RESEARCH ON COLLISION OF BEAM-TYPE STRUCTURES BASED ON HERTZ-DAMP MODEL

YANG YONGQIANG[1], LI SHUANG[2] AND XIE LILI[1,2]

[1] *Institute of Engineering Mechanics, China Earthquake Administration, Harbin. China*
[2] *School of Civil Engineering and Architecture, Harbin Institute of Technology, Harbin. China*
Email: yq_yang@foxmail.com

ABSTRACT :

Collision of structural components is one of the leading reasons which result in structure collapse during earthquakes. Because of the limitations of physical experiment, numerical method becomes an effective strategy in solving collision problem. The paper derived a constraint condition on displacement imposed by collision and established a dynamic equation for general collision system. The presented method, which combined Hertz-damp model with finite element method, can account for the nonlinear stiffness as well as the energy dissipated during collision procedure. A computer program is developed and a numerical example of beam-type structure is provided to verify the method. It is shown that the theory is correct based on the gained time-history of collision load and structural displacement responses. This study provides a useful theoretical basis for collision analyses of beam-type structures.

KEYWORDS: collision, Hertz-damp model, beam-type structure, nonlinear stiffness, energy dissipated

1. INTRODUCTION

Structural collision has been reported from most of major earthquakes affecting metropolitan areas of the world, which can result in or contribute to building damage. The investigation of the earthquake that struck Mexico City in 1985 showed that collision was present in over 40% of 330 collapsed or severely damaged buildings surveyed, and in 15% of all cases it led to collapse. During the 1989 Loma Prieta earthquake, collision was present over a wide geographical area including the cities of San Francisco, Oakland, Santa Cruz and Watsonville and observed more seriously at sites within 90 km from the epicenter. Shear failure of column, partial collapse of the wall, and veneer spalling were reported at many sites in downtown of San Francisco. Reconnaissance reports from the 1995 Kobe earthquake identify pounding as a major cause of fracture of the bearing supports and potential contributor to the collapse of several bridge decks. Collision between a six-story building and two-story building in Golcuk, Turkey during the 1999 Kocaeli earthquake contributed to column failure above the third-floor slab in the taller building, and shear failure of two second-floor piers in the smaller building. Pounding of abutments and deck joint was also observed in several highway bridges during the same earthquake. These earthquakes illustrated the significant hazard of collision during earthquake (Kasai and Maison, 1997; Muthukumar and DesRoches, 2006).

At the beginning of new century, Xie Lili (2000) had proposed the final goals of earthquake engineering: first, to reduce earthquake disaster, the sign is to promote the seismic ability of cities. Second, to clear the damage of earthquake and the damage mechanics of structure, the sign is to realize replay of seismic disaster, which couldn't be solved by existing research achievements. Because the lab experiments such as shaking table test and pseudo-dynamic test cannot replay seismic disaster, numerical simulation becomes an effective strategy in solving this problem. Thus, numerical simulation of seismic damage has been promoted to the research focus in seismic engineering field. Building collapse is one of typical seismic damage. For the numerical simulation of building collapse, the related research—numerical simulation of structural collision became an important subject which should be studied firstly. However, most of the previous studies were focus on pounding of adjacent structures, and few studies were about collision between different portions of one building during

Supported by: Project 90715038 supported by NSFC; Joint Seismological Science Foundation of China (C07025)

collapse. Although good simulated result was obtained to this problem, Zhang and Liu (1999, 2004) could not get the real time-history curve of collision load for the limitation of the method used by them. Therefore, this paper proposes to develop a method in which the real time-history curve of collision load can be gained. And this paper studies most common collision process of beam-type structures during structural collapse, and hopes to provide support to study of numerical simulation of building collapse.

2. COLLISION MODELS

Collision is a highly non-linear phenomenon, which results in several uncertainties in its mathematical modeling. Primarily, two modeling techniques have been used—the Stereomechanical Approach and the Contact Element Approach. The stereomechanical approach assumes that impact is instantaneous and uses the momentum conservation principle and the coefficient of restitution e to modify the velocities of the colliding bodies after impact. The theory assumes a direct, central impact and does not consider transient stresses and deformations in the impacting bodies. The stereomechanical approach divides the collision process into before-collision stage and after-collision stage, and neglects collision stage. Although the model cannot give out the time-history curve of collision load and is no longer valid if the impact duration is large enough so that significant changes occur in the configuration of the system, it needs fewer impact parameters and can be easily solved. The contact element approach is a force-based approach, where a contact element is activated once impact occurs. Typically, a linear spring is used to represent the axial stiffness of the colliding structures and the force during impact. This model can give out the time-history curve of collision load. But this model needs more impact parameters, which takes difficult to the solution (Muthukumar and DesRoches, 2006).

2.1. Stereomechanical model

Since this is not a force-based approach, the effect of impact is accounted by adjusting the velocities of the colliding bodies. The coefficient of restitution (*e*) is defined as the ratio of separation velocities of the bodies after impact to their approaching velocities before impact. The equation of this model can be pressed as

$$\begin{cases} v_1' = v_1 - (1+e)\dfrac{m_2 v_1 - m_2 v_2}{m_1 + m_2} \\ v_2' = v_2 + (1+e)\dfrac{m_1 v_1 - m_1 v_2}{m_1 + m_2} \end{cases} \tag{2.1}$$

where v_1', v_2' are the velocities of the colliding masses (m_1, m_2) after impact, v_1, v_2 are the velocities before impact and e is the coefficient of restitution. The coefficient of restitution e can be gotten through free fall experiment, and generally the value range of e is 0.5-0.75 during impact. Azevedo (1996) pointed out that e is 0.65 for reinforced concrete impact specially, and the value is used to numerical simulation of impact by many researchers.

2.2. Contact element model

There are several kinds of contact element approach, such as linear spring model, Kelvin model, Hertz model, and Hertz-damp model, of which Hertz-damp model based on a nonlinear spring with a nonlinear hysteresis damper can account for the nonlinear stiffness as well as the energy dissipated during collision process. Muthukumar, DesRoches (2006) and Jankowski (2005) considered that Hertz-damp model is suitable for simulation of collisions caused by earthquakes after comparing different collision models. Contact force-displacement relationship for Hertz-damp model is illustrated in figure 1, and the contact force can be pressed as

$$\begin{cases} F_c = k_h(u_1 - u_2 - g_p)^n + c_h(\dot{u}_1 - \dot{u}_2), & u_1 - u_2 - g_p \geq 0 \\ F_c = 0, \quad u_1 - u_2 - g_p < 0 \end{cases} \tag{2.2}$$

where k_h is the spring stiffness, c_h is the damping coefficient, $u_1 - u_2 - g_p$ is the relative penetration, $\dot{u}_1 - \dot{u}_2$ is the penetration velocity. Based on the Hertz Contact Theory, the spring stiffness k_h can be calculated with material properties of colliding objects and characteristics of contact surfaces. Damping coefficient c_h can be pressed as

$$c_h = \xi(u_1 - u_2 - g_p)^n \tag{2.3}$$

Equating the energy loss during impact to the energy dissipated by the damper, an expression for the damping constant ξ can be found in terms of the spring stiffness k_h, the penetration velocity $\dot{u}_1 - \dot{u}_2$, the coefficient of restitution e and the relative approaching velocity $v_1 - v_2$ as follows:

$$\xi = \frac{3k_h(1-e^2)}{4(v_1 - v_2)} \tag{2.4}$$

Hence, the contact force can be expressed as

$$\begin{cases} F_c = k_h(u_1 - u_2 - g_p)^n \left[1 + \dfrac{3(1-e^2)}{4(v_1 - v_2)}(\dot{u}_1 - \dot{u}_2)\right], & u_1 - u_2 - g_p \geq 0 \\ F_c = 0, \quad u_1 - u_2 - g_p < 0 \end{cases} \tag{2.5}$$

This paper selected Hertz-damp model to simulate structural collision for the importance of history-time of contact force and its good effects.

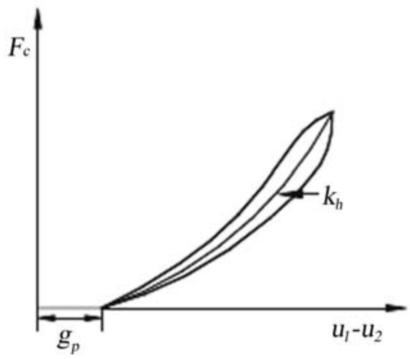

Figure 1 Contact force-displacement relationship for Hertz-damp model

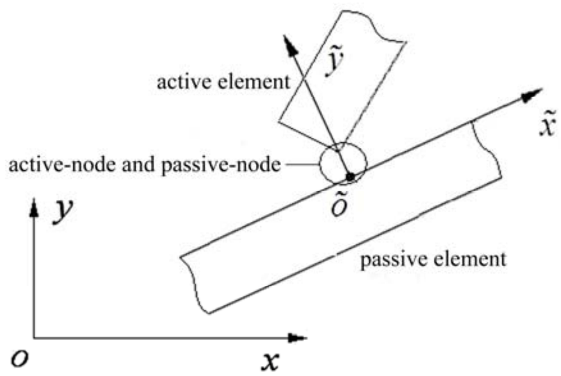

Figure 2 Collision coordinate system

3. DYNAMIC EQUATION FOR COLLISION SYSTEM OF GENERAL STRUCTURES

The convergence condition for collision problems is always non-overlapping of material on contact surfaces. Dynamic equation of structure is used to simulate structural collision after adding constrains according to characteristics of different collision models. As to Hertz-damp model used in this study, it is to add displacement constraints to the equation. The history-time of contact force and the response of colliding structures can be achieved through solving the equations.

3.1. Constraint conditions imposed by collision

Collision-pair was consisted of active-node of active element and passive-node of passive element, as shown in figure 2. Impact model can be classed into two types: point-line impact (two dimensions) and point-surface impact (three dimensions), this paper used point-line impact model. Took the contact point as the origin, the paper set up collision coordinate system along normal direction and tangential direction of contact surface. At the beginning of collision, the gap between the two colliding objects was zero, and the displacement condition of Hertz-damp model can be expressed as

$$\{\tilde{u}\}_1 - \{\tilde{u}\}_2 - \{\tilde{\delta}\} = 0 \tag{3.1}$$

where $\{\tilde{u}\}_1$, $\{\tilde{u}\}_2$ are the displacement vectors of active-node and passive-node, $\{\tilde{\delta}\}$ is the compression deformation vector , wave line upper vectors means under collision coordinate system.

It is necessary to consider propagation process of stress wave in simulating structural collision in theory. But the size of structures in civil engineering is generally smaller relative to the velocity of stress wave, sp the travel time is very short and it is reasonable to neglect the travel time (Zhang, 1999). According to element displacement shape function, the displacement of collision points can be expressed by elements node displacement under elemental coordinate system. Then Equation (3.1) is changed into

$$\left[\tilde{T}\right]^{\mathrm{T}}\left([T]_1 [N]\big|_1 \{\bar{q}\}_1^e - [T]_2 [N]\big|_2 \{\bar{q}\}_2^e - \{\delta\}\right) = 0 \tag{3.2}$$

where $\left[\tilde{T}\right]$ is the transformation matrix from the collision coordinates to global coordinates; $[T]_1$ and $[T]_2$ are the transformation matrix from the active elemental coordinates to global coordinates and the transformation matrix from the passive elemental coordinates to global coordinates; $[N]\big|_1$ and $[N]\big|_2$ are the value matrix of active and passive element displacement shape function at collision point; $\{\bar{q}\}_1^e$ and $\{\bar{q}\}_2^e$ are the node displacement vectors under each elemental coordinate system; $\{\tilde{\delta}\}$ is the compression deformation vector under global coordinate system. Then Equation (3.2) can be expressed in elements node displacement under global coordinates as follows:

$$\left[\tilde{T}\right]^{\mathrm{T}}\left[\left([T]_1 [N]\big|_1 \left[\bar{T}\right]_1^{e,\mathrm{T}} [T]_1^{e,\mathrm{T}} - [T]_2 [N]\big|_2 \left[\bar{T}\right]_2^{e,\mathrm{T}} [T]_2^{e,\mathrm{T}}\right)\{q\} - \{\delta\}\right] = 0 \tag{3.3}$$

where $\left[\bar{T}\right]_1^e$ and $\left[\bar{T}\right]_2^e$ are transformation matrix of active and passive element displacement vectors from the elemental coordinates to global coordinates; $[T]_1^e$ and $[T]_2^e$ are the transformation matrix from active and passive element node displacement vectors to global node displacement vector. Here assume

$$[R_s] = \left[\tilde{T}\right]^{\mathrm{T}}\left([T]_1 [N]\big|_1 \left[\bar{T}\right]_1^{e,\mathrm{T}} [T]_1^{e,\mathrm{T}} - [T]_2 [N]\big|_2 \left[\bar{T}\right]_2^{e,\mathrm{T}} [T]_2^{e,\mathrm{T}}\right)$$

Equation (3.3) can be simplified as

$$[R_s]\{q\} - \{\tilde{\delta}\} = 0 \tag{3.4}$$

Equation (3.4) is constraint conditions imposed by a single collision. Integrating the constraint conditions imposed by all collision, the constraint conditions became as

$$[A]_s\{q\}-\{\delta\}_s=0 \tag{3.5}$$

where $[R]_s$ is the total constraint coefficient matrix of colliding; $\{q\}$ is the global displacement vector; $\{\delta\}_s$ is the global compression deformation vector. Equation (3.5) is the expression of displacement constraint imposed by collision.

3.2. Dynamic equation

Using D'Alembert Principle to establish structural dynamic equation without considering collision, as follows:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F\} \tag{3.6}$$

Expression of collision force can be deduced in the same method with displacement constraint condition, as follows:

$$\{F_c\}^E=[T]^e[\bar{T}]^e[N]\|^{\mathrm{T}}[T]^{\mathrm{T}}[\tilde{T}]\{\tilde{F}_c\} \tag{3.7}$$

where $\{\tilde{F}_c\}$ is the collision load under the elemental coordinate system; $\{F_c\}^E$ is the equivalent nodal load. Integrating the all equivalent nodal load, Equation (3.8) can be gained

$$\{F_c\}^E=\sum\{F_c\}_i^E\text{ , }\quad i=1,2,3,\cdots \tag{3.8}$$

Dynamic equation considering collision force can be expressed as

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F\}+\{F_c\}^E \tag{3.9}$$

Combining Equation (3.5) and (3.9)，dynamic equation of colliding system can be expressed as

$$\begin{cases}[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F\}+\{F_c\}^E\\ [R]_s\{q\}-\{\delta\}_s=0\end{cases} \tag{3.10}$$

For convenience of solving, the form of equation was transferred as

$$\begin{bmatrix}M & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{q}\\ \ddot{\delta}_s\end{Bmatrix}+\begin{bmatrix}C & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\dot{q}\\ \dot{\delta}_s\end{Bmatrix}+\begin{bmatrix}K & 0\\ R_s & -I\end{bmatrix}\begin{Bmatrix}q\\ \delta_s\end{Bmatrix}=\begin{Bmatrix}F\\ 0\end{Bmatrix}+\begin{Bmatrix}F_c\\ 0\end{Bmatrix}^E \tag{3.11}$$

Structural collision can be simulated by solving Equation (3.11). Equation (3.11) is based on finite element method, so it meets the requirement just as the displacement and deformation are small. If some member separated from the structure, displaced large and collided with the structure, Equation (3.11) is no longer suitable. The article separates the calculation process of displacement of separated member into two stages: a. when colliding, since the collision time is very short, the displacement of separated member is small during this stage, so Equation (3.11) is suitable; b. when no colliding, the displacement of separated member is large, Equation (3.11) is unsuitable, but the deformation of it is small, so its motion can be calculated by Kinematics Equation approximately.

3.3. Solution of dynamic equation

Among the numerical solutions of dynamic equation, Newmark-β method is most widely applied. When $\gamma \geq 0.5$ and $\beta \geq 0.25(0.5+\gamma)^2$, Newmark-$\beta$ method is convergent unconditionally. Selection of integral step Δt is dependent on the solution accuracy. Concretely speaking, it is determined by the cycles of several vibration models which contribute to the main response of structure when no collision. When colliding, for the collision time is short, the collision stage may be finished in one integral step Δt above mentioned and cannot get time-history curve of collision load, so Δt must be small enough during this stage. According to the experience, the value rang of Δt is $(0.001 \sim 0.005)/c$, in the equation c is the velocity of stress wave, $c = \sqrt{EI/\rho A}$, where EI is the sectional flexural stiffness of contact element, ρ is the density of contact element, A is the sectional area of contact element (Xing and Zhu, 1995).

4. MODEL VERIFICATION

For the verification of the above theory, the paper selected a typical case of collision, as shown in figure 3. The substructure is a simply-supported beam, its parameters as: the standard compressive strength $f_{ck} = 13.4\text{N/mm}^2$, the span length $L = 4.0\text{m}$, the section size $200\text{mm} \times 600\text{mm}$, the density $\rho = 2500\text{kg/m}^3$, the damping constant $\xi = 0.05$, the coefficient of restitution $e = 0.65$, Rayleigh Damping. The parameters of dropping beam as: the length $L_0 = 2.0\text{m}$, the dip angle $\theta = 45^\circ$, the others are the same with the above. The gravity center of dropping beam corresponds to the center of substructure in horizontal direction, the nearest gap $h = 0.3\text{m}$.

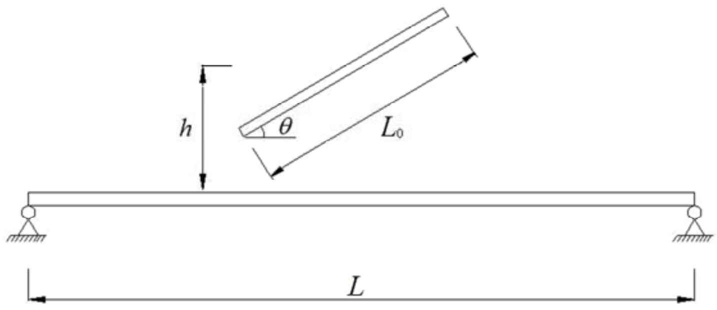

Figure 3 Collision of beam-type structures

A computer program was developed to simulate the example numerically by the above theories, and the results were shown as in figure 4 and figure 5. In the figures, the left end of the dropping beam collides with the substructure at *t*=25ms, then the dropping beam rebounds and rotates. Because the velocity of rotation is slow, the left end collides with the substructure again at *t*=55ms. The two collisions accelerates the rotational velocity of the dropping beam, and the right end collides with the substructure at *t*=71ms. The collision loads and the duration times of the three contacts are about 350kN to 550kN and 4 to 5ms that are close to the experimental results by Jan G.M. Van Mier (1992). For the rotation of the dropping beam, the angle between dropping beam and the substructure becomes small. Accordingly, the equivalent mass of the end decreases, that reduces the collision load and shortens the duration time. In addition, because collision can result in energy dissipation and transfer energy from the dropping beam to the substructure, the rebound height and the peak value of collision load decrease with the increase of collision times. The method is verified by well agreement between the results of numerical simulation and the real collision process.

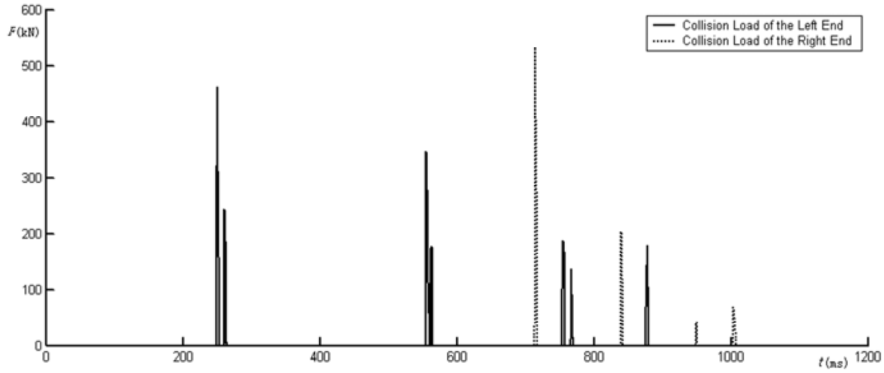

Figure 4 History-time curve of collision-load

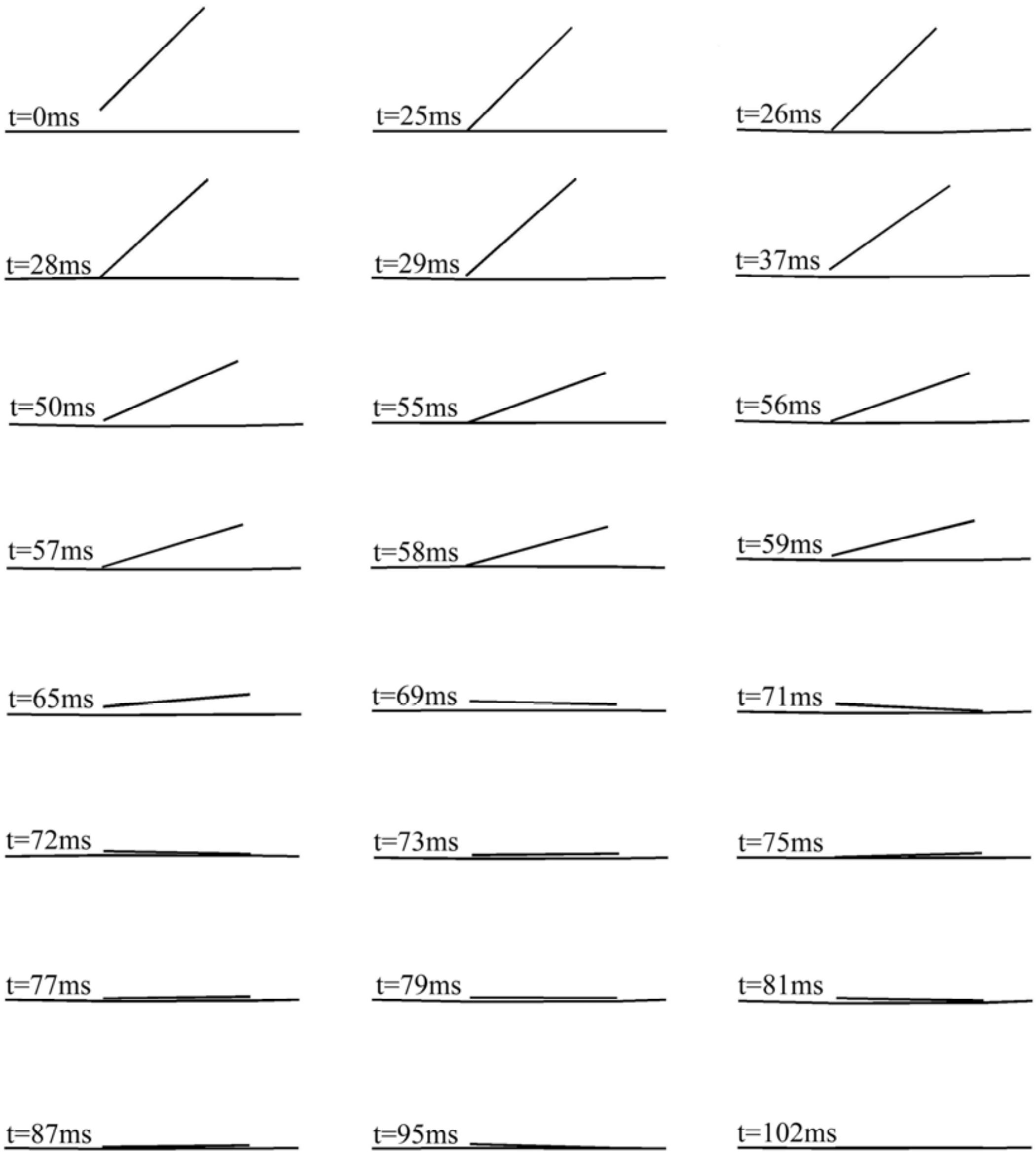

Figure 5 Motion locus of collision system

5. CONCLUSIONS

The paper simulated the collision process of beam-type structures based on Hertz-damp model and finite element method. The presented method can account for the influence of mass distribution of the dropping beam as well as the nonlinear stiffness and the energy dissipated during collision procedure. The presented model, which is used to simulate the collision during the plane frame structure collapse, can be generalized to spatial case. The study is limited to the normal collision simulation and how to simulate the tangential collision and the material fracture caused by collision is what needed further researches.

REFERENCES

Kazuhiko Kasai and Bruce F. Maison (1997). Building pounding damage during the 1989 Loma Prieta earthquake. *Engineering Structures* **19:3**, 195-207.

Susendar Muthukumar and Reginald DesRoches (2006). A Hertz contact model with non-linear damping for pounding simulation. *Earthquake Engineering and Structural Dynamics* **35:7**, 811-828.

Xie Li-Li (2000). Some challenges to Earthquake Engineering in New Millennium. *China-U.S.A. Millennium Symposium on Earthquake Engineering*.

Zhang Leiming and Liu Xila (2004). Analysis of pounding between adjacent structures. *Journal of Shanghai Jiaotong University* **38:6**, 941-946.

Zhang Leiming (1999). Research on collapse mechanisms of structures under disaster load. [Doctoral dissertation]. Beijing, Tsinghua University.

Azevedo J. and Bento R. (1996). Design criteria for buildings subjected to pounding. *Eleventh World Conference of Earthquake Engineering*. Acapulco, Mexico, 23-28 June 1996, Paper No. 1063.

Jankowski R (2005). Non-linear viscoelastic modeling of earthquake-induced structural pounding. *Earthquake Engineering and Structural Dynamics* **34:6**, 595-611.

XingYifeng and Zhu Dechao (1995). The feasibility of the determination of rigid/elastic impact loads between structures with the method of mode superposition. *Chinese Journal of Theoretical and Applied Mechanics* **27:5**, 560-566.