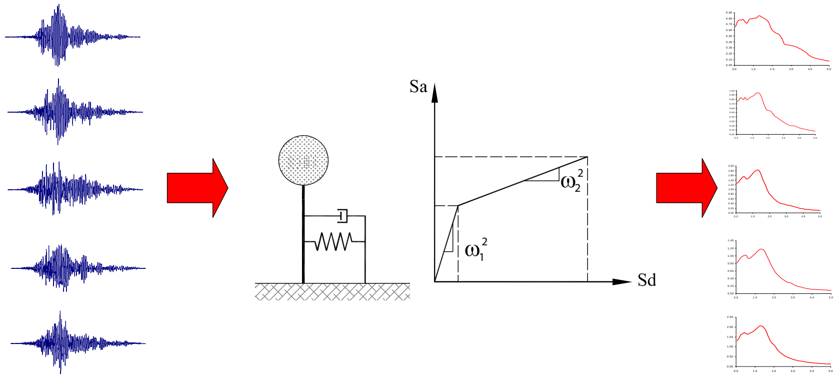

Figure 3.1 Filtering of synthetic records through single degree of freedom systems in the application of the Monte Carlo method.

The statistic information necessary to define the pdfs required to obtain the probability of exceedence of a given performance index is obtained from the strength spectra calculated for all simulated records. This work presents the results of the evaluation of eqs. 2.6 and 2.7 which define the conditional probability given that the structure has an uncertain vibration period T or that the structure has an uncertain strength, R, correspondingly. With these results it is possible to construct a family of curves of period (T_μ) *vs*. strength per unit mass, for different rates of exceedence of a life safety performance level defined by a ductility demand μ = 4, considering first the structural period as uncertain, Fig. 3.2a, and then the strength, fig. 3.2b.

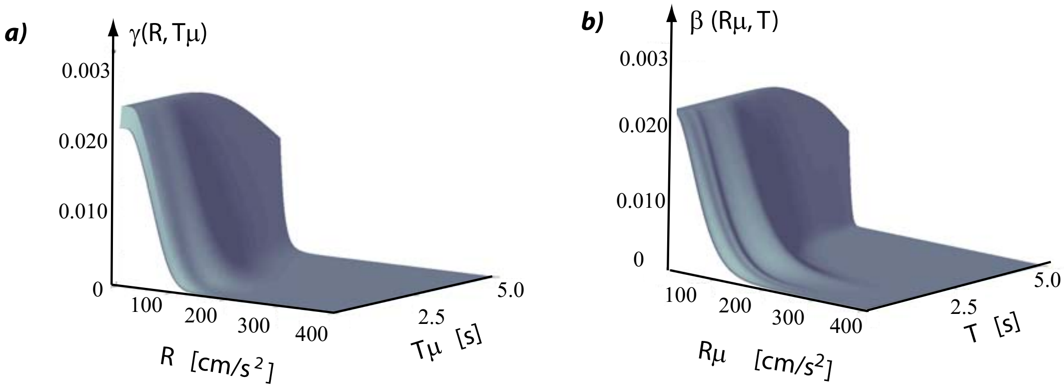

Figure 3.2 Surfaces defining the rate of exceedence of a given performance level independently considering the uncertainties in a) structural period and b) strength.

From the surfaces shown in Fig. 3.2 it is possible to obtain uniform hazard spectra associated to different rates of exceedence of the considered performance level by plotting the intersections of the different horizontal planes associated to the constant rates of exceedence considered. Fig. 3.3 presents the uniform hazard spectra for a rate of exceedence $\nu = 0.01$, for a return period of 100 years. These spectra were derived for the two cases previously described in this work, *i.e.*, in Fig. 3.3a the uniform hazard spectrum obtained by considering in the evaluation of the rate of exceedence of the performance level the real structural period uncertain; similarly in Fig. 3.3b the uniform hazard spectrum for the case in which strength is considered uncertain. To define the variability of these variables and their log-normal pdfs for the real structural period and strength the corresponding standard deviations $\zeta_T = 0.3$, and $\zeta_R = 0.308$ were used.

It may be observed in Figs. 3.3a and b that when compared with the spectra obtained with deterministic structural properties, the smooth spectra considering uncertain structural period shows some shape modifications. On the other hand, when uncertain strength is considered the spectral ordinates of the resulting smooth spectra tend to increase in a uniform way.

To illustrate the differences found when the uncertainties in structural period and strength are considered, Fig. 3.3a also shows for the sake of comparison the corresponding uniform hazard spectra without consideration of these uncertainties.

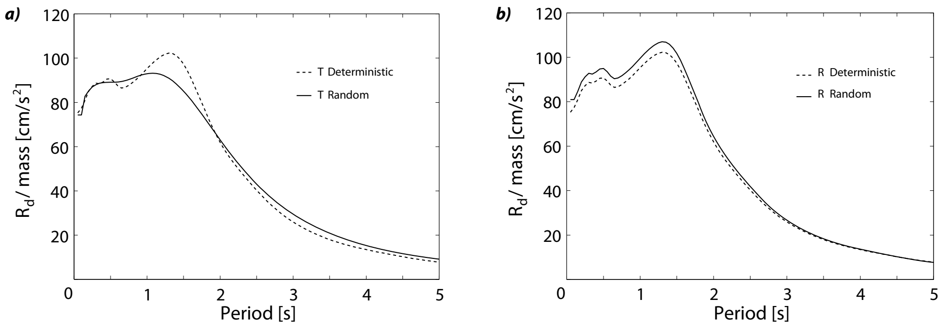

Figure 3.3 Comparison of uniform hazard spectra deterministic structural properties and spectra with uncertain: a) period and b) strength

It is important to mention that up to now the uncertainties in natural period and structural strength have been considered in independent formulations, these conditions are not realistic as in actual structures these properties are correlated and therefore to be of practical use they should be included simultaneously in a unique formulation. However, they are independently considered in this paper to evaluate their relative influence on the ordinates of the uniform hazard spectra. The effect of the simultaneous consideration of the uncertainties of these correlated variables and on the post yielding stiffness on the characteristics of the uniform hazard spectra has also been investigated and reported by Niño, (2008).

4. SIMULTANEOUS CONSIDERATION OF ALL UNCERTAINTIES IN THE DETERMINATION OF UNIFORM HAZARD SPECTRA.

For practical purposes, in the calculation of uniform hazard spectra it is necessary to consider the simultaneous effects of the uncertainties of the two previously investigated variables, particularly as they are known to be correlated. Thus, considering that the rate of exceedence of a performance level as a function of earthquake magnitude is evaluated in independent form with eq. 2.4, and that eqs. 2.6 and 2.7, were used to evaluate the corresponding independent effects of the uncertainties in the structural period and strength, an equation to evaluate the simultaneous effect of the uncertainties on the vibration period and the strength, may be expressed as:

$$\nu\left(R_\mu,T_\mu\right)=\int_0^\infty\int_0^\infty \eta\left(R,T\right)p_{r,T}\left(R,T\left|R_\mu,T_\mu\right.\right)dR\,dT \tag{4.1}$$

where $\nu(R_\mu, T_\mu)$ is the rate of exceedence of the performance level which considers besides the uncertainties of the seismic demand those of the structural period and the strength, and $p_{r,T}(R, T\,|R_\mu, T_\mu)$ is the bivariate pdf characterizing the joint variability of these two structural properties.

$$p_{r,T}\left(R,T\right)=\frac{1}{2\pi RT\zeta_R\zeta_T\sqrt{1-\rho^2}}\exp\left[-\frac{q}{2\left(1-\rho^2\right)}\right] \tag{4.2}$$

where q is equal to

$$q=\left[\left(\frac{\ln\left(R\right)-\ln\left(R_\mu\right)}{\zeta_R}\right)^2-2\rho\left(\frac{\ln\left(R\right)-\ln\left(R_\mu\right)}{\zeta_R}\right)\left(\frac{\ln\left(T\right)-\ln\left(T_\mu\right)}{\zeta_T}\right)+\left(\frac{\ln\left(T\right)-\ln\left(T_\mu\right)}{\zeta_T}\right)^2\right] \tag{4.3}$$

and ρ is the correlation factor between period and strength ($-1\leq\rho\leq 1$).
As well as these two structural properties, there are others which also considerably influence structural performance under seismic action; this is the case of post yielding stiffness (particularly in considerable rigid structures) and the over-strength.

5. CONCLUSIONS

The consideration of the uncertainties in important structural properties as structural period of vibration and the strength on the estimation of the performance of structures subjected to design seismic demands through uniform hazard design spectra represents an advantage over the way in which up till now these demands have been defined in terms of a rate of exceedence of a seismic intensity, generally given spectral acceleration instead of a rate of exceedence of a performance index as is done in this paper. Furthermore, in this new definition of seismic demands, the uncertainties in all relevant structural parameters may be explicitly considered. Uncertainties in other important variables which have been considered in this investigation are those associated to the post yielding stiffness and to bias of the lateral strength to be larger than the nominal strength used in design. In this research, the relative importance of the different uncertain properties on the uniform hazard spectra was evaluated first by considering each of them independently and then all together. The obtained showed their important influence when considered all together (Niño, 2008).

In this paper it is shown that the proposed procedure is a significant improvement compared with those proposed in documents such as Vision 2000, SEAOC (2005), and that it is a useful tool for the research field and further developments of the philosophy and procedures for the performance based seismic design and retrofit of structures.

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