The 14th World Conference on Earthquake Engineering
October 12-17, 2008, Beijing, China

Investigation Into The Floor Diaphragm Flexibility In Rectangular Reinforced Concrete Buildings And Error Formula

M. Moeini [1], B. Rafezy [2] and W. P. Howson [3]

[1] *M. Sc of Structural Engineering, The Sahand University of Technology, Tabriz, Iran*
[2] *Assistant Professor, Dept. of Civil Engineering, The Sahand University of Technology, Tabriz, Iran*
[3] *Associate Professor, Cardiff School of Engineering, Cardiff University, Cardiff, UK*
Email: Ma.moeini@Gmail.com, Rafezyb@sut.ac.ir , Howson@cf.ac.uk

ABSTRACT :

Building structures are typically designed on the assumption that the floor systems serve as rigid diaphragms that span between the vertical resisting elements. Such an assumption is normally perfectly adequate for the seismic analysis of most buildings, but some structural forms, typically those comprising long, thin floor plans and perimeter lateral resisting elements, can exhibit significant in plane flexibility in their floor systems. The dynamic behavior of this latter class of structures is dissimilar to the behavior expected of typical structures and can lead to unexpected force and drift patterns. The main purpose of this paper is to use the finite element method to investigate the influence of floor diaphragm flexibility on the behavior of concrete structures. Initially a parametric study is undertaken on a variety of reinforced concrete structures with rectangular plan form, perimeter shear walls and slabs that contain openings. In the second part of the paper, a number of response spectrum analyses are performed on the structures assuming both rigid and flexible diaphragm assumptions. A regression analysis is then performed on the results to obtain an error formula that can be used to estimate the error involved in using the rigid diaphragm assumption on structures similar to those tested.

KEYWORDS: Rigid diaphragm, Flexible diaphragm, Finite element, Response-spectrum analysis.

1. INTRODUCTION

Floor diaphragms are essential structural elements in the lateral force resisting system of building structures. In the analysis of multistory buildings subjected to lateral loads, a common assumption is that the floor system undergoes no deformation in its own plane [1, 2]. At the mass center of each rigid floor, there is a master node having three degrees of freedom to represent the two in-plane translations and one out-of-plane rotation of all the other nodes or so-called slave nodes. These slave nodes contain three degrees of freedom, two in-plane rotations and one out-of-plane translation [1, 2]. Muto [3] used a beam with bending and shear deformation effects to simulate the behavior of flexible floors. Jain [4] also used this beam to evaluate the effect of flexible floors on the seismic response of buildings. Saffarini and Qudaimat [5] analyzed 37 RC[1] buildings to compare the difference between rigid-floor and flexible-floor analyses. They found that the rigid-floor assumption is accurate for buildings without shear walls, but it can cause errors for building systems with shear walls. A quantitative investigation of the difference between flexible and rigid floor analyses of buildings with shear walls was not given and appears to be largely absent from the literature. Ju and Lin [6] confirmed the results of [5] and performed a quantitative investigation.
They then proceeded to deduce an error formula using a regression analysis of the rigid-floor and flexible-floor analyses from 520 rectangular, U-shaped, and T-shaped buildings. The effect of slab openings was not considered and also appears to be absent in the literature. Fleishman and Farrow [7] investigated the dynamic behavior of perimeter lateral-system structures with flexible diaphragms.

[1] Reinforced concrete

Modern structural systems often employ isolated or perimeter lateral-systems with long floor spans in which the rigid diaphragm treatment is not accurate. For these structures, diaphragm flexibility can modify dynamic behavior. The dynamic behavior of such structures is dissimilar to the behavior expected of typical structures. This difference can lead to unexpected force and drift patterns [7]. Due to the widespread use of the rigid-floor building analysis assumption, the quantitative study of the error caused from this scheme is a very important topic. However, it is extremely difficult to obtain an accurate error formula, since there is no general theoretical solution for the structural analysis of buildings. Thus, a statistical method is adopted to obtain an approximate error formula. In this paper, the finite element method is used to analyze a number of buildings with and without shear walls. From the results of a number of response-spectrum analyses, the rigid-floor model is found to be sufficiently accurate for regular and irregular buildings without shear walls. However, the difference between rigid-floor and flexible-floor analyses can be large for buildings with shear walls. In order to estimate the difference between those two types of analyses for buildings with perimeter shear wall and internal symmetric slab-opening, an error formula is developed statistically. Using this formula, one can estimate the error in the result when the rigid-floor assumption is used. Furthermore, it is easy to use, since only the geometric data of the shear wall and the slab are required.

2. STRUCTURAL MODELLING AND ANALYSIS FRAMEWORK

The total number of degrees of freedom in a three-dimensional (3D) building analysis is equal to three times the total number of slave nodes and master nodes in the mesh. For the equivalent static lateral force method, the horizontal forces are often applied to the master nodes in a rigid-floor analysis. However, it is difficult to add these horizontal forces to the nodes of a building with the flexible-floor assumption. For example, adding these horizontal forces only to the node at the mass center of each floor will cause a stress concentration near the mass center. Thus, to compare the results of the rigid and flexible-floor analyses, dynamic analysis is probably a better choice since the earthquake loading can be applied to the base of the building without any differentiation between the rigid and flexible-floor analyses. Forced dynamic analyses include time-history and response-spectrum analyses. For time-history analysis, it is not easy to compare the complex analysis results between the rigid- and flexible-floor analyses. For example, the two results may differ due to a significant time shift, so comparing them at a certain time will cause errors. The response-spectrum analysis does not have this problem, since only the maximum responses are calculated. Hence, this method with the response spectrum of the 2800 code (Fig. 1) is used to perform the two types of building analyses. For the dynamic analysis, the mode superposition method is used, and the first 30 modes are used to develop the input data for the response-spectrum analysis. The effective masses in the x-translation, y-translation and z-rotation for these 30 modes are always larger than 95% of their total masses in the building analyses.

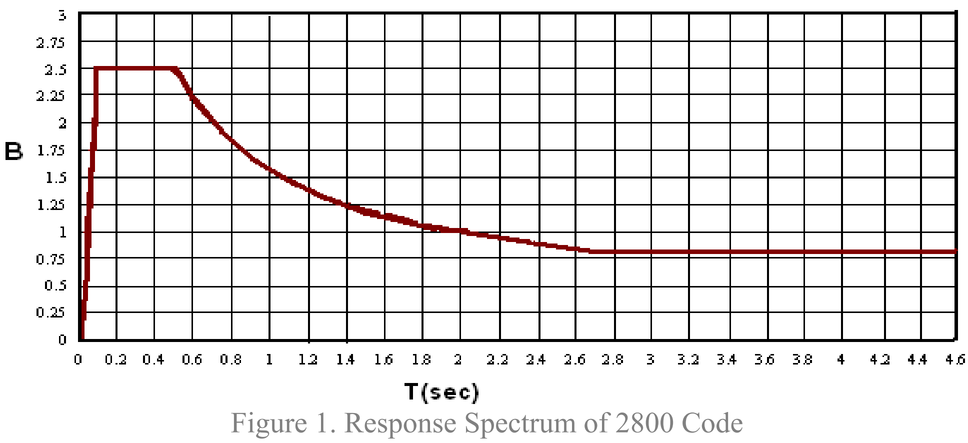

Figure 1. Response Spectrum of 2800 Code

3. CHARACTERISTICS OF THE BUILDINGS CONSIDERED

The RC structures considered all have a rectangular plan form and perimeter shear walls. The nesting of the parameters in the parametric study is given in Fig. 2, while the slabs are shown in Fig. 3. For the purpose of analysis, the slabs are divided into the following groups according to the size of the opening, as follows

- Group 1, no opening.
- Group 2, opening area of approximately 11%.
- Group 3, opening area of approximately 20%.
- Group 4, opening area of approximately 50%.

In each group there are three different aspects ratios (1:1, 1:3 and 1:5); for each aspect ratio there are four slab thicknesses (5, 10, 15 and 20 cm); for each slab thickness there are four different story numbers (1, 5, 10 and 20-story) and each building is analyzed according to the rigid and flexible diaphragm assumption. Fig. 2 is only drawn for first group.

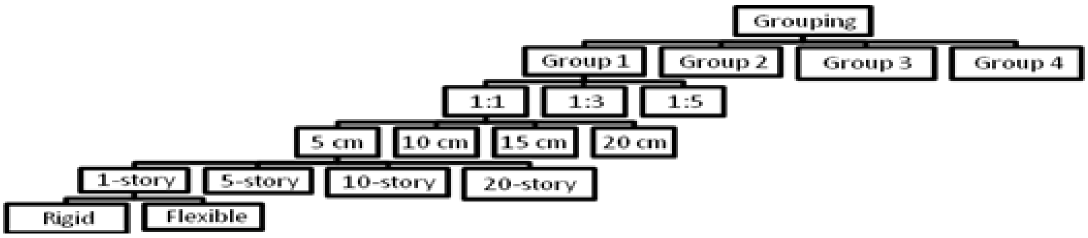

Figure 2. Style of Buildings Grouping

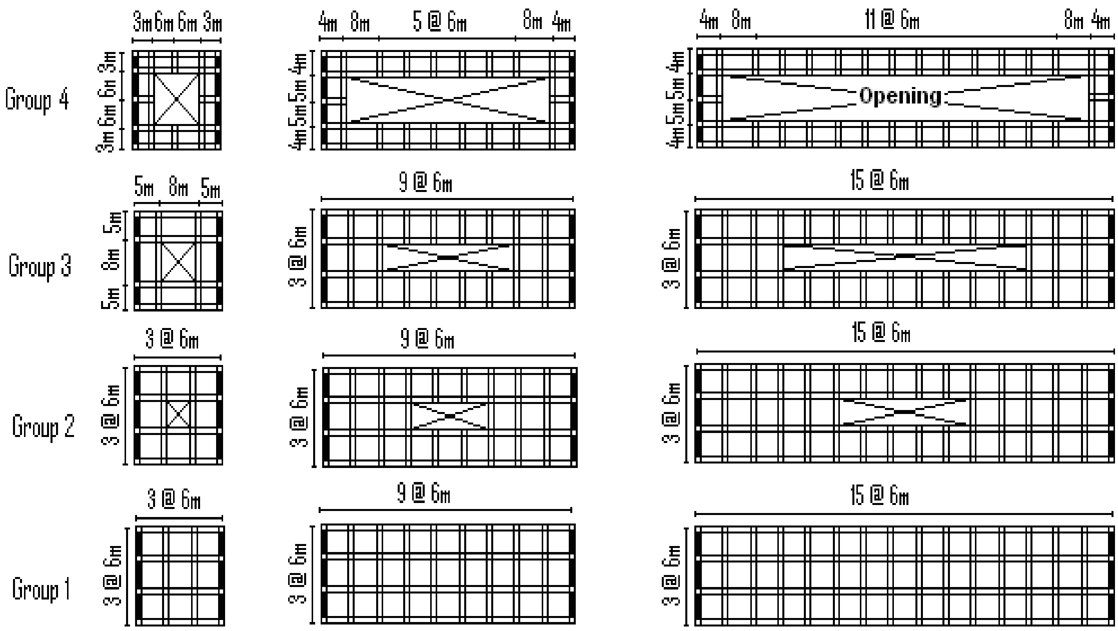

Figure 3. Plan shape, opening area and shear wall positions

Table 1. Member sizes, story height and shear wall thickness

Number of stories	Beam size (cm)	Column size (cm)	Story height (m)	Shear wall thickness (cm)
1	50X80	50X50	3.2	15
5	50X80	80X80	3.2	15
10	50X80	100X100	3.2	30
20	50X80	120X120	3.2	30

Table 2. Properties of buildings in all groups

	Aspect ratio	Plan dimension (m)	Opening dimension (m)	Approximate opening area %
Group 1	1:1	18X18	-	-
	1:3	18X54	-	-
	1:5	18X90	-	-
Group 2	1:1	18X18	6X6	11%
	1:3	18X54	6X18	11%
	1:5	18X90	6X30	11%
Group 3	1:1	18X18	8X8	20%
	1:3	18X54	6X30	19%
	1:5	18X90	6X54	20%
Group 4	1:1	18X18	12X12	45%
	1:3	18X54	10X46	48%
	1:5	18X90	10X82	51%

For simplicity, buildings are labeled according to the following procedure. A building with properties, "Group 1, aspect ratio 1:3, slab thickness 5cm and 5-story", would be labeled 1130505. It is clear that for the number of groups, aspect ratio, slab thickness and number of stories, the total number of buildings analyzed is (4*3*4*4=192) 192. Since each building is analyzed with rigid and flexible diaphragms, the total number of analyses is 384. Fig. 3 indicates the plan shape, opening in slab and the position of perimeter shear walls. For the analytical modelling and dynamic analyses of the structures considered, the computer program SAP2000 [8] was used. The floor diaphragms and shear walls were modelled with shell elements.

4. ANALYSIS OF THE PARAMETRIC STUDY

The effects of the various parameters are now shown in the following graphs. In these graphs, the error between the results from the rigid and flexible floor analyses is defined by equation (1) and plotted along the Y-coordinate.

$$\text{Error \%} = \text{diff \%} = 100 \times \left| \frac{\Delta_{\text{flexible}} - \Delta_{\text{story}}}{\Delta_{\text{flexible}}} \right| \qquad (1)$$

Where Δ_{flexible} = lateral deformation of the diaphragm in the top story under the flexible floor assumption, Δ_{story} = lateral deformation of the diaphragm in the top story according to the rigid floor assumption (average story drift).

4.1. Number of Stories

Fig. 4 shows the variation of the error with the number of stories. It is clear that increasing the number of stories reduces the lateral stiffness of the building and hence the effect of diaphragm flexibility. Fig. 4 shows the results for buildings 3131001, 3131005, 3131010 and 3131020.

4.2. Floor thickness

Fig. 5 shows the variation of the error with the floor thickness. It can be seen that increasing the floor thickness increases the in-plane floor stiffness and thus reduces the effect of diaphragm flexibility. Fig. 5 shows the results for buildings 3150501, 3151001, 3151501 and 3152001.

4.3. Aspect ratio

Fig. 6 shows the variation of the error with aspect ratio. Again, it can be seen that increasing the aspect ratio reduces the in-plane floor stiffness and thus increases the importance of diaphragm flexibility. Fig. 6 shows the results for buildings 2110520, 2130520 and 2150520.

4.4. Opening in slab

Fig. 7 shows the variation of the error with the opening in slab. Once more it can be seen that increasing the size of the opening in the slab reduces the in-plane floor stiffness and thus increases the importance of diaphragm flexibility. Fig. 7 shows the results for buildings 1131505, 2131505, 3131505 and 4131505.

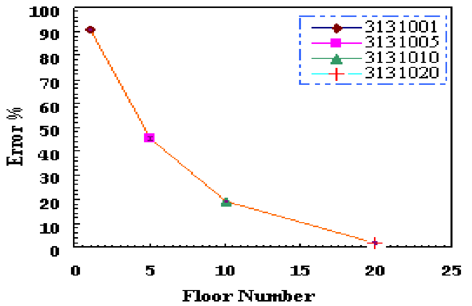

Figure 4. Variation of the error with number of stories

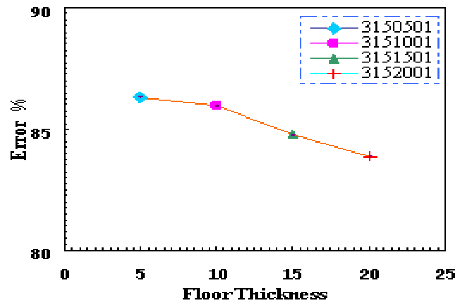

Figure 5. Variation of the error with floor thickness

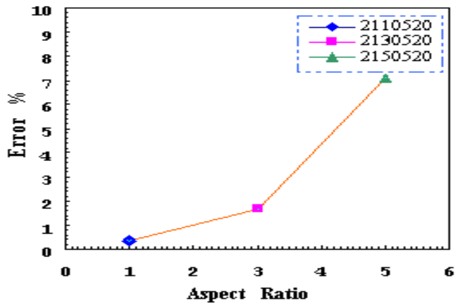

Figure 6. Variation of the error with aspect ratio

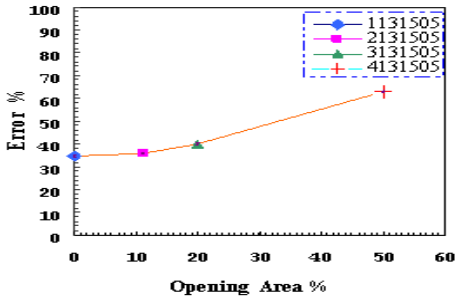

Figure 7. Variation of the error with opening in the slab

5. ERROR FORMULA

A statistical method is now used to find a regression curve for estimating the error of the rigid-floor building analysis. First, the error value between the rigid-floor and flexible-floor analyses is defined to be the Y-coordinate, while the sample input value, defined as the displacement difference ratio of the flexible-floor and the rigid-floor analyses, is the X-coordinate. Each pair of rigid-floor and flexible-floor building analyses produces one error value and one sample input value. Regression analysis is then used to determine the error formula from the 384 building analyses.

The error value defined in (2) is used to calculate the difference in analysis results between the rigid-floor and the flexible-floor models for shear-wall buildings. All the four column end moments are compared in this equation. The weightings of these column moments are also used in (2) to magnify the importance of large moments and suppress the effect of small moments.

$$\text{Error }\% = 100\times\frac{\sum_{i=1}^{n}\sum_{j=1}^{4}\left(\left|M_{rij}-M_{fij}\right|/\left|M_{fij}\right|\right)\times\left|M_{fij}\right|}{\sum_{i=1}^{n}\sum_{j=1}^{4}\left|M_{fij}\right|}=100\times\frac{\sum_{i=1}^{n}\sum_{j=1}^{4}\left|M_{rij}-M_{fij}\right|}{\sum_{i=1}^{n}\sum_{j=1}^{4}\left|M_{fij}\right|} \qquad (2)$$

Where n = total number of columns in the building; index j = index of the bending moments of the two axes at the two column ends; M_{rij} = moments of column i using the rigid-floor building analysis; and M_{fij} = moments of column i using the flexible-floor building analysis.

An appropriate definition for the sample input value is very important, since the variation of the error formula is highly dependent on this definition. However, a complex definition should be avoided for practical purposes. The error calculated from (2) should be proportional to the displacement difference ratio (R) between the flexible-floor and rigid-floor analyses. In this paper, the following steps are used to define this displacement difference ratio (R):

1. The floor is assumed to be a simply supported beam subjected to a unit uniform load (load/length) along the length direction (Fig. 8). Since the shear walls are symmetric, only one of the two symmetric shear walls is chosen. This shear wall is assumed to be a cantilever beam subjected to a concentrated load at the wall top (Fig. 8). The magnitude of the concentrated load is L/2, which is transferred from the unit uniform load of the floor, where L is the floor length shown in Fig. 8.

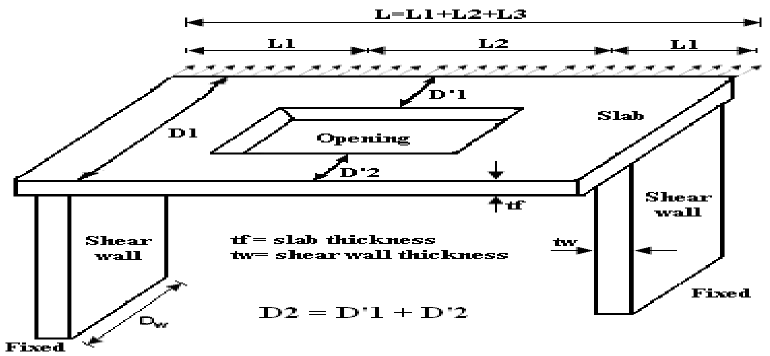

Figure 8. Model and Dimensions of Floor and Shear Walls

2. The approximate averaged displacements of the rigid-floor and flexible-floor analysis are assumed as follows

$$\Delta_{rigid} - \Delta_w \tag{3}$$

$$\Delta_{flexible} = \Delta_w + \Delta_f/2 \tag{4}$$

Where Δ_{rigid} = approximate averaged displacement of the floor according to the rigid-floor assumption; $\Delta_{flexible}$= approximate averaged displacement of the floor according to the flexible-floor assumption; Δ_w= displacement at the top of the cantilever beam (wall) according to the assumption of step 1; and Δ_f= displacement at the center of the simply supported beam (floor) according to the assumption of step 1.

3. From (3) and (4), the displacement difference ratio is defined as:

$$\mathrm{R} = \frac{\Delta_{\mathrm{flexible}} - \Delta_{\mathrm{rigid}}}{\Delta_{\mathrm{flexible}}} = \frac{\Delta_{\mathrm{f}}}{2\Delta_{\mathrm{w}} + \Delta_{\mathrm{f}}} \tag{5}$$

For rectangular slabs with internal symmetric slab-opening, Δ_w and Δ_f can be obtained from (6) and (7) which were derived using an energy method. If D1= D2 and L1= 0 in (7), Δ_f of a rectangular slab (without opening) is obtained from (8).

$$\Delta_w = \frac{2H^3L}{ED_w{}^3t_w} + \frac{\alpha HL}{2GD_w t_w} \tag{6}$$

$$\begin{aligned}\Delta_f = {} & \frac{(4LL_1{}^3 - 3L_1^4)}{24Et_f D_1{}^3/12} + \frac{L[(L/2)^3 - L_1{}^3]}{6Et_f D_2{}^3/12} - \frac{[(L/2)^4 - L_1{}^4]}{8Et_f D_2{}^3/12} \\ & + \frac{\alpha(LL_1 - L_1{}^2)}{2Gt_f D_1} + \frac{\alpha[(LL_2/2) - L_1L_2 - (L_2{}^2/4)]}{2Gt_f D_2}\end{aligned} \tag{7}$$

$$\Delta_{f-\mathrm{Rec}} = \frac{5L^4}{32ED_w{}^3t_f} + \frac{\alpha L^2}{8GD_w t_f} \tag{8}$$

Where E = Young's modulus; G = shear modulus; t_f= thickness of the floor slab; t_w= thickness of the shear wall; α= ratio of the effective shear area of the slab, which can be approximated to 6/5; H = total shear-wall height; and L, L1, L2, D, D1, D2, and D_w= dimensions of the floor slab and shear wall as shown in Fig. 8.

Fig. 9 indicates the results from 384 building analyses, where each points represents the error evaluated from a rigid-floor and a flexible-floor building analysis. From this figure and the values of errors, in order to obtain appropriate accuracy in the linear regression analysis, the analysis should be performed in the following two regions of R, $0 < R < 0.85$ and $0.85 \le R < 1$, as shown in Fig. 10.

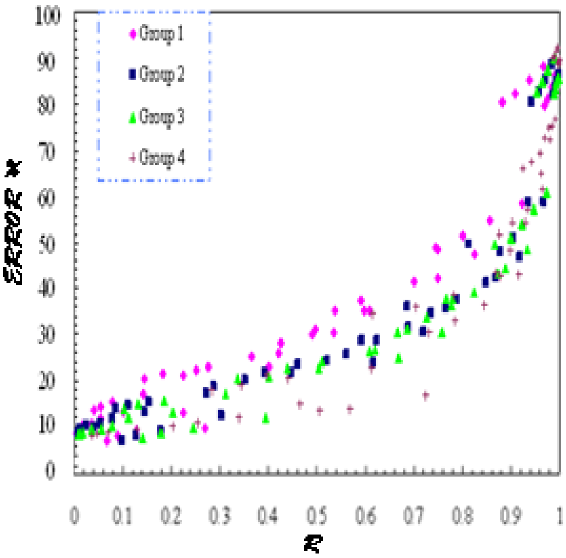

Figure 9. The results of analyses

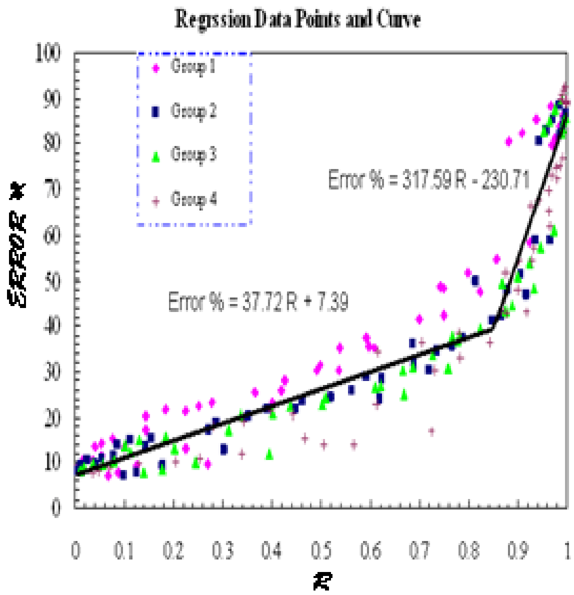

Figure 10. Regression data points and curve

$$\text{Error}\,\% = \begin{cases} 37.72\,\text{R} + 7.39 & , \quad 0 < R \le 0.85 \\ 317.59\,\text{R} - 230.71 & , \quad 0.85 < R < 1 \end{cases} \tag{9}$$

Fig.10 indicates that all the data points are concentrated along the banded lines without too much variation and the straight lines are shown in (9) are a good regression model for the regions ($0 < R < 0.85$) and ($0.85 \le R < 1$). The average and maximum variations of equation (9) are shown in Table 3, where the variation is defined as the absolute value of the error percentage from (9) minus the real value. The values of this table indicate satisfactory accuracy in the regression analysis. Thus, using (5) as a regression base is acceptable. Equations (9) are useful for estimating the error in columns moments, when the wide-spread rigid-floor method is used to simulate buildings with continuous symmetric shear walls and internal slab openings. Furthermore, it is easy to use, since only the geometric data of the shear walls and slab are required.

In the first region ($0 < R < 0.85$), when $R < 0.3$, the percentage error obtained from (9) is less than 20%, which indicates that the diaphragm is effectively behaving rigidly. In the remainder of the region, the percentage error is always less than 40% and the diaphragm can be classed as moderately flexible. In the second region ($0.85 \le R < 1$), the percentage of error obtained from (9) is always more than 40% and the diaphragm can be classed as highly flexible. Thus, in general, it is recommended that for rectangular RC structures with continuous symmetric shear walls and internal slab-opening, the behavior of the diaphragm should be considered to be sufficiently flexible that the flexible-floor analysis be used in place of the rigid-floor analysis when $R \ge 0.3$.

Table 3. Average and Maximum Variations of Eq. (9)

R	Variation of Eq. (9)	
	Average	Maximum
0.00<R<0.05	1.36	4.56
0.05<R<0.10	2.44	5.07
0.10<R<0.15	3.21	7.59
0.15<R<0.20	4.24	7.43
0.20<R<0.30	3.92	7.78
0.30<R<0.40	3.41	10.20
0.40<R<0.50	3.39	9.75
0.50<R<0.60	5.81	14.95
0.60<R<0.70	4.40	7.82
0.70<R<0.80	5.33	17.74
0.80<R<0.85	4.46	11.57
0.85<R<0.90	7.08	31.28
0.90<R<0.95	10.33	24.83
0.95<R<1.00	5.75	17.25

6. SUMMARY AND CONCLUSIONS

For buildings without shear walls, the rigid-floor model is as accurate as the flexible model even for irregular floor systems. This is due to the fact that the in-plane stiffness of the slab is much larger than the out-of-plane column stiffness. For buildings with shear walls, the rigid-floor and flexible-floor analyses can differ greatly due to the very large lateral stiffness of the shear wall system. In such cases, the in-plane stiffness of the slab is relatively insignificant, and the slab in-plane deformation cannot be ignored. An error formula has been generated using a regression analysis on the results of rigid-floor and flexible-floor analyses from 196 rectangular reinforced concrete buildings with internal symmetric slab-openings. Using this formula, one can estimate the error involved in applying the rigid-floor assumption for this class of building. Furthermore, the formula is easy to use, since only the geometric data of the shear wall and the slab are required.

REFERENCES

[1] MacLeod, I. A. "Shear *wall- frame interaction: A design aid*," Engrg. Bull. No. 3, Portland Cement Association, Skokie, IL, 1970.

[2] Wilson, E. L., Hollings, J. P., and Dovey, H. H. "*Three dimensional analysis of Building systems (expanded version)*", Rep. No. EERC 75-13, University of California, Berkeley, calif, 1972.

[3] Muto,K. "*A seismic design analysis of buildings*" , Marunzen Co., Ltd., Tokyo 241-260, Japan, 1974.

[4] Jain, S. K. "*Seismic response of buildings with flexible floors*", J. Engrg. Mech. ASCE110, 125-129, 1984.

[5] Saffarini, H.; Qudaimat,M., " *In-plane floor deformations in RC structures,* "Journal of Structural Engineering , vol. 118, No. 11, 3089- 3102, 1992.

[6] Ju, S .H. ; Lin ,M .C. "Comparison *of building analyses assuming rigid or flexible floors,* " J. Struct. Engrg. ASCE 125, 25-39, 1999.

[7] R.B.Fleischman; K.T.Farrow, " *Dynamic behavior of perimeter lateral-system structures with flexible diaphragms"*, Earthquake engineering and structural dynamics 2001; 30:745-763.

[8] SAP2000, Advanced 9.0.3, Structural Analysis Program User Manual, Copyright 1976-2004 Computer and Structural, Inc.