CYCLIC DEFORMATION CAPACITY, RESISTANCE AND EFFECTIVE STIFFNESS OF RC MEMBERS WITH OR WITHOUT RETROFITTING

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ABSTRACT :

A database of over 2000 concrete beams, rectangular columns and walls is used to develop and calibrate rules for the calculation of the yield moment and curvature and the values of the chord rotation at a yielding end when the member yields or fails in flexure, in terms of the geometry and the mechanical characteristics of the member and its reinforcement, including the effect of retrofitting by Fibre Reinforced Plastics (FRP) or concrete jackets. The yield moment and the chord rotation at yielding are used then to determine the effective stiffness to the member yield point. These rules, some in an earlier version, have been adopted in Part 3 of Eurocode 8 for the seismic assessment and retrofitting of existing concrete buildings.

KEYWORDS: cyclic deformation capacity, effective stiffness, RC members, seismic retrofitting

1. INTRODUCTION

In displacement-based seismic design or assessment/retrofitting of buildings member deformation demands are the basis for the design of new members or the assessment/retrofitting of existing ones. So, quantitative information on member deformations at yielding and at ultimate conditions under cyclic loading is needed, as well realistic values of the effective stiffness to member yielding - a key parameter for the estimation of inelastic deformation demands through analysis. This paper gives expressions for the calculation of these parameters for concrete members, in terms of the geometry and the mechanical characteristics of the member and its reinforcement, including the effect of retrofitting by Fibre Reinforced Plastics (FRP) or concrete jackets. The deformation measure used here for members is the chord rotation, θ, at its end(s) (i.e., the angle between the normal to the end section and the chord connecting the member ends at the displaced position of the member).

2. DEFORMATIONS AND EFFECTIVE STIFFNESS AT YIELDING

2.1 Members with longitudinal bars continuous in the plastic hinge region

The force-deformation relation of a structure or a member in primary (monotonic) loading, or the envelope of its response to cyclic loading are normally approximated as multilinear (often bilinear, at least until resistance starts decaying fast with increasing deformation); the first is corner considered as effective yield point. For a structure this point is beyond first yielding of any member. For a member or section, it is slightly past yielding of the extreme tension steel or significant nonlinearity of the extreme compression fibres. Therefore, the “theoretical” yield moment, M_y, and curvature, φ_y, computed from 1st principles (the plane-section hypothesis, equilibrium, material σ-ε laws), as e.g., in Panagiotakos and Fardis (2001), should be multiplied by a factor of 1.025 for slender beams/columns or 1.03 for slender walls, derived from 1850 or 110 tests, respectively (Biskinis, 2007).

The member’s chord rotation when its end section yields, θ_y, comprises deformations due to: (a) flexure, (b) shear; and (c) the fixed-end rotation due to slippage (pull-out) of longitudinal bars from their anchorage beyond the end section, which, although it takes place outside the member length, shows up as member deformation.

Theoretically, the part of θ_y due to flexure is equal to $\varphi_y L_s/3$, where L_s is the shear-span (moment-to-shear ratio at the member end). However, diagonal cracking near the yielding end spreads yielding of the tension reinforcement up to the point where a diagonal crack starting from the end section intersects this reinforcement.

Diagonal cracking shifts the value of the tension reinforcement force from the section to which this value corresponds on the basis of the bending moment and axial force diagrams to one where the bending moment is lower. This is the "shift rule" in dimensioning the tension reinforcement at the Ultimate Limit State for bending with axial force. Although the magnitude of the shift depends on the amount of transverse reinforcement and on the inclination of the diagonal compression struts to the member axis, a default value of the shift equal to the internal lever arm, z, is commonly used. The shift increases the theoretical contribution of flexure in θ_y from $\varphi_y L_s/3$ to approximately $\varphi_y(L_s+z)/3$. The increase takes place only if diagonal cracking precedes flexural yielding of the end section. So, the term z is added to L_s only when the shear force that causes diagonal cracking (V_{Rc}) is less than the shear force at flexural yielding of the end section, $V_{My}= M_y/L_s$).

The fixed-end rotation due to pull-out of longitudinal bars from their anchorage beyond the end section is:

$$\theta_{y,slip} = \frac{\varphi_y d_{bL} f_y}{8\sqrt{f_c}} \tag{2.1}$$

where the yield stress of steel, f_y, and the concrete strength, f_c, are in MPa, and d_{bL} is the mean diameter of the tension reinforcement. Eq. 2.1 can be derived assuming that the bond stress (in MPa) along that anchorage length is equal to $\sqrt{f_c}$, i.e., lower than the ultimate bond stress of ribbed bars - corresponding to a slip equal to s = 0.6mm of about $2\sqrt{f_c}$ in unconfined concrete for "good" bond conditions according to the CEB/FIP Model Code 90. It gives good average agreement with values measured or inferred from about 160 tests (Biskinis, 2007):
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Using the values above for the contributions of flexure and the fixed-end rotation to θ_y, the following expressions were fitted by Biskinis and Fardis (2004) and Biskinis (2007) to about 1650 tests with yielding in flexure:

– for beams or rectangular columns:
$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0013\left(1 + 1.5\frac{h}{L_s}\right) + a_{sl}\frac{\varphi_y d_{bL} f_y}{8\sqrt{f_c}} \tag{2.2a}$$

– for rectangular walls:
$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.002 \cdot \max\left(0, \left(1 - \frac{L_s}{8h}\right)\right) + a_{sl}\frac{\varphi_y d_b f_y}{8\sqrt{f_c}} \tag{2.2b}$$

In the 1st term of Eqs. 2.2 the value of φ_y is the "theoretical" yield curvature (from 1st principles) times the correction factor of 1.025 or 1.03 for beams/columns or walls, respectively; a_v is a zero-one variable:
– $a_v = 0$, if $V_{Rc} > V_{My} = M_y/L_s$ and
– $a_v = 1$, if $V_{Rc} \leq V_{My} = M_y/L_s$;
In beams or columns the internal lever arm, z, is the distance between tension and compression reinforcement: $z= d-d_1$, while in rectangular walls is $z = 0.9d$. The shear force at diagonal cracking, V_{Rc}, may be taken from Eurocode 2: for b_w (width of web) and d (effective depth) in m, f_c in MPa, ρ_1 denoting the tension reinforcement ratio and the axial load N (>0 for compression, but for tensile N $V_{R,c}$=0) in kN, $V_{R,c}$ (in kN) is:

$$V_{R,c} = \left\{\max\left[180(100\rho_1)^{1/3},\ 35\sqrt{1+\sqrt{\frac{0.2}{d}}}\, f_c^{1/6}\right]\left(1+\sqrt{\frac{0.2}{d}}\right) f_c^{1/3} + 0.15\frac{N}{A_c}\right\} b_w d \tag{2.3}$$

h in the 2nd term of Eq. 2.2 is the section depth; in the 3rd term, (the fixed-end rotation from Eq. 2.1):
– $a_{sl} = 1$ if slippage of longitudinal bars from their anchorage zone beyond the end section is possible, or
– $a_{sl} = 0$ otherwise.

With M_y and θ_y established as outlined above, the effective elastic stiffness of the shear span, L_s, is:

$$EI_{eff} = \frac{M_y L_s}{3\theta_y} \tag{2.4}$$

2.2 Members with ribbed (deformed) bars and straight ends lap-spliced in the plastic hinge region

At least in European building practice the vertical bars of columns or walls are lap-spliced at floor levels, starting at the top surface of the floor slab. If lapping is short, member properties are adversely affected. About 130 tests on members with rectangular section and ribbed longitudinal bars with straight ends lapped starting at the end section show that in the calculation of the yield moment and curvature, M_y, φ_y, from 1st principles both bars in a pair of lap-spliced compression bars should count as compression reinforcement (Biskinis and Fardis, 2004, Biskinis, 2007). Moreover, if the straight lap length l_o is less than a value $l_{oy,min}$, given by:

$$l_{oy,min}=0.3d_{bL}f_{yL}/\sqrt{f_c}, \qquad (2.5)$$

with f_{yL} and f_c in MPa, then (Biskinis and Fardis, 2004, Biskinis, 2007):

- M_y and φ_y are calculated with yield stress of the tensile longitudinal reinforcement equal to $f_{yL}l_o/l_{oy,min}$;
- the 2nd term in the right-hand-side of Eqs. 2.2 for θ_y is multiplied by the ratio of the yield moment M_y as modified due to the lap splicing, to the yield moment outside the lap splice.

3. FLEXURE-CONTROLLED ULTIMATE CHORD ROTATION

3.1 Members with longitudinal bars continuous in the plastic hinge region

The ultimate deformation of a concrete member is defined conventionally. It is commonly considered to be reached when an increase in deformation cannot increase anymore the lateral force resistance of the member. With this definition, more than 1000 uniaxial cyclic tests and over 300 monotonic ones were used in Biskinis and Fardis (2004) and Biskinis (2007) to fit empirical models for the flexure-controlled ultimate chord rotation of concrete members having detailing for earthquake resistance ("conforming" to modern seismic codes) and continuous longitudinal bars. The 1st model, Eq. 3.1a, is for the total ultimate chord rotation, θ_u, while the others, Eqs. 3.1b, 3.1c, are for its plastic component, $\theta_u^{pl}=\theta_u-\theta_y$, with the elastic component, θ_y, from Eqs. 2.2.

$$\theta_u=\alpha_{st}(1-0.43a_{cy})\left(1+\frac{a_{sl}}{2}\right)\left(1-\frac{3}{8}a_{w,r}\right)\left(1-\frac{3}{13}a_{w,nr}\right)(0.3^{\nu})\left[\frac{\max(0.01;\omega_2)}{\max(0.01;\omega_1)}f_c\right]^{0.225}\left(\frac{L_s}{h}\right)^{0.35}25^{\left(\frac{\alpha\rho_s f_{yw}}{f_c}\right)}1.25^{100\rho_d} \qquad (3.1a)$$

$$\theta_u^{pl}=\alpha_{st}^{pl}(1-0.52a_{cy})\left(1+\frac{a_{sl}}{1.6}\right)\left(1-\frac{a_{w,r}}{2.5}\right)\left(1-\frac{a_{w,nr}}{6}\right)(0.25)^{\nu}\left(\frac{\max(0.01;\omega_2)}{\max(0.01;\omega_1)}\right)^{0.3}f_c^{0.2}\left(\frac{L_s}{h}\right)^{0.35}25^{\left(\frac{\alpha\rho_s f_{yw}}{f_c}\right)}1.275^{100\rho_d} \qquad (3.1b)$$

$$\theta_u^{pl}=\alpha_{st}^{hbw}(1-0.525a_{cy})(1+0.6a_{sl})\left(1-0.05\max\left(1.5;\min\left(10;\frac{h}{b_w}\right)\right)\right)(0.2)^{\nu}\left(\frac{\max(0.01;\omega_2)}{\max(0.01;\omega_1)}\frac{L_s}{h}\right)^{\frac{1}{3}}f_c^{0.2}25^{\left(\frac{\alpha\rho_s f_{yw}}{f_c}\right)}1.225^{100\rho_d} \qquad (3.1c)$$

In Eqs. 3.1:

- a_{st}, a^{pl}_{st}, a^{hbw}_{st}: coefficients for the steel type: $a_{st}= a^{pl}_{st}= 0.0185$ and $a^{hbw}_{st}= 0.022$ for ductile hot-rolled or heat-treated (tempcore) steel, and $a_{st}= 0.0115$,$a^{pl}_{st}= 0.009$, $a^{hbw}_{st}= 0.0095$ for cold-worked steel;
- a_{cy}: zero-one variable for the type of loading, $a_{cy}= 0$ for monotonic loading $a_{cy}= 1$ for cyclic;
- a_{sl}: zero-one variable for slip, defined already for the purposes of Eq. 2.2;
- $a_{w,r}$: zero-one variable for rectangular walls, $a_{w,r}= 1$ for rectangular walls, $a_{w,,r}= 0$ otherwise;
- $a_{w,nr}$: zero-one variable for non-rectangular walls, $a_{w,nr}= 1$ for walls with T-, H-, U- or hollow rectangular section and $a_{w,nr}= 0$ for other members;
- $\nu=N/bhf_c$ with b = width of compression zone, N = axial force, positive for compression);
- ω_1: mechanical reinforcement ratio of tension and web longitudinal bars, $\omega_1=(\rho_1 f_{yL}+\rho_v f_{yv})/f_c$;
- ω_2: mechanical reinforcement ratio of compression longitudinal reinforcement, $\rho_2 f_{y2}/f_c$;
- f_c: uniaxial (cylindrical) concrete strength (MPa)
- $L_s/h=M/Vh$: shear span ratio at the section of maximum moment;
- α: confinement effectiveness factor derived according to Sheikh and Uzumeri (1982):

$$\alpha = \left(1 - \frac{s_h}{2b_c}\right)\left(1 - \frac{s_h}{2h_c}\right)\left(1 - \frac{\sum b_i^2}{6b_c h_c}\right) \quad (3.2)$$

with s_h denoting the centreline spacing of stirrups, b_c and h_c the dimensions of the confined core to the centreline of the hoop and b_i the centreline spacing (along the section perimeter) of longitudinal bars (indexed by i) that are laterally engaged by a stirrup corner or a cross-tie.

– $\rho_s = A_{sh}/b_w s_h$: ratio of transverse steel parallel to the direction of loading;
– f_{yw}: yield stress of transverse steel;
– ρ_d: steel ratio of diagonal reinforcement (if there is any) in each diagonal direction;
– b_w: width of one web, even in cross-sections with one or more parallel webs.

Eqs. 3.1a, b or c provide practically the same accuracy. However, Eqs. 3.1b or c are more readily extendible to members with lap-splicing of longitudinal bars in the plastic hinge region (see Sect. 3.2) and/or wrapping of the end(s) with FRP (see Sect. 4) that affect differently θ_u^{pl} and θ_y and should be accounted for accordingly. Eq. 3.1c distinguishes walls and members with T-, H-, U- or hollow rectangular section via the aspect ratio, h/b_w, of each web.

3.2 Members with continuous longitudinal bars but without detailing for earthquake resistance

About 50 tests on concrete members with continuous ribbed (deformed) longitudinal bars and about 25 tests with continuous smooth (plain) longitudinal bars, but without seismic detailing (notably with not well closed and sparse stirrups) show that the values of M_y, θ_y and of the effective elastic stiffness to yielding, as well as the flexure-controlled ultimate chord rotation under monotonic loading are not affected by poor detailing and can still be described by the models described so far. However, the flexure-controlled cyclic ultimate chord rotation is adversely affected (Biskinis, 2007):

- Old-type members with ribbed (deformed) bars, cyclic loading:

$$\theta_u = \theta_{u,Eq.(3.1a)}/1.2, \text{ or} \quad (3.3a)$$
$$\theta_{u,pl} = \theta_{u,Eq.(3.1b) \text{ or } Eq.(3.1c)}/1.2 \quad (3.3b)$$

- Old-type members with smooth (plain round) bars, cyclic loading:

$$\theta_u = 0.9\,\theta_{u,Eq.(3.3a)}, \text{ or} \quad (3.4a)$$
$$\theta_{u,pl} = 0.9\,\theta_{u,Eq.(3.3b)} \quad (3.4b)$$

3.3 Members with ribbed (deformed) bars and straight ends lap-spliced in the plastic hinge region

The results of about 70 tests to flexure-controlled failure on members having rectangular section and ribbed longitudinal bars with straight ends lapped over a length l_o, show that the ultimate chord rotation is affected as follows (Biskinis and Fardis, 2004, Biskinis, 2007):

- As in the case of M_y, φ_y, θ_y in Sect. 2.2, both bars in a pair of lap-spliced compression bars count in the compression reinforcement ratio in ω_2 in Eqs. 3.1 for the ultimate chord rotation;
- If Eqs. 3.1b or c are used and the lap-length, l_o, is less than the value $l_{oy,min}$ given by Eq. 2.5, the rules outlined in Sect. 2.2 are applied for the calculation of θ_y (namely, φ_y is calculated with yield stress of tensile reinforcement equal to $f_{yL} l_o/l_{oy,min}$ and the 2nd term in the right-hand-side of Eqs. 2.2 is multiplied by the ratio of M_y as modified due to the lap splicing to the value of M_y outside the lap splice);
- If the straight lap length l_o is less than the following value $l_{ou,min}$:

$$l_{ou,min} = d_{bL} f_{yL}/[(1.05 + 14.5\alpha_l \rho_s f_{yw}/f_c)\sqrt{f_c}] \quad (3.5)$$

θ_u^{pl}, from Eqs. 3.1b or c, or Eqs. 3.3b or 3.4b if relevant, is multiplied by $l_o/l_{ou,min}$.

$$\theta_u^{pl} = (l_o/l_{ou,min})[\theta_u^{pl}{}_{Eqs..3.1b \text{ or } 3.1c}, \text{ or } \theta_u^{pl}{}_{Eqs..3.3b \text{ or } 3.4b}] \text{ if } l_o < l_{ou,min} \quad (3.6)$$

In Eq. 3.5 f_{yL}, f_{yw} and f_c are all in MPa; ρ_s is the ratio of transverse steel parallel to the direction of loading (as in Eqs. 3.1) and d_{bL} is the mean diameter of the tension bars (as in Eqs. 2.1, 2.2), while in rectangular columns:

$\alpha_l = (1\text{-}0.5s_h/b_o)(1\text{-}0.5s_h/h_o)n_{restr}/n_{tot}$, (3.7)

where:
- s_h: stirrup spacing,
- b_o, h_o: dimensions of the confined core to the hoop centreline,
- n_{tot}: total number of longitudinal bars along the cross-section perimeter;
- n_{restr}: number of lap-spliced bars which are laterally restrained by a stirrup corner or a cross-tie.

4. FRP-WRAPPED MEMBERS

4.1 FRP-wrapped RC columns with continuous vertical bars

4.1.1 Moment, chord rotation and effective stiffness at yielding

The yield curvature and moment, φ_y and M_y, of FRP-wrapped columns may still be computed from 1st principles, but with the unconfined concrete strength, f_c, replaced by the value f_c^* increased due to FRP confinement. If the Lam and Teng (2003) model is applied, which uses an effective strength of the FRP, $f_{u,f}$, equal to:

$$f_{fu,\,L\&T} = E_f(k_{eff}\varepsilon_{u,f}) \tag{4.1}$$

where E_f and $\varepsilon_{u,f}$ are the FRP's Elastic modulus and failure strain, respectively, k_{eff} is an FRP effectiveness factor, equal to k_{eff} = 0.6 for CFRP or GFRP according to Lam and Teng (2003); for AFRP k_{eff} is taken here the same as for CFRP and GFRP (k_{eff} = 0.85 is proposed in Lam and Teng (2003) for AFRP on the basis of few test results). The FRP effective strength is multiplied by an effectiveness factor for confinement by FRP (cf. Eq. 3.2), which, for rectangular sections with cross-sectional dimensions b_x, b_y and corners rounded to a radius R, is equal to:

$$a_f = 1 - \frac{(b_x - 2R)^2 + (b_y - 2R)^2}{3b_x b_y} \tag{4.2}$$

The increase of concrete strength according to Lam and Teng (2003) is not sufficient to capture the enhancement of yield moment due to confinement by FRP. The so-computed value of M_y is, on average, 7% less than the experimental value in about 165 FRP-wrapped test columns. So, a correction factor of 1.07 should be applied on the values of φ_y, M_y computed for FRP-wrapped columns from 1st principles and the Lam and Teng (2003) model. This applies also to the value of φ_y used in Eqs. 2.2 for the calculation of θ_y. Tests on columns that had suffered serious damage (ranging from yielding to exceedance of ultimate deformation) before repair and FRP-wrapping show that such repair and FRP-wrapping re-instates the full yield moment but cannot prevent an about 25% reduction in the value of the effective flexural stiffness to yielding, as given by Eqs. 2.1, 2.2.

4.1.2 Flexure-controlled deformation capacity

The ultimate chord rotation, θ_u, of FRP-wrapped members is expressed as the chord rotation at yielding, θ_y, computed as in Sect. 2.1 with the modifications in Sect. 4.1.1, plus a plastic part, θ_u^{pl}. It has been proposed by Biskinis and Fardis (2004) and Biskinis (2007) and adopted by Eurocode 8, Part 3 to extend the empirical model for θ_u, Eqs. 3.1b to FRP wrapped members by including in the exponent of the 2nd term from the end the effect of confinement by the FRP, adding to it the term $a_f\rho_f f_{f,e}/f_c$ where:
- $\rho_f = 2t_f/b_w$ is the geometric ratio of the FRP parallel to the loading direction,
- a_f is the confinement effectiveness factor of the section by the FRP, given by Eq. 4.2, and
- $f_{f,e}$, is the effective stress of the FRP:

$$f_{f,e} = \min\left(f_{fu,nom}; \varepsilon_{u,f}E_f\right)\left(1 - \min\left[0.5; 0.7\min\left(f_{fu,nom}; \varepsilon_{u,f}E_f\right)\frac{\rho_f}{f_c}\right]\right) \tag{4.3}$$

with $f_{fu,nom}$ and E_f denoting the nominal strength and the Elastic modulus of FRP and $\varepsilon_{u,f}$ a limit strain:

- $\varepsilon_{u,f}$ = 0.015 for CFRP or AFRP; and
- $\varepsilon_{u,f}$ = 0.02 for GFRP.

With this modification Eqs. 3.1b or c underpredict the results of about 95 tests of FRP-wrapped members by about 8.5% on average.

The proposal above may be improved as follows: the term added to the exponent of the 2nd term from the end of Eqs. 3.1b or c to reflect effective confinement by the FRP is:

$$\left(a\frac{\rho f_u}{f_c}\right)_{f,eff} = a_f \min\left[1.0; \min\left(f_{fu,nom}; \varepsilon_{u,f}E_f\right)\frac{\rho_f}{f_c}\right]\left(1-0.4\min\left[1.0; \min\left(f_{fu,nom}; \varepsilon_{u,f}E_f\right)\frac{\rho_f}{f_c}\right]\right) \quad (4.4a)$$

where the limit strain is always equal to $\varepsilon_{u,f}$ = 0.015. With this modification, Eqs. 3.1b or c underpredict the results of about 95 tests of FRP-wrapped members by about 5% on average and have smaller prediction scatter. An even better fit to these tests (underprediction of 2.5%) is achieved if the FRP-confinement term added to the exponent of the 2nd term from the end of Eqs. 3.1b or c is based on the Lam and Tend effective FRP strength of Eq. 4.1:

$$\left(a\frac{\rho f_u}{f_c}\right)_{f,eff} = a_f c_f \min\left[0.4; \frac{\rho_f f_{fu,L\&T}}{f_c}\right]\left(1-0.5\min\left[0.4; \frac{\rho_f f_{fu,L\&T}}{f_c}\right]\right) \quad (4.4b)$$

where c_f = 1.8 for CFRP and c_f = 0.8 for GFRP or AFRP.

The predictions are safe sided for members intact when wrapped with FRP, even when they do not have seismic detailing. However, in 16 members that were pre-damaged, then repaired, FRP-wrapped and re-tested cyclically, the modification of Eqs. 3.1b or c on the basis of Eqs. 4.3 or 4.4 gives an average overprediction of 4%. Pre-damage seems to have a noticeable adverse effect on the ultimate chord rotation of FRP-wrapped members.

4.2 FRP-wrapped RC columns with lap-spliced deformed (ribbed) vertical bars

As in the case of members without FRP-wrapping, in the calculation φ_y, M_y and of the plastic part of the flexure controlled ultimate chord rotation, θ_u^{pl}, both bars in a pair of lap-spliced compression bars should count as compression reinforcement. Moreover, if the straight lap length, l_o, is less than a minimum value $l_{oy,min}$, given by:

$$l_{oy,min}=0.2d_{bL}f_{yL}/\sqrt{f_c}, \quad (4.5)$$

then φ_y and M_y should be calculated with the yield stress of the tensile longitudinal reinforcement, f_{yL}, multiplied by $l_o/l_{oy,min}$. As in members without FRP-wrapping, the 2nd term in the right-hand-side of Eqs. 2.2 for θ_y should be multiplied by the ratio of the value of M_y as modified for the effect of lapping to its value without it. Experimental values of M_y and θ_y in about 25 FRP-wrapped columns with lap splices are in good average agreement with the predictions of these rules for the effect of bar lapping (Biskinis, 2007).

After correcting the value of θ_y for the effect of lap-splicing and FRP-wrapping as outlined above, the plastic part of the flexure controlled ultimate chord rotation, θ_u^{pl}, may be estimated as in Sect. 3.3 and Eq. 3.6:

$$\theta_u^{pl} = (l_o/l_{ou,min})[\theta_u^{pl}{}_{Eqs..3.1b\ or\ 3.1c}] \text{ if } l_o < l_{ou,min} \quad (4.6)$$

with $l_{ou,min}$ from:

$$l_{ou,min} = \frac{d_{bL}f_{yL}}{\left(1.05+14.5\frac{4}{n_{tot}}a_f\frac{\rho_f f_{f,e}}{f_c}\right)\sqrt{f_c}} \qquad (f_{yL}, f_{f,e}, f_c \text{ in MPa}) \quad (4.7)$$

where a_f and $f_{f,e}$ were defined via Eqs. 4.2, 4.3, $\rho_f = 2t_f/b_w$ is the geometric ratio of the FRP parallel to the loading direction and n_{tot} denotes the total number of lapped longitudinal bars along the cross-section perimeter. So, $4/n_{tot}$ is the fraction of the total number of lap splices confined by the FRP, as in rectangular columns only the four corner bars, are confined by the FRP wrapped around the corner. Note that Values of θ_u in about 25 FRP-wrapped test columns with lap splices show that Eqs. 4.6, 4.7 predict well on average the cyclic ultimate chord rotation.

The parallel to the substitution in Sect. 4.1.2 of Eqs. 4.4 for the term $\alpha_f \rho_f f_{f,e}/f_c$ with $f_{f,e}$ from Eq. 4.3 is to modify Eq. 4.7 as:

$$l_{ou,\min} = \frac{d_{bL} f_{yL}}{\left(1.05 + 14.5 \frac{4}{n_{tot}} \left(a \frac{\rho f_u}{f_c}\right)_{f,eff}\right)\sqrt{f_c}} \quad (f_{yL}, f_{f,u}, E_f, f_c \text{ in MPa}) \qquad (4.8)$$

with $(\alpha \rho f_u/f_c)_{f,eff}$ from Eqs. 4.4. However, although adding the term of Eqs. 4.4 to the exponent of the 2nd term from the end of Eqs. 3.1b or c, instead of the term $\alpha_f \rho_f f_{f,e}/f_c$ with $f_{f,e}$ from Eq. 4.3 improves the accuracy of the prediction of θ_u for members with continuous bars and FRP wrapping, it doesn't do so if the bars inside the wrapping are lap-spliced. For such members Eq. 4.7 seems to be the best alternative.

Note that for the value of θ_u^{pl} before its reduction due to the lap splice the exponent of the 2nd term from the end reflects confinement by the steel ties as well as by the FRP (i.e., $\alpha_f \rho_f f_{f,e}/f_c$ or $(\alpha \rho f_u/f_c)_{f,eff}$ is added to $\alpha \rho_s f_{yw}/f_c$ due to confinement by the steel ties), while confinement of lapped bars by the FRP alone and not by the steel ties is taken into account in Eqs. 4.7, 4.8.

All rules above were developed and calibrated on the basis of members with FRP wrapping applied over a length exceeding that of the lap. Accordingly, they should be applied only when such wrapping extends over a length from the end section of the member at least, e.g., 125% of the lapping.

5. MEMBERS WITH CONCRETE JACKETS

From a database of about 55 tests on members with or without lap splices which had been retrofitted via concrete jacketing, simple rules and expressions have been developed for the calculation of key properties of columns (Bousias et al, 2007, Biskinis, 2007). According to these rules, the jacket may be considered as integral with the old member; the jacketed member is taken with:

1. external cross-sectional dimensions and f_c value those of the jacket,
2. the axial load applied on the full section of the jacketed member,
3. tension and compression reinforcement those of the jacket (in walls the jacket reinforcement is supplemented with the old vertical bars near the end of the wall section, taking into account differences in yield stress with the new reinforcement and any lap splicing at floor levels),
4. longitudinal bars of the old member between the new tension and compression reinforcement included in a "web" reinforcement ratio (supplementing any longitudinal bars of the jacket between the tension and compression reinforcement)
5. confinement provided only by to the transverse reinforcement in the jacket.

The idea behind points 1 and 2 is that, for common ratios of jacket thickness to depth of the jacketed section, when yielding takes place and a plastic hinge forms at the end section, the compression zone there is almost fully in the jacket, carrying the full axial load. Also, it is the jacket that mainly controls the shear resistance and the bond along the longitudinal reinforcement of the jacket.

If the jacket longitudinal bars do not continue for full anchorage past the member end section but stop there, the yield moment and the flexure-controlled cyclic deformation capacity of the jacketed member may be estimated

considering the jacketed member as monolithic, with:
i. cross-sectional dimensions, longitudinal reinforcement and f_c value those of the old member,
ii. the axial load applied on the section of the old member,
iii. confinement provided by the jacket and its transverse steel (with the transverse reinforcement ratio, $\rho_s = A_s/s_h b_w$ determined using the value of A_s/s_h in the jacket and the width of the old column as b_w and the confinement effectiveness factor a taken equal to 1.0, as if the jacket serves as a cushion distributing confinement to the full extent of the old section).

With these rules the yield moment and curvature, M_y, φ_y, of the jacketed member are, on average, well estimated. The chord rotation of the jacketed member at yielding, θ_y, is on average underestimated by about 5% (and the effective stiffness to yielding, EI_{eff}, overestimated by about 5%). This 5% difference should be taken into account. Any special connection measures at the interface do not seem to have a systematic effect on M_y or EI_{eff} of the jacketed member.

The ultimate chord rotation, computed as the sum of $1.05\theta_y$ plus the value of θ_u^{pl} from Eqs. 3.1b or c, both for the equivalent monolithic member with properties according to 1 to 5 or i to iii above, as relevant, underestimates by 8% on average the experimental value. Unlike the yield moment and the effective stiffness, the experimental ultimate chord rotation, θ_u, exhibits a statistically significant effect of bonding measures at the interface of the jacket and the old member It is underestimated by the proposed rules by 27% on average for the few specimens employing either dowels at the interface, or U-bars welded to the new and the old longitudinal bars, but only by 8% for the more numerous specimens with neither of these measures. Welded U-bars seem to be slightly more beneficial for the ultimate chord rotation than dowels, possibly due to their anti-buckling action. Roughening of the old surface alone does not seem to positively affect the ultimate chord rotation. So, neglecting the favourable effect of any positive connection measure at the interface of the old and the new concrete is therefore safe-sided for the ultimate chord rotation, θ_u.

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