PREDICTION METHOD OF CRACK WIDTH AND SPACING IN REINFORCED CONCRETE BASED ON BOND ANALYSIS

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ABSTRACT :

This research presents a prediction method of crack width and spacing in reinforced concrete. The crack width can be calculated by numerical calculation based on bond analysis, however, it is necessary to assume the local bond stress versus slippage relationship for calculation. The local bond stress – slippage relationship is the dominant factor of controlling crack width and spacing, so to comprehend the relationship is absolutely imperative for predicting the crack width and spacing.

In this study, the local bond stress – slippage relationship obtained from pullout loading test is modeled by bi-linear model and the crack width and spacing is calculated by bond analysis based on the model. The analytical results of the crack width can represent the experimental results. The parametric bond analysis is performed to investigate the influence of elastic modulus and local bond stress versus slippage relationship on the crack width. In consequence, it is identified that not only elastic modulus but also the local bond stress – slippage relation ship influences the crack width. On the basis of these results, the calculation figure for crack width and spacing is proposed as the prediction method for them.

KEYWORDS: Bond Analysis, Bond stress-slippage relationship, Crack width, Crack spacing, AFRP

1. INTRODUCTION

The crack width is one of the important factors to the design of reinforced concrete structures. The cracks usually take place due to dry shrinkage, the expansion of silica minerals caused by alkali-aggregate reaction and rebar corrosion, etc. However, in Japan where earthquakes occur frequently, the crack caused by earthquake responses is the most important issue.

Recently, the study on the crack width has been continued and the many evaluate formulas of the crack width are proposed. However, almost of them are developed by the regression analysis of experimental data. Therefore, the results predicted by these formulas may vary by the type of specimens and the method of loading and so on. In addition, almost of reinforcement used in the experiments is deformed steel bar. Though deformed steel bar is generally utilized as reinforcement of the concrete structures, insulation property or non-corroded durability are required to a part of structures in some circumstances. For example, aramid fiber-reinforced polymer (AFRP) reinforcing bars provide excellent electrical isolation and high-specific strength, so AFRP reinforcing bars are applied for a part of structures. Therefore, the evaluate formulas which can predict not only for deformed steel bar but also other reinforcing bar are needed.

The local bond stress versus slippage relationship between the concrete and the reinforcing bar can give the crack width and spacing by bond analysis in which the equilibrium of axial forces and compatibility conditions between concrete and reinforcing bar is determined. This analytical method is not influenced by the types of reinforcing materials and the calculated value has universality. Therefore, to establish the prediction formula for crack width, bond analytical results are compared with experimental results and the difference of crack properties caused by the local bond stress - slippage relationship of each reinforcing bars is investigated.

2. BOND ANALYSIS

2.1. Local Bond Stress – Slippage Relationship

It is important to assume the local bond stress (τ) versus slippage (s) relationship for predicting the crack width and spacing. Generally the $\tau - s$ relationships are given by simple experiment. Figure 1 shows an example of experiment method. The specimen is set with steel plate. The monotonic loading test is performed to downward. In addition, Teflon sheet is set between specimen and steel plate to not restrict lateral displacement of concrete. Measurement items are tensile load and reinforcement slippage of free end.

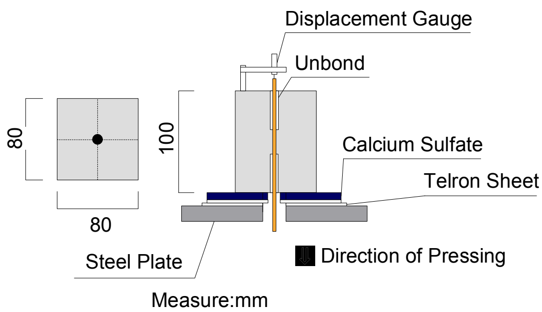

Figure 1 Method of pullout loading test

2.2. Proposal of Local Bond Stress – Slippage Relationship Model

It is necessary to present a model that can represent the local bond stress – slippage relationship for bond analysis. In this study, the $\tau - s$ relationship as shown in the below is proposed.

$$
\begin{aligned}
&\tau_x = k_1 \cdot s_x (s_x < s_{\max}), \tau_x = k_2 \cdot s_x (s_x > s_{\max}) \\
&k_1 = \frac{\tau_{\max}}{s_{\max}}, k_2 = \frac{\tau_{\max}}{s_{\max} - s_u} \\
&s_u = s_{\max} + \frac{2}{\tau_{\max}} \cdot \left(G_f - \frac{\tau_{\max} \cdot s_{\max}}{2} \right)
\end{aligned}
\tag{2.1}
$$

where, τ_x = local bond stress, τ_{max} = maximum local bond stress, s_x = slippage of reinforcement bar, s_{max} = slippage at τ_{max}, s_u = slippage at $\tau_x = 0$ $(x \neq 0)$, G_f = fracture energy (area enclosed by $\tau - s$ relationship)

τ_{max}, s_{max} and G_f are obtained directly from the experimental local bond stress versus slip relationship. Figure 2 shows an example of modeling the experimental $\tau - s$ relationship.

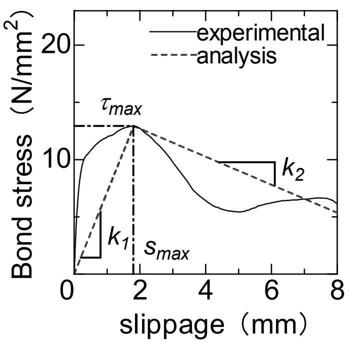

Figure 2 Example of modeling

2.3. Basic Equation for Bond Analysis

The following second differential equation expresses the bond between reinforcement and concrete neglecting concrete deformation:

$$\frac{d^2 s_x}{dx^2} = \frac{\phi}{E_s A_s} \tau_x \tag{2.2}$$

where, s_x = slippage of reinforcement, ϕ = perimeter of reinforcement, E_s = elastic modulus of reinforcement, A_s = area of reinforcement, τ_x = local bond stress

If τ_x is expressed as Eq.(2.1) that is the function of s, the slippage of arbitrary position of bar can be calculated by solving Eq.(2.2). However, the solution obtained by solving Eq.(2.2) is very complicate. Therefore, generally the solution of Eq.(2.2) is obtained in numerical calculations.

2.4. Method of Bond Analysis

Figure 3 shows an example of analysis target and the order of crack occurence. Figure 4 shows the algorithm of bond analysis. In numerical calculation, the difference between tensile load and minimum load at cracking section which corresponds to center position is solved. When the difference is over the tensile strength of concrete, the length of analysis target is divided in halves and bond analysis restarts based on renewed length. When the average bond stress passes a peak, bond analysis is terminated.

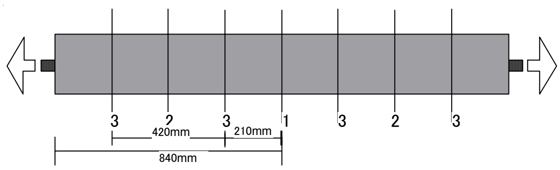

Figure 3 Example of bond analysis object and Flow of crack initiation

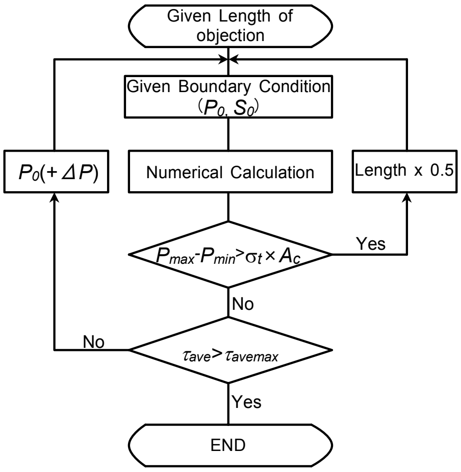

Figure 4 Algorithm of bond analysis

where, σ_t = concrete tensile strength, A_c = effective area of concrete, τ_{ave} = average bond stress, τ_{avemax} = maximum average bond stress, P_{max} = tensile load, P_{min} = minimum load, P_0 =initial load at cracking section, s_0 = initial slippage at cracking section

Figure 5 shows the bond analysis result using the local bond stress versus slip relationship shown in figure 2 and an experimental result. When the tensile load decreases sharply, crack takes place. Totally the bond analysis can almost represent the test result. The tensile load of both in analysis and in experiment is upper than that of bare reinforcement when the total deformation is same. This phenomenon is due to the tension stiffening effect. The tension stiffening effect is occurred when the concrete share the tensile load through bond stress. The efficiency of the share depends on the local bond stress – slippage relationship between concrete and reinforcement.

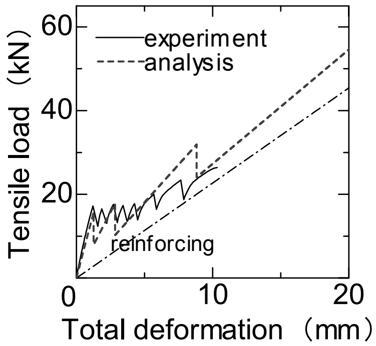

Figure 5 Result of bond analysis

3. CRACK WIDTH

3.1. Influencing Factor to Crack Width

Table 1 shows the mechanical properties of reinforcing bars to be analyzed for a parametric investigation. Figure 6 shows the two models of local bond stress – slippage relationship based on the mechanical properties in Table 1. Figure 7 shows the analysis results of crack width using the models in figure 6.

The analysis results of crack width in figure 7 show good agreement to the experimental results. The two analysis results in figure 7 are remarkably different. Elastic modulus and the local bond stress – slippage relationship have influence to crack width. Therefore the effectiveness of two factors is investigated by bond analysis.

Table 1 Mechanical properties of reinforcing bars

Kind	Diameter(mm)	Effective cross section area(mm^2)	Elastic modulus (GPa)
TE-F6	6.43	28.3	53.0
SD-D10	D10	71.3	197.0

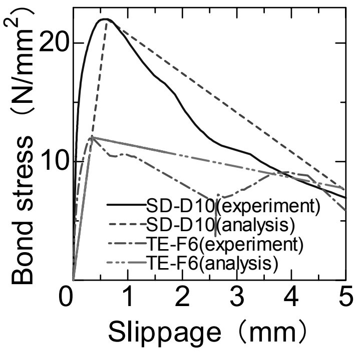

Figure 6 Model of local bond stress – slippage relationship

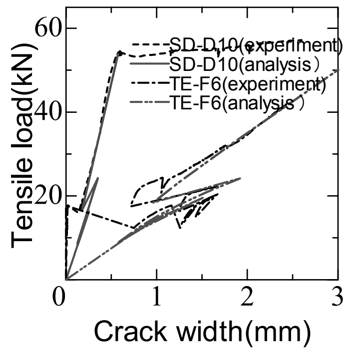

Figure 7 Analysis results of crack width

Table2 shows the two types of mechanical properties of reinforcing bars. Figure 8 shows the bond stress – slippage relation ships, model 1 and mode 2. The two types of model are different in k_1, k_2 and s_{max}. Figure 9 shows the analysis results of average stress of reinforcing – crack width relationship. The average stress of reinforcing is calculated by averaging the reinforcing stress from the point at cracking section to loaded end. In figure 9, if the local bond stress – slippage is the same, the crack width obtained from higher elastic modulus bar is small than that of lower elastic modulus when the average stress is the same. On the other hand, Figure 10 shows the analysis results of average strain – crack width relationship to remove the influence of elastic modulus. In Figure 10, if the model in Figure 6 is the same, the average strain – crack width relationship is also the same in spite of the difference of elastic modulus. However, if the model is not same, in the same average strain, the crack width of model 1 is smaller than that of model 2. Initial stifness, k_1 of model 1 is larger than model 2, consequently, model 1 can reduce the extent of crack width comparing with model 2.

Table 2 Mechanical properties of reinforcing bars

Kind	Diameter(mm)	Effective cross section area(mm^2)	Elastic modulus (GPa)
No.1	10	71.3	53.0
No.2	10	71.3	197.0

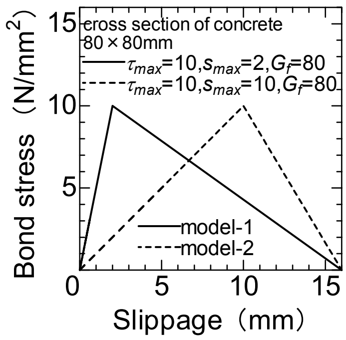

Figure 8 Model of local bond stress - slippage relationship

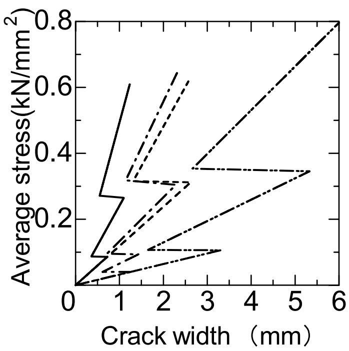

Figure 9 Average stress - crack width

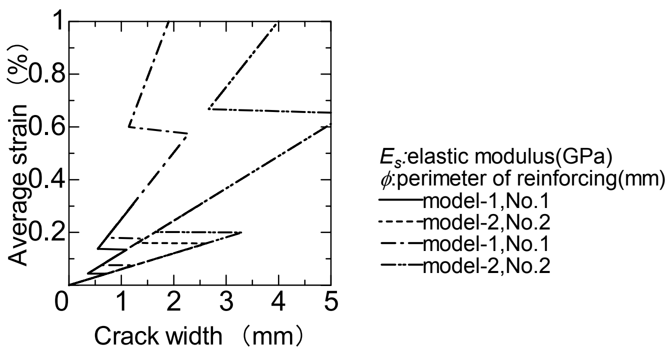

Figure 10 Average strain - crack width

3.2. The Calculation Figure for Crack Width

It is convenience to predict the crack width from reinforcing strain of loaded end. As to the variant reinforcing bars, if the local bond stress versus slippage relationships of them are the same, strain of reinforcing versus crack width relationships are also the same. Therefore, in this study, the predicting figures for crack width as to the two types of local bond stress – slippage models in Figure 6 are proposed.

Figure 11 shows the predicting figures. Each dotted line in Figure 11 shows the relationship between reinforcing strain and crack width according to the crack spacing. The white plots on the dotted lines show the crack occurrence in case of various values of *p*, reinforcement ratio (cross sectional area of reinforcing bar divided by cross sectional area of concrete. The full lines which link white plots show the crack width as to the arbitrary strain at loaded end.

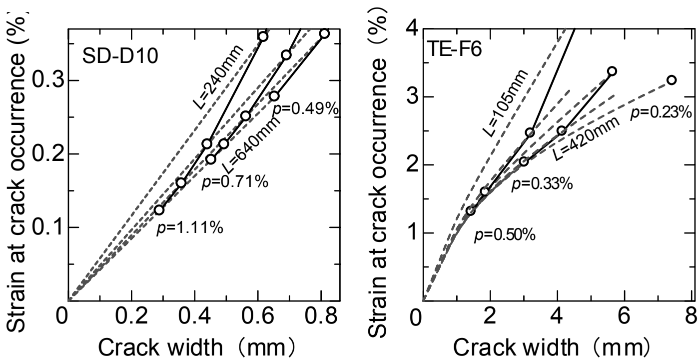

Figure 11 Strain at crack initiation - crack width relationship

4. CONCLUSIONS

1. The local bond stress versus slippage relationship obtained by experiment is modeled. The bond analysis based on the model can represent the tensile load versus total deformation relationship and tensile load versus crack width relationship.
2. The crack width is mainly influenced by elastic modulus and the local bond stress – slippage relationship.
3. The predicting figure for crack width is proposed. The crack width as to the arbitrary strain of reinforcing bar at loaded end can be estimated by the predicting figure.