DESIGN EQUATIONS FOR EVALUATION OF STRENGTH AND DEFORMATION CAPACITY FOR UNSTRENGTHENED AND FRP STRENGTHENED RC RECTANGULAR COLUMNS UNDER COMBINED BIAXIAL BENDING AND AXIAL LOAD

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ABSTRACT:
When assessing the seismic performance of existing reinforced concrete buildings designed according to obsolete codes, one can identify potentially dangerous situations that could result in catastrophic failures. A typical inadequacy lies in the so-called "strong beam-weak column" situation: columns are no so strength to force plastic hinge formation in beams and, if this is extended to all columns at a given floor, can lead to the development of a soft-storey mechanism. Such weaknesses should be eliminated by upgrading all weak columns in the zones of potential formation of plastic hinges so to increase their flexural capacity and to force plastic hinge formation in the beams. With this aim one or more layers of FRP could be wrapped longitudinally along the column at the end zones. Nevertheless, the flexural strengthening can reduce the element's deformation capacity. This paper, starting from previous authors works propose a simplified procedure for the assessment of flexural strength and deformation capacity of rc unstrengthened and FRP strengthened reinforced concrete rectangular columns. Approximate closed form equations are developed by which the flexural strength and section ultimate curvature are evaluated as a function of the normalized acting axial load and the geometrical and mechanical section's parameter. The proposed approach is compared with a fibre approach.

KEYWORDS: RC columns, FRP–strengthening, interaction domain, biaxial bending, closed-form equations, seismic upgrade

1. EXACT APPROACH

1.1. Evaluation of section's strength capacity

Classical methods for check of RC members under combined biaxial bending and axial load are based on the construction of the 3D failure domain (N, M_x, M_y). The boundary of the failure domain (the so called '*interaction failure surface*') defines the limit terns (N_{Rd}, M_{xRd}, M_{yRd}) that cause ultimate limit state achievement. Its construction is performed point by point by integration of stresses associated to the strain distribution, corresponding to a flexural failure mode for the section. Reinforced concrete section analysis at the ultimate limit state is based on the following usual hypotheses:

- plane sections remain plane (linear strains),
- perfect bond between steel and concrete,
- no tensile strength in concrete,
- non–linear stress–strain laws for steel and concrete.

The strain state over the section is therefore uniquely defined by the concrete compression strain ε_c and by the steel tensile strain ε_s. Flexural failure occurs when one of the following conditions is met: the concrete ultimate strain, $\varepsilon_{c_{max}} = \varepsilon_{cu}$, or the steel tensile ultimate strain, $\varepsilon_{s_{max}} = \varepsilon_{su}$:

$$ultimate \begin{cases} \varepsilon_{c_{max}} = \varepsilon_{cu}, & \varepsilon_{su} \le \varepsilon_{s_{max}} \le 0 \\ 0 \le \varepsilon_{c_{max}} \le \varepsilon_{cu}, & \varepsilon_{s_{max}} = \varepsilon_{su} \end{cases} \tag{1.1}$$

The limit terns N_{Sd}, M_{xRd}, M_{yRd}, corresponding to boundary points on the section failure surface are calculated continuously modifying the neutral depth for every value of the neutral axis angle and solving at each step the equilibrium equations:

$$N_{Sd} = \int_{A_c} \sigma_c dA_c + \int_{A_s} \sigma_s dA_s \tag{1.2}$$

$$M_{xRd} = \int_{A_c} \sigma_c y dA_c + \int_{A_s} \sigma_s y dA_s \tag{1.3}$$

$$M_{yRd} = \int_{A_c} \sigma_c x dA_c + \int_{A_s} \sigma_s x dA_s \tag{1.4}$$

where σ_c is the concrete stress, σ_s is the stress in steel reinforcement, A_c is the concrete compressed area and A_s the steel reinforcement area.

1.2. Evaluation of section's ultimate curvature at constant axial load

Section curvature associated with an axial load and bending moment can be evaluated on the basis of the same hypothesis used for determination of resisting moments and from the requirements of strain compatibility and equilibrium of forces. The ultimate curvature, φ_u, can be evaluated as:

$$\varphi_u = \frac{\varepsilon_{cm}}{\xi_u h} \tag{1.5}$$

where ξ_u is the non dimensional neutral axis depth of the compressed zone at failure, h is the section height, measured orthogonally to the neutral axis, and ε_{cm} is the strain at the extreme compressed fiber:

$$\varepsilon_{cm} = \begin{cases} \varepsilon_{cu} & concrete\ failure \\ \varepsilon_{su} \dfrac{\xi_u}{1-\xi_u-\delta} & steel\ failure \end{cases} \tag{1.6}$$

To evaluate the ultimate curvature the neutral axis depth, ξ_u, must be calculated by iteratively solving the translational equilibrium equation (1.2).

2. APPROXIMATE APPROACH

2.1. Evaluation of section's strength capacity

2.1.1. Unstrengthened sections

Construction of the 3D failure domain involves some computational difficulties mainly due to the integration of stress over the compressive portion of concrete and to the analytical and graphical representation of the surface and the comparison with the acting external forces. A simplified method, which allows to avoid the numerical integration required for solution of the equilibrium equations, is fully described in Monti *et al.*, 2006; it is based on the analytical approximation of the failure surface by sections at constant axial load proposed by Bresler, 1960, and expressed as:

$$\left(\frac{m_{ux}}{m_{0x}}\right)^{\alpha} + \left(\frac{m_{uy}}{m_{0y}}\right)^{\alpha} = 1 \tag{2.1}$$

where m_{ux}, m_{uy} = normalized uniaxial resisting moments under the normalized applied axial load n_{Sd}; m_{0x},

m_{0y} = normalized resisting moments about the section main axis x, y, given by:

$$n_{Sd} = \frac{N_{Sd}}{0.85 \cdot f_{cd} \cdot b \cdot h}, \quad m_x = \frac{M_x}{0.85 \cdot f_{cd} \cdot b \cdot h^2}, \quad m_y = \frac{M_y}{0.85 \cdot f_{cd} \cdot b^2 \cdot h} \tag{2.2}$$

where: b = section width, h = section height, f_{cd} = design compressive strength of concrete (0.85 account for long–term loads), α = exponent depending on the cross section geometry, the steel reinforcement percentage and the axial load n_{Sd}. The α exponent is evaluated as a function of the cross–section's mechanical and geometrical parameters and the normalized applied axial load n_{Sd}:

$$\alpha = c \cdot \left(\frac{b}{h}\right)^{\gamma} \cdot \mu_{sx}^{\eta_{sx}} \cdot \mu_{sy}^{\eta_{sy}} \cdot n_{Sd}^{\omega} \tag{2.3}$$

where μ_{Sx}, μ_{Sy} = mechanical ratio of steel reinforcement laid parallel to x and y section axis given, respectively, by:

$$\mu_{sx} = \frac{A_{sx} \cdot f_{yd}}{0.85 \cdot f_{cd} \cdot b \cdot h}, \quad \mu_{sy} = \frac{A_{sy} \cdot f_{yd}}{0.85 \cdot f_{cd} \cdot b \cdot h}, \tag{2.4}$$

with A_{Sx}, A_{Sy} = area of reinforcement laid parallel to x and y axis; f_{yd} = design yield strength of steel. The values of parameters in the equation (2.3) have been obtained through the least–squares method; they are shown in Table 2.1.

Table 2.1. Parameters for calculation of α exponent for unstrengthened sections

n_{Sd}	c	γ	η_{sx}	η_{sy}	ω
> 0	1.45	-0.02	-0.06	0.03	-0.025
= 0	1.35	-0.02	-0.03	-0.20	
< 0	1.70	-0.09	-0.01	-0.50	0.30

Fig. 1 shows the comparison between the corresponding failure domains for several values of the basic parameters (b/h, μ_{sx}, μ_{sy}, n_{Sd}). It can be seen that the simplified equations correctly represent the interaction diagram of the section.

2.1.2. *FRP-strengthened sections*

FRP–strengthened RC section analysis at the ultimate limit state is based on the same usual hypotheses adopted for the unstrengthened section, with the addition of the following:

- perfect bond between FRP and concrete,
- no compressive strength in FRP,
- linear stress–strain law for FRP.

The strain state over the section is uniquely defined by the concrete compression strain ε_c and by the FRP tensile strain ε_f. Flexural failure occurs when one of the following conditions is met: the concrete ultimate strain, $\varepsilon_{c_{max}} = \varepsilon_{cu}$, or the FRP ultimate strain, $\varepsilon_{f_{max}} = \varepsilon_{fd}$:

$$ultimate \begin{cases} \varepsilon_{c_{max}} = \varepsilon_{cu}, & \varepsilon_{fd} \le \varepsilon_{f_{max}} \le 0 \\ 0 \le \varepsilon_{c_{max}} \le \varepsilon_{cu}, & \varepsilon_{f_{max}} = \varepsilon_{fd} \end{cases} \tag{2.5}$$

The explicit relationship developed for the α exponent in equation (2.3) is here modified to include the FRP reinforcement mechanical parameters:

$$\alpha = c \cdot \left(\frac{b}{h}\right)^{\gamma} \cdot \mu_{sx}^{\eta_{sx}} \cdot \mu_{sy}^{\eta_{sy}} \cdot \mu_{fx}^{\eta_{fx}} \cdot \mu_{fy}^{\eta_{fy}} \cdot n_{Sd}^{\omega} \tag{2.6}$$

where μ_{fx} and μ_{fy} are the mechanical ratio of FRP-strengthening laid parallel to x and y section axis given, respectively, by:

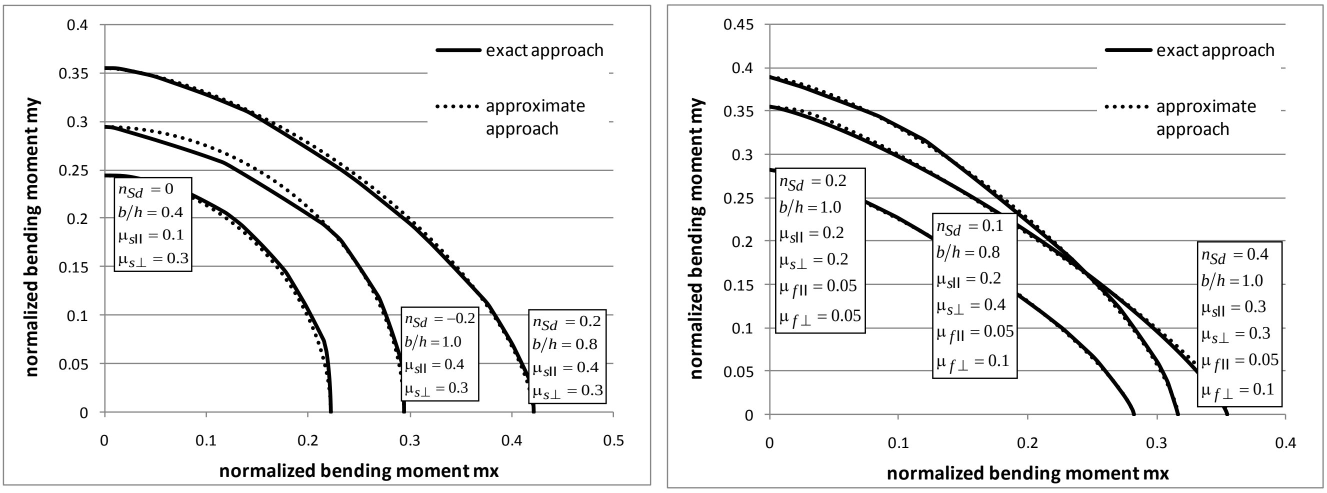
$$\mu_{fx} = \frac{A_{fx} \cdot f_{fd}}{0.85 \cdot f_{cd} \cdot b \cdot h}, \quad \mu_{fy} = \frac{A_{fy} \cdot f_{fd}}{0.85 \cdot f_{cd} \cdot b \cdot h}, \tag{2.7}$$

with A_{fx}, A_{fy} =area of FRP reinforcement laid parallel to x and y axis; f_{fd} = design strength of FRP. The values of the parameters in the equation (2.6) have been obtained through the least–squares method; they are shown in Table 2.2.

Table 2.2. Parameters for calculation of α exponent for FRP-strengthened sections

n_{Sd}	c	γ	η_{sx}	η_{sy}	η_{fx}	η_{fy}	ω
> 0	1.2	-0.04	-0.06	0	0	0.02	-0.03

Fig. 1 shows the comparison between the corresponding failure domains for several values of the basic parameters (b/h, μ_{sx}, μ_{sy}, μ_{fx}, μ_{fy}, n_{Sd}). It can be seen that the simplified equations correctly represent the interaction diagram of the section.



Ustrengthened sections FRP-strengthened sections

Fig. 1. Comparison between exact (fiber method) and approximate approaches for an RC section under combined biaxial bending and axial load.

2.2. Evaluation of section's ultimate curvature

The approximate approach for resisting moment evaluation can be extended to evaluation of section ultimate curvature. Section ultimate curvature associated to the acting axial load n_{Sd} and resisting bending moment m_{Rd} can be evaluated by:

$$\left(\frac{\varphi_{xu}}{\varphi_{x0u}(n_{Sd})}\right)^{\alpha_{\varphi u}} + \left(\frac{\varphi_{yu}}{\varphi_{y0u}(n_{Sd})}\right)^{\alpha_{\varphi u}} = 1 \tag{2.8}$$

where: φ_{xu}, φ_{yu} = section curvatures about the section main axis related to the components m_{xRd}, m_{yRd} of the normalized resisting bending moment m_{Rd} under the acting axial load n_{Sd}; $\varphi_{x0u}(n_{Sd})$, $\varphi_{y0u}(n_{Sd})$= section curvatures about the section main axis at ultimate. The $\alpha_{\varphi u}$ exponent can be evaluated as a function of the cross-section's mechanical and geometrical parameters and the normalized applied axial load n_{Sd}:

$$\alpha_{\varphi u} = \begin{cases} c_{\varphi u} \cdot \left(\frac{b}{h}\right)^{\gamma_{\varphi u}} \cdot \mu_{sx}^{\eta_{sx\varphi u}} \cdot \mu_{sy}^{\eta_{sy\varphi u}} \cdot n_{Sd}^{\omega_{\varphi u}} & \textit{unstrengthened sections} \\ c_{\varphi u} \cdot \left(\frac{b}{h}\right)^{\gamma_{\varphi u}} \cdot \mu_{sx}^{\eta_{sx\varphi u}} \cdot \mu_{sy}^{\eta_{sy\varphi u}} \cdot \mu_{fx}^{\eta_{fx\varphi u}} \cdot \mu_{fy}^{\eta_{fy\varphi u}} \cdot n_{Sd}^{\omega_{\varphi u}} & \textit{FRP-strengthened sections} \end{cases} \tag{2.9}$$

The values of the parameters in the equation (2.9) have been obtained through the least–squares method for different kind of rectangular cross sections (obtained by varying the basic parameters); these values are shown in Table 2.3. and Table 2.4.

Table 2.3.Parameters for calculation of $\alpha_{\varphi u}$ exponent for unstrengthened sections

n_{Sd}	c	γ	η_{sx}	η_{sy}	ω
> 0	1.3	0.01	0.004	0.07	0.2
= 0	0.95	0.03	0.03	0.2	
< 0	0.73	0.05	0.0015	0.17	-0.055

Table 2.4. Parameters for calculation of $\alpha_{\varphi u}$ exponent for FRP-strengthened sections

n_{Sd}	c	γ	η_{sx}	η_{sy}	η_{fx}	η_{fy}	ω
> 0	1.16	-0.004	0.01	0.04	0	-0.025	0.18

2.3. *Closed-form equations for uniaxial bending capacities of sections with double symmetric steel reinforcement*

For application of equation (2.1) the uniaxial resisting moments must be evaluated beforehand. To this aim the translational equilibrium equation must be iteratively solved to find the neutral axis depth under the acting axial load n_{Sd}. In order to avoid the iterative solution, the neutral axis depth can be expressed as a function of the acting axial load by simplified closed form equations. According to the notations of Fig.2, a simplified model with an equivalent area of reinforcing steel uniformly distributed around the section's side is used; by this way the equilibrium equations, for a section with two-way steel reinforcement and FRP sheets around each sides, can be written in a non dimensional form as follows:

$$\alpha_c \xi_c + \frac{2\mu_{s\perp}}{1-2\delta_\perp}\left(\alpha_{s\perp}^{+}\xi_{s\perp}^{+} - \alpha_{s\perp}^{-}\xi_{s\perp}^{-}\right) + \mu_{s\parallel}\left(\alpha_{s\parallel}^{+} - \alpha_{s\parallel}^{-}\right) - 2\mu_{f\perp}\alpha_{f\perp}^{-}\xi_{f\perp}^{-} - \mu_{f\parallel}\alpha_{f\parallel}^{-} = n_{Sd} \tag{2.11}$$

$$\begin{aligned} m_{0\perp} = {} & \alpha_c \xi_c \left(0.5 - k_c \xi_c\right) + \frac{2\mu_{s\perp}}{1-2\delta_\perp}\left[\alpha_{s\perp}^{+}\xi_{s\perp}^{+}\left(0.5-\delta_\perp - k_{s\perp}^{+}\xi_{s\perp}^{+}\right) + \alpha_{s\perp}^{-}\xi_{s\perp}^{-}\left(0.5-\delta_\perp - k_{s\perp}^{-}\xi_{s\perp}^{-}\right)\right] + \\ & + \mu_{s\parallel}\left(0.5-\delta_\perp\right)\left(\alpha_{s\parallel}^{+} + \alpha_{s\parallel}^{-}\right) + 2\mu_{f\perp}\left[\alpha_{f\perp}^{-}\xi_{f\perp}^{-}\left(0.5 - k_{f\perp}^{-}\xi_{f\perp}^{-}\right)\right] + 0.5\mu_{f\parallel}\alpha_{f\parallel}^{-} \end{aligned} \tag{2.12}$$

In the previous equations the non dimensional parameters ξ define the depth of the equivalent stress blocks normalized with respect to the section height h; the subscript identifies the material (c for concrete, s for steel and f for FRP), the superscript identifies the compression or the tension, while the symbols $\perp$ and $\parallel$ define the direction with respect to the neutral axis. The symbols α identify the equivalent stress blocks, while k are for the relevant resultants depths. The symbol $\delta_\perp$ indicates the cover ratio evaluated orthogonally to the neutral axis. The parameters $\mu_{s\parallel}$ and $\mu_{s\perp}$, $\mu_{f\parallel}$ and $\mu_{f\perp}$ represent the mechanical ratios of parallel and orthogonal steel and FRP reinforcement, respectively, where $A_{s\parallel}$ and $A_{s\perp}$, $A_{f\parallel}$ and $A_{f\perp}$ are their areas.

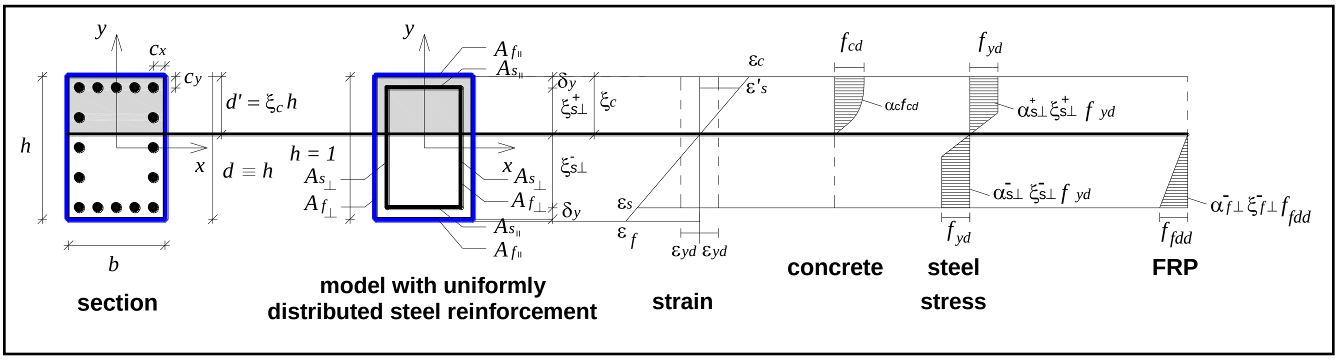


Fig. 2. Non-dimensional quantities depicting the geometry and the strain and stress state (in concrete, steel and FRP) of a RC section with two-way steel reinforcement.

The coefficients in the equilibrium equations (2.11) and (2.12) have been calculated for the different modes by which sectional failure can occur (their values/expressions are given in Table 2.5, Table 2.6 and Table 2.7) defining, for each mode, a simplified expression to evaluate the non–dimensional neutral axis depth ξ_c. Three different sectional failure modes can be defined:

- $\varepsilon_c \le \varepsilon_{cu}$ and $\varepsilon_t = \varepsilon_u$ (mode 1);
- $\varepsilon_c = \varepsilon_{cu}$ and $\varepsilon_u < \varepsilon_t \le \varepsilon_{yd}$ (mode2);
- $\varepsilon_c = \varepsilon_{cu}$ and $\varepsilon_{yd} < \varepsilon_s \le 0$ (mode3).

where ε_t is the strain at the extreme tensile fiber and ε_u is the ultimate tensile strain ($\varepsilon_u = \varepsilon_{su}$, for unstrengthened sections and $\varepsilon_u = \varepsilon_{fd}$ for FRP- strengthened sections). Mode 1 can be subdivided into two sub–modes: mode 1a and mode 1b, which differ in the compression steel state, either elastic or yielded; mode 3 is only for unstrengthened sections, because of FRP-strengthening is effectiveness only for yielded sections.

Table 2.5. Values and expressions of the coefficients in equations (2.11) and (2.12) for different failure modes

Failure Mode	α_c	$\alpha^+_{s\parallel}$	$\alpha^-_{s\parallel}$	$\xi^+_{s\perp}$	$\alpha^+_{s\perp}$	$\xi^-_{s\perp}$	$\alpha^-_{s\perp}$
1°	0.8	$\frac{\varepsilon_{su}}{\varepsilon_{yd}}\frac{\xi_c-\delta_\perp}{1-\xi_c-\delta_\perp}$	1	$\xi_c-\delta_\perp$	$0.5\frac{\varepsilon_u}{\varepsilon_{yd}}\frac{\xi_c-\delta_\perp}{1-\xi_c-\delta_\perp}$	$1-\xi_c-\delta_\perp$	$1-0.5\frac{\xi_c-\delta_\perp}{1-\xi_c-\delta_\perp}$
1b		1			$1-0.5\frac{\varepsilon_{yd}}{\varepsilon_{su}}\frac{1-\xi_c-\delta_\perp}{\xi_c-\delta_\perp}$		
2					$1-0.5\frac{\varepsilon_{yd}}{\varepsilon_{cu}}\frac{\xi_c}{\xi_c-\delta_\perp}$		$1-0.5\frac{\varepsilon_{yd}}{\varepsilon_{cu}}\frac{\xi_c}{1-\xi_c-\delta_\perp}$
3			$\frac{\varepsilon_{cu}}{\varepsilon_{yd}}\frac{1-\xi_c-\delta_\perp}{\xi_c}$				$0.5\frac{\varepsilon_{cu}}{\varepsilon_{yd}}\frac{1-\xi_c-\delta_\perp}{\xi_c}$

Table 2.6. Values and expressions of the coefficients in equations (2.11) and (2.12) for different failure modes

Failure Mode	k_c	$k^+_{s\perp}$	$k^-_{s\perp}$
1	0.4	$\frac{1}{3}$	$1-\frac{0.5-0.67(1-\alpha^-_{s\perp})^2}{\alpha^-_{s\perp}}$
2		$1-\frac{0.5-0.67(1-\alpha^+_{s\perp})^2}{\alpha^+_{s\perp}}$	$\frac{1}{3}$
3			

Table 2.7. Values and expressions of the coefficients for FRP-strengthening in equations (2.11) and (2.12) for different failure modes

Failure Mode	$\bar{\alpha}_{f\parallel}$	$\bar{\xi}_{f\perp}$	$\bar{\alpha}_{f\perp}$	$\bar{k}_{f\perp}$
1a	1	$1-\xi_c$	0.5	$\frac{1}{3}$
1b	1	$1-\xi_c$	0.5	$\frac{1}{3}$
2	$\frac{\varepsilon_{cu}}{\varepsilon_{fd}}\frac{(1-\xi_c)}{\xi_c}$	$1-\xi_c$	$0.5\frac{\varepsilon_{cu}}{\varepsilon_{fd}}\frac{(1-\xi_c)}{\xi_c}$	$\frac{1}{3}$

The equations of the neutral axis depth are given for different failure modes in Table 2.8; in failure mode 1a, 2 and 3 they are get by applying the secant method. The parameter A is given by the following expression:

$$A=-\frac{\mu_{f\perp}(1-\xi_{(2-3)})\left[\frac{\varepsilon_{cu}}{\varepsilon_{fd}}\frac{(1-\xi_{(2-3)})}{\xi_{(2-3)}}-1\right]+\mu_{f\parallel}\left[\frac{\varepsilon_{yd}}{\varepsilon_{fd}}\frac{(1-\xi_{(2-3)})}{0.95-\xi_{(2-3)}}-1\right]}{(\xi_{(2-3)}-\xi_{(1b-2)})} \tag{2.13}$$

Table 2.8. Values/expressions of non-dimensional neutral axis depth ξ_c for different failure modes.

Failure mode	ξ_c
1a	$\frac{\left(n_{sd}+\frac{2\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}+\mu_{f\parallel}+\mu_{s\parallel}\right)}{\left(0.8+\frac{4\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}\right)+\frac{\mu_{s\parallel}}{\xi_{(1a-1b)}}}$
1b	$\frac{n_{sd}+\frac{2\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}+\mu_{f\parallel}}{0.8+\frac{4\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}}$
2	$\frac{n_{sd}+\frac{2\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}+\mu_{f\parallel}+\xi_2 A}{0.8+\frac{4\mu_{s\perp}}{1-2\delta_\perp}+\mu_{f\perp}+A}$
3	$\frac{n_{Sd}+\frac{\mu_{s\perp}}{1-2\delta_\perp}(1+\delta_\perp)+\mu_{s\parallel}\frac{\varepsilon_{cu}}{\varepsilon_{yd}}}{0.8+\frac{2\mu_{s\perp}}{1-2\delta_\perp}\left(1.5-0.5\frac{\varepsilon_{yd}}{\varepsilon_{cu}}\right)+\mu_{s\parallel}\frac{\varepsilon_{cu}+\varepsilon_{yd}}{\varepsilon_{yd}(1-\delta_\perp)}}$

To define the relevant failure mode of the section the normalized acting axial load n_{Sd} must be compared with the values, n_i, at the failure modes boundaries, that can be evaluated by using the expressions given in Table 2.10, with the values of the neutral axis depth given in Table 2.10. Once the relevant failure mode of the section has been defined and the neutral axis depth ξ_c has been evaluated, the normalized resisting moment $m_{0\perp}(n_{Sd})$ can be evaluated using equation (2.12) with coefficients given in Table 2.5.

Table 2.9. Values and expressions of non–dimensional neutral axis depth for different failure mode boundaries.

Failure mode boundaries	0–1a	1a–1b	1–2	2–3	3–4
ξ_c	0	$\frac{\varepsilon_{yd}\cdot d+\varepsilon_u\delta_\perp}{\varepsilon_{yd}+\varepsilon_u}$	$\frac{\varepsilon_{cu}\cdot d}{\varepsilon_{cu}+\varepsilon_{su}}$	$\frac{\varepsilon_{cu}(1-\delta_\perp)}{\varepsilon_{cu}+\varepsilon_{yd}}$	$(1-\delta_\perp)$

Table 2.10. Expressions of non–dimensional axial load for different failure mode boundaries.

Failure – mode boundaries	n_i
0 – 1a	$-\frac{2\mu_{s\perp}}{1-2\delta_{\perp}}-\mu_{s\|}$
1a – 1b	$0.8\xi_{(1a-1b)}+\frac{2\mu_{s\perp}}{1-2\delta_{\perp}}\left(2\xi_{(1a-1b)}-1\right)$
1b – 2	$0.8\xi_{(1b-2)}+\frac{2\mu_{s\perp}}{1-2\delta_{\perp}}\left(2\xi_{(1b-2)}-1\right)$
2 – 3	$0.8\xi_{(2-3)}+\frac{2\mu_{s\perp}}{1-2\delta_{\perp}}\left[\xi_{(2-3)}\left(1.5-0.5\frac{\varepsilon_{yd}}{\varepsilon_{cu}}\right)-0.5\left(1+\delta_{\perp}\right)\right]$
3 – 4	$0.8\xi_{(3-4)}+\frac{2\mu_{s\perp}}{1-2\delta_{\perp}}\left[\xi_{(3-4)}\left(1-0.5\frac{\varepsilon_{yd}}{\varepsilon_{cu}}\right)-\delta_{\perp}\right]+\mu_{s\|}$

The ultimate curvature, φ_u, can be evaluated as a function of the non-dimensional acting axial load, n_{Sd}. It is given by:

$$\varphi_u=\frac{\varepsilon_{cm}}{\xi_u h} \tag{2.14}$$

where ξ_u is the non dimensional neutral axis depth of the compressed zone at failure and ε_{cm} is the strain at the extreme compressed fiber.

3. CONCLUSIONS

A method has been proposed that arrives at defining closed-form equations for performing the assessment of existing RC columns with two-way steel reinforcement, under combined biaxial bending and axial load, and the design of the FRP flexural strengthening. Starting from the load contour method originally proposed by Bresler (1960) and from a previous authors work (Monti *et. al*. 2006), an efficient procedure for estimating the strength/deformation section capacity has been developed. In addition, simple closed-form equations for computing section uniaxial resisting moments and ultimate curvature has been defined. The results obtained testing the approximate approaches on rectangular rc sections with different geometrical and mechanical characteristics are compared with that obtained from an exact one, which makes use of the discretization fibre method. The curves obtained with the first show very little deviation from the exact ones. The proposed method lends itself to a straightforward assessment of rectangular concrete columns: starting from the assigned axial load, the failure mode is directly found and the corresponding moment/curvature capacity computed.

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