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THEORETICAL DEVELOPMENTS AND NUMERICAL VERIFICATION OF A DISPLACEMENT-BASED DESIGN PROCEDURE FOR STEEL BRACED STRUCTURES

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ABSTRACT :

The paper discusses concepts and presents procedures for the development of a displacement-based design (DBD) methodology for steel braced structures. One basic concept in DBD is the independence of yield deformations of structural members from their strength. This independence allows the yield deformation to be computed before selecting member cross section properties, i.e. at the starting phase of the design. Using appropriate sub-structuring techniques, yield displacements of the whole structure can also be computed independent of member strength. In addition, post-yield displacements can be computed before member cross sections have been completely detailed. This only requires that the ultimate limit state for the whole structure (e.g. the achievement of a limiting ductility in one or more members) is clearly identified. Once inelastic target displacements have been computed, earthquake engineering methods can be applied to compute the strength required in order that displacements are not exceeded under the design-level earthquake. The case of inverted V-bracing is taken as case study, because of specific features emphasizing problems that could be encountered in the application of the new methodology. Both static and dynamic inelastic response analyses under a set of selected acceleration records have been carried out, with reference to a 10-storey frame. Results are very encouraging about the potentialities of the novel methodology, showing that maximum displacements, drifts and ductility demand are within the limits imposed at the design stage.

KEYWORDS: Capacity design, displacement-based design, dynamic time-history analysis, inverted V-bracing, pushover analysis, steel structures

1. INTRODUCTION

Seismic design of structures is currently codified by structural Codes and Standards of practice, using a so-called "force-based" approach. In its most basic form, this approach does not significantly differ from the design method used for any other external action (such as gravity loads and wind loads). In fact, earthquake actions are simulated by means of "equivalent" static forces, whose intensity and distribution is fixed by the Code as a function of (i) the earthquake shaking intensity at the site of interest and (ii) the structural type (for example, braced structures are assigned different forces than moment resisting frames). In this procedure, the structure displacements are only the final output of the design process, to be calculated once all the member cross sections have been fixed.
As it is well-known, earthquake-induced forces can be approximated by means of Equation 1 (Chopra 1995):

$$F_i = \frac{m_i \left(u_i / u_r\right)}{\sum_1^N m_i \left(u_i / u_r\right)} V_b = \frac{m_i \phi_i}{\sum_1^N m_i \phi_i} V_b \qquad (1)$$

where F_i is the earthquake force at the *i-th* floor level, m_i and u_i are mass and displacement at the *i-th* floor, u_r is a reference displacement (e.g. the roof displacement) and V_b is the required base shear strength. Equation 1 says that earthquake forces are approximated by the product of two terms: 1) a height-wise distribution factor and 2) the base shear. The height-wise distribution factor depends on the shape of lateral floor displacements ($\phi_i = u_i/u_r$).
Equation 1 clearly shows that displacement shapes ($\phi_i = u_i/u_r$) are needed to define forces. Besides, basically starting

from the approximation that one mode of vibration is dominant (which is implicit in using Equation 1), proposals have been made for substituting the real multi-storey building with a substitute single degree of freedom structure (Shibata and Sozen, 1976). In this context, displacement-based design (DBD) methodologies have been proposed, aiming at an explicit and case-by-case definition of the base shear strength required to meet a selected seismic performance objective (Priestley *et al.*, 2007). In the DBD procedures, displacements are used as the main input to the design process, as opposite to the force-based procedures. The displacement shape ($\phi_i = u_i / u_r$) is usually fixed using some mixture of theory and empirical knowledge of the specific structural type under analysis. Frequently, the displacement shape is derived based on the statistical characterization of the response of a large number of 'typical' frames. A displacement-based design method incorporating definition of the displacement shape directly in the design process is discussed in this paper. The method is based on sound mechanics of elastic structures, but aims at controlling both the elastic and inelastic deformation patterns. This is achieved by pre-selecting the inelastic parts of the structure and recognizing that the remaining parts of the structure can still be dealt with as normal elastic structures subjected to imposed displacements at the connection points between elastic and inelastic regions. The case of inverted V-bracing is taken as case study, because of specific features emphasizing problems that could be encountered in the application of the new methodology.

2. BASIS OF THE DESIGN METHOD

2.1. General considerations

Figure 1a illustrates a schematic representation of backbone response curves for steel braces. N_{pl} is the axial force corresponding to a fully yielded cross section. χ is the reduction factor for strength in compression, in case of first loading. Values of χ are given by structural codes as function of the normalized slenderness $\bar{\lambda}$. χ_{r} gives the residual strength in compression, after large compression deformation and it may be assumed in the form $\chi_{\mathrm{r}} = a + b\bar{\lambda}^{-c}$ (Tremblay, 2002). $\chi_{\mathrm{r}}^{\mathrm{t}}$ is an intermediate value between χ_{r} and χ , and it is also function of the axial shortening of the brace. h_{p} is a coefficient accounting for maximum strain hardening of the brace in tension (Tremblay, 2002). Δ_{y} indicates the axial elongation corresponding to N_{pl} , while Δ_{c} is the inelastic deformation capacity of the brace. Tremblay (2002) gives information about the ductility available in the last deformation excursion before failure $\mu_{\mathrm{F}} = c + d\bar{\lambda}$. Assuming a symmetric cyclic loading history, the monotonic ductility capacity, taking into account cyclic degradation effects, may conservatively be estimated as equal to $\Delta_{\mathrm{c}} / \Delta_{\mathrm{y}} = \mu_{\mathrm{F}} / 2$.

A typical pushover response of a structure with slender braces is illustrated in Figure 1b. The following limit states may be identified:

1. Compression brace buckling.
2. Tension brace yielding.
3. Compression brace failure (maximum available ductility).

For stockier braces the transition from buckling to ultimate state is not so smooth as indicated in Figure 1b.
The three limit states are also illustrated in Figure 2, in terms of member forces.

The design (target) displacement, to be reached under the design-level earthquake, may be located in an intermediate position between brace buckling and the ultimate state, according to the design choice and/or necessity.

Figure 3 illustrates a reference state of the structure. It is obtained assuming that one brace at every story is fully yielded and hardened in tension, while neglecting the other (compression) brace. This is a virtual state, which is found convenient to be introduced because beam and column forces can be computed independent of the displacement. It is easy to see that beam bending moments are maximized in the reference state ("capacity design"). It may also be proved that compression column axial forces are maxima at the last two storey levels. This is not true for lower stories. However, the use of a tension force equal to the maximum value $h_{\mathrm{p}} N_{\mathrm{pl,br,i}}$ at every storey practically compensates for neglecting compression brace residual strength. In fact, in the real structure, maximum column axial forces would be reached with only some (or also none) of the tension brace axial forces reaching $h_{\mathrm{p}} N_{\mathrm{pl,br,i}}$, while the remaining tension braces being characterized by forces smaller than $h_p N_{pl,br,i}$ and the compression braces by some residual strength $\chi_r^t N_{pl}$. Besides, the difference in column axial forces computed for the assumed reference state (Fig. 3) and the theoretical upper bound situation may be small, because of small residual strength in the buckled brace. Therefore, because of simplicity and convenience in the analytical formulation, the scheme of member forces shown in Figure 3 will be used as reference situation for strength design of beams and columns. Anyway, any other preferred reference state (i.e. any other capacity design criteria) could be assumed, without affecting the DBD design procedure which is outlined in the following Sections.

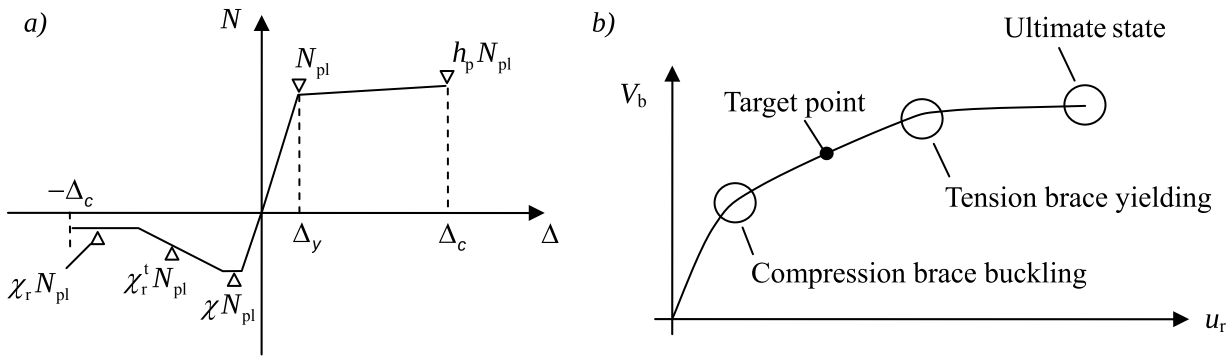

Figure 1. Schematic response of braces (a) and braced structures (b).

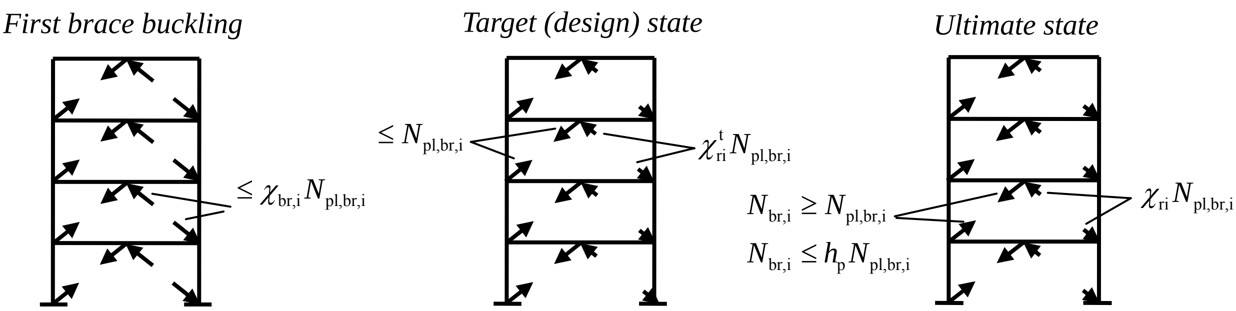

Figure 2. Buckling, target and ultimate state in terms of brace forces.

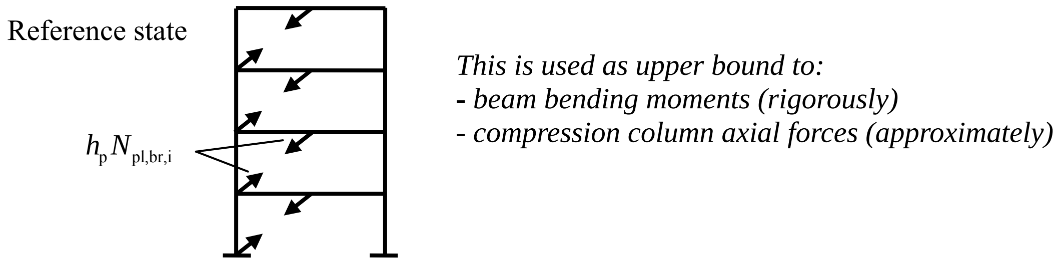

Figure 3. Reference state.

2.2. Outline of the design procedure

Basic concepts behind the proposed design method have been presented in Della Corte (2006).
The design procedure can be summarized with the following list of main steps:

1. Compute yielding (i.e. first brace buckling) displacements.
2. Compute target inelastic displacements.
3. Compute the required base shear strength at the target (design) state (e.g. by the equivalent viscous damping approach (Priestley *et al.*, 2007)).
4. Reduce the base shear force from the target to the buckling state.

5. Design braces in such a way that incipient brace buckling occurs under the base shear force computed at step 4.
6. Design columns and beams using forces in the reference state (upper bound to beam and column forces, Figure 3).

Key concepts of the procedure are briefly discussed hereafter, starting with a simple one-storey structure.
Buckling displacements (u_y) can be computed by means of a function having as input variables the brace and column axial strains (ε_{br} and ε_c , subscript "y" indicates that the relevant quantity is computed at the buckling state):

$$u_y = f(\varepsilon_{br,y}, \varepsilon_{c,y}) \quad (2)$$

Analogously, a function of the brace and column axial strains, as well as beam deflection (v_b), can give displacements of the structure in the post-buckling stage (subscript "d" indicates that the relevant quantity is computed at the design (target) state):

$$u_d = g(\varepsilon_{br,d}, \varepsilon_{c,d}, v_{b,d}) \quad (3)$$

Derivation of Equations (2) and (3) can be based solely on kinematics of the structure, i.e. it does not require consideration of equilibrium of forces. It can be shown that an additional kinematics relationship can be written, relating the beam vertical displacement to the brace and column axial strains in the design state:

$$v_{b,d} = g_1(\varepsilon_{br,d}, \varepsilon_{c,d}) \quad (4)$$

Once Equations (2), (3) and (4) are known, using the brace and column axial strain as design variables, buckling and target displacements can easily be computed, independent of forces. For example, with reference to buckling displacements, one can fix the brace axial strain equal to the value corresponding to incipient buckling $\varepsilon_{br,y} = \chi_{br}\varepsilon_y$ while taking the column axial strain equal to a fraction of its buckling value $\varepsilon_{c,y} = \rho_c \chi_c \varepsilon_y$, with $\rho_c < 1$. The coefficient ρ_c must be appropriately and carefully selected: in fact, after brace buckling the base shear increases, thanks to the increase in the tension brace force, thus producing an increase in the compression column force. Therefore, the column axial strain at buckling must be fixed looking at the ultimate response of the structure. The coefficient ρ_c cannot be rigorously defined at the starting stage of the design, because it depends on the actual cross section given to the braces. Therefore, it is rather to be assumed as a reasonable value and eventually changed with a few iterations on the design solution.
In case of multistory buildings, it is also required to fix the height-wise distribution of braces' and columns' axial deformations. Furthermore, when going from ideal situations, with only earthquake loads, to more general cases, with also vertical loads taken into account, one additional difficulty is encountered. Vertical loads may produce initial strain (and stress) in the braces. Consequently, the sequence of brace buckling may be far away from the assumption made considering only earthquake effects. This is because the incidence of the initial strain due to gravity loads is not uniform over the height of the building, being larger at the upper stories where the earthquake shear effects are smaller. Therefore, it may be foreseen that a sufficiently accurate representation of the frame inelastic response must necessarily take into account the effect of gravity loads.

3. MATHEMATICAL FORMULATION

3.1. Pre-buckling displacements

The explicit form of Equation (2) can be obtained using the sub-structuring technique illustrated in Figure 4. The storey drift is obtained as the superposition of a rigid story rotation, which is due to the axial shortening and elongation of columns below the storey under examination, and the effect of brace deformations.

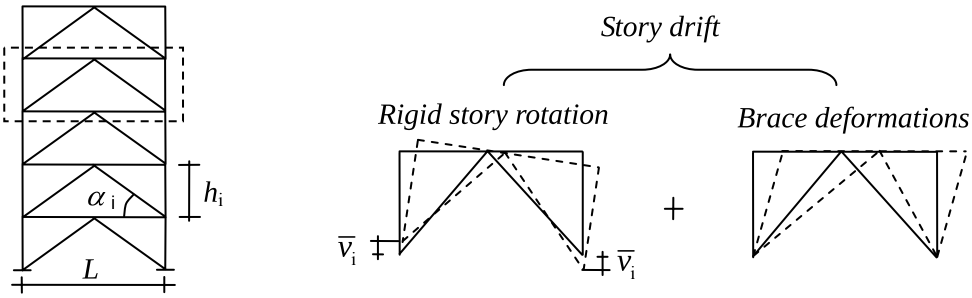

Figure 4. Frame sub-structuring and schematization for calculation of buckling displacements.

Consequently, the drift corresponding to the first significant non-linear event in the braced structure, indicated hereafter as "yield" drift, can be expressed as given by Equation (5):

$$\theta_{\mathrm{i,y}} = \frac{2\bar{v}^{\mathrm{E}}_{\mathrm{i,y}}}{L} + \frac{2\varepsilon^{\mathrm{E}}_{\mathrm{br,i,y}}}{sen2\alpha_{\mathrm{i}}} \tag{5}$$

where:

- $\varepsilon^{\mathrm{E}}_{\mathrm{br,i,y}}$ is the brace axial strain at the *i*-th storey, due to earthquake loads;
- L, and α_{i} are the braced bay length and the angle of the brace on the horizontal axis, respectively (Fig. 6).
- $\bar{v}^{\mathrm{E}}_{\mathrm{i,y}} = \sum_{j=1}^{i-1} \varepsilon^{\mathrm{E}}_{\mathrm{c,j,y}} h_{\mathrm{j}}$ is the vertical displacement at the base of the *i*-th storey column, due to the axial shortening/elongation of columns ($\varepsilon^{\mathrm{E}}_{\mathrm{c,i,y}}$ is the column axial strain at the *i*-th story, due to earthquake loads, h_{j} is the interstorey height, Fig. 4).

3.2. Post-buckling displacements

In the post-buckling range, the drift increases because of the increase of column and brace axial deformations, but also because of beam flexural deformations due to the unbalanced vertical force transmitted by the tension and compression braces (Fig. 5).

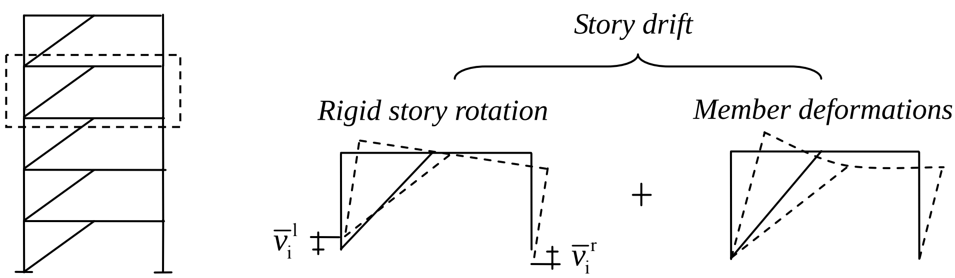

Figure 5. Frame sub-structuring and schematization for calculation of design (target) displacements.

Using the scheme of Figure 5, Equation (6) could be derived for the *i*-th storey drift in the post-buckling range:

$$\theta_{\mathrm{i,d}} = \frac{\bar{v}^{\mathrm{r}}_{\mathrm{i,d}} - \bar{v}^{\mathrm{l}}_{\mathrm{i,d}}}{L} + \frac{2\varepsilon^{\mathrm{tension}}_{\mathrm{br,i,d}}}{sen2\alpha_{\mathrm{i}}} + \frac{v_{\mathrm{b,i,d}}}{h_{\mathrm{i}}} tg\alpha_{\mathrm{i}} + \frac{\varepsilon^{\mathrm{r}}_{\mathrm{c,i,d}} + \varepsilon^{\mathrm{l}}_{\mathrm{c,i,d}}}{2} tg\alpha_{\mathrm{i}} \tag{6}$$

where:

- $\varepsilon^{\mathrm{tension}}_{\mathrm{br,i,d}}$ is the tension brace axial strain, at the *i*-th storey;

- $v_{\mathrm{b,i,d}}$ is the beam mid-span vertical deflection, at the *i*-th storey;
- $\varepsilon^{\mathrm{r}}_{\mathrm{c,i,d}}$ and $\varepsilon^{\mathrm{l}}_{\mathrm{c,i,d}}$ are the right and left column axial strains, at the *i*-th storey (taken positive if they are compression strains);
- $\overline{v}^{\mathrm{r}}_{\mathrm{i,d}}$ and $\overline{v}^{\mathrm{l}}_{\mathrm{i,d}}$ are vertical displacements, at the base of the *i*-th storey right and left column, due to elongation/shortening of columns from the first to the (*i*-1)-th storey.

and other symbols have the meaning already declared.
Using the scheme of Figure 5, an additional Equation can be derived, relating the beam mid-span vertical deflection to the braces axial deformations, as seen into Equation (7) ($\mu_{\mathrm{i,d}}$ is the design ductility for the *i*-th compression brace):

$$\varepsilon^{\mathrm{tension}}_{\mathrm{br,i,d}} + 2\left[\frac{v_{\mathrm{b,i,d}}}{h_{\mathrm{i}}} + \left(\frac{\varepsilon^{\mathrm{r}}_{\mathrm{c,i,d}} + \varepsilon^{\mathrm{l}}_{\mathrm{c,i,d}}}{2}\right)\right] sen^2\alpha_{\mathrm{i}} = \mu_{\mathrm{i,d}}\varepsilon_{\mathrm{y}} \quad (7)$$

3.3. Relationship between design and yielding limit state strength

At step 4 of the design procedure previously outlined, the base shear force demand calculated for the design state must be scaled to the first-buckling situation in order to design braces and, subsequently, columns and beams. The relationship between the base shear force at the design situation and the same quantity evaluated at buckling is given by Equation (8):

$$\frac{V_{\mathrm{b,d}}}{V_{\mathrm{b,y}}} = \frac{(\overline{\varepsilon} + \chi_{\mathrm{r}})N_{\mathrm{pl,br,1}}\cos\alpha_1}{\frac{1}{\Omega_1}N_{\mathrm{pl,br,1}}\cos\alpha_1} = (\overline{\varepsilon} + \chi_{\mathrm{r}})\Omega_1 \quad (8)$$

where $\overline{\varepsilon}$ is a design parameter governing the distance of the target point from the ultimate state (Fig. 2) and $\Omega_1 = \frac{N_{\mathrm{pl,br,1}}}{2N_{\mathrm{br,1,y}}} = \frac{\varepsilon_{\mathrm{y}}}{2\varepsilon_{\mathrm{br,1,y}}} = \frac{1}{2\chi_{\mathrm{br,1}}}$ ($N_{\mathrm{br,1,y}}$ is the 1st-storey brace axial force at buckling).

3.4. Design criteria

The main design input to the DBD procedure is the normalized slenderness of braces and columns. For braces, a linear distribution with decreasing values from the top downwards has been found effective, such as for example $\overline{\lambda}_{\mathrm{br,i}} = a - b(N - i)$ where *N* is the number of storeys, *a* and *b* are two design parameters. The slenderness distribution chosen for the columns is much less critical and a first-trial uniform distribution may be found appropriate.
Application of the DBD procedure also requires selection of design values for the brace and column axial deformations at the relevant limit states. The assumed strain distributions have significant consequences on the final displaced shape (Eq. 5) and consequent strength assignments (Eq. 1). One possible distribution, which has been found effective to get every brace buckling in compression, is given by Equation (9):

$$\varepsilon^{\mathrm{E}}_{\mathrm{br,i,y}} = \rho_{\mathrm{br,i}}\chi_{\mathrm{br,i}}\varepsilon_{\mathrm{y}} = \chi_{\mathrm{br,N}}\varepsilon_y \quad (9)$$

Equation (9), jointly with the assumed brace slenderness distribution, implies that the first brace to buckle under the action of lateral forces is that one at the top storey $\varepsilon^{\mathrm{E}}_{\mathrm{br,N,y}} = \chi_{\mathrm{br,N}}\varepsilon_{\mathrm{y}}$ and subsequently the other braces going from the top downwards. Equation (9) assumes that the initial brace strain due to gravity loads is a small fraction of the total brace strain.
The column deformation at the brace buckling limit state must be selected in such a way to avoid column buckling at the ultimate state. This implies the need to select values appropriately small at the brace buckling state, in order to permit the required strength increase to be developed at the ultimate state (Fig. 1b). Equation (10) was found effective.

$$\varepsilon^{\mathrm{E}}_{\mathrm{c,i,y}} = \rho_{\mathrm{c,i,y}}\chi_{\mathrm{c,i}}\varepsilon_{\mathrm{y}} = \frac{\chi_{\mathrm{c,i}}\varepsilon_{\mathrm{y}}}{\Omega_1 h_{\mathrm{p}}\left(N^{\mathrm{G+E}}_{\mathrm{c,i,max}} / N^{\mathrm{E}}_{\mathrm{c,i,max}}\right)} \tag{10}$$

The column axial force due to gravity loads is not known at the starting of the design process. Therefore, a trial value must be assumed and then corrected if larger values are calculated. This implies a few iterations in the design process. The target displacements can be calculated using Equations (6) and (7), with the following approximate and simplified design assumptions for member deformations (γ_{bk} is a partial safety factor against column buckling):

$$\varepsilon^{\mathrm{tension}}_{\mathrm{br,i,d}} = \bar{\varepsilon}\varepsilon_{\mathrm{y}} \;;\; \varepsilon^{\mathrm{comp}}_{\mathrm{br,i,d}} = \mu_{\mathrm{d,i}}\varepsilon_{\mathrm{y}} \;;\; \varepsilon^{\mathrm{r}}_{\mathrm{c,i,d}} = \frac{\bar{\varepsilon}}{h_{\mathrm{p}}}\frac{1}{\left(N^{\mathrm{G+E}}_{\mathrm{c,i,max}} / N^{\mathrm{E}}_{\mathrm{c,i,max}}\right)}\frac{\chi_{\mathrm{c,i}}\varepsilon_{\mathrm{y}}}{\gamma_{\mathrm{bk}}} \;;\; \begin{matrix} \varepsilon^{\mathrm{l}}_{\mathrm{c,i,d}} = 0 & \text{for } i = 1, n-1 \\ \varepsilon^{\mathrm{l}}_{\mathrm{c,i,d}} = -\varepsilon^{\mathrm{r}}_{\mathrm{c,i,d}} & \text{for } i = n \end{matrix} \tag{11}$$

The most appropriate value to be assigned to the parameter $\bar{\varepsilon}$ may depend on the specific design case. A first-trial calculation with $\bar{\varepsilon} = 1$ is suggested, but there could be cases where a value smaller than 1 is more appropriate.

4. RESULTS FROM AN EXAMPLE OF APPLICATION

4.1. General data

A ten-storey one-bay frame has been designed. The length of the bay is equal to 6 m, while the interstorey height is 3.5 m exception made for the first story where it is equal to 4 m. The floor mass is equal to about 55 kNs^2/m at the roof level, 51 kNs^2/m at other floors. The design spectrum is taken from EC8: type 1, ground type C, PGA = 0.35g. The European S 275 steel has been used (expected average yield stress equal to 316 MPa). European I-shaped wide-flange cross sections have been used for beams and columns, while circular hollow sections have been adopted for braces. The cross section size obtained at every floor level for the frame members is given in Table 1.
Subsequently to the design, the structure has been analyzed using both static and dynamic analysis. In particular, dynamic time-histories analysis has been carried out using a set of 7 acceleration records, selected and scaled in order to get an average displacement response spectrum as close as possible to the design spectrum.

Table 1. Cross sections of members obtained for the analyzed case.

Storey #	Columns	Beams	Braces
	Wide-flange sections	*Wide-flange sections*	*Circular hollow sections (diameter x thickness)*
10	HE B 180	HE B 500	96 x 4
9	HE B 180	HE B 500	101.6 x 6
8	HE B 260	HE B 650	114.3 x 7
7	HE B 260	HE B 650	114.3 x 9
6	HE B 360	HE M 600	114.3 x 10
5	HE B 360	HE M 600	127 x 10
4	HE B 550	HE M 650	139.7 x 10
3	HE B 550	HE M 650	139.7 x 10
2	HE M 550	HE M 700	139.7 x 10
1	HE M 550	HE M 700	152.4 x 10

4.2. Numerical results

Figure 6 summarizes the results of the static and dynamic analysis of the designed V-bracing. Namely, Figure 6a shows the first vibration mode of the structure and compares it with the prediction made at the design stage (normalized pre-buckling displacements). Figure 6b shows a similar comparison, but with reference to the design (post-buckling) displacements. Figure 6c summarizes the peak displacement demand from time-history response analysis. The average displacement demand is smaller than the target displacements, because of two main reasons: (i) the actual stiffness and strength of the structure are larger than the design values because commercial profiles of members do not exactly match the design output; (ii) the Rayleigh modeling of viscous damping, which has been based on the initial stiffness rather than the tangent stiffness. The scattering in displacement response is mainly attributed to record-to-record variability of displacement spectra. Finally, Figure 6d illustrates the average ductility demand to braces together with target and capacity values. It is interesting to note that the structure over-strength has a beneficial effect on the limitation of local ductility demand due to higher mode effects.

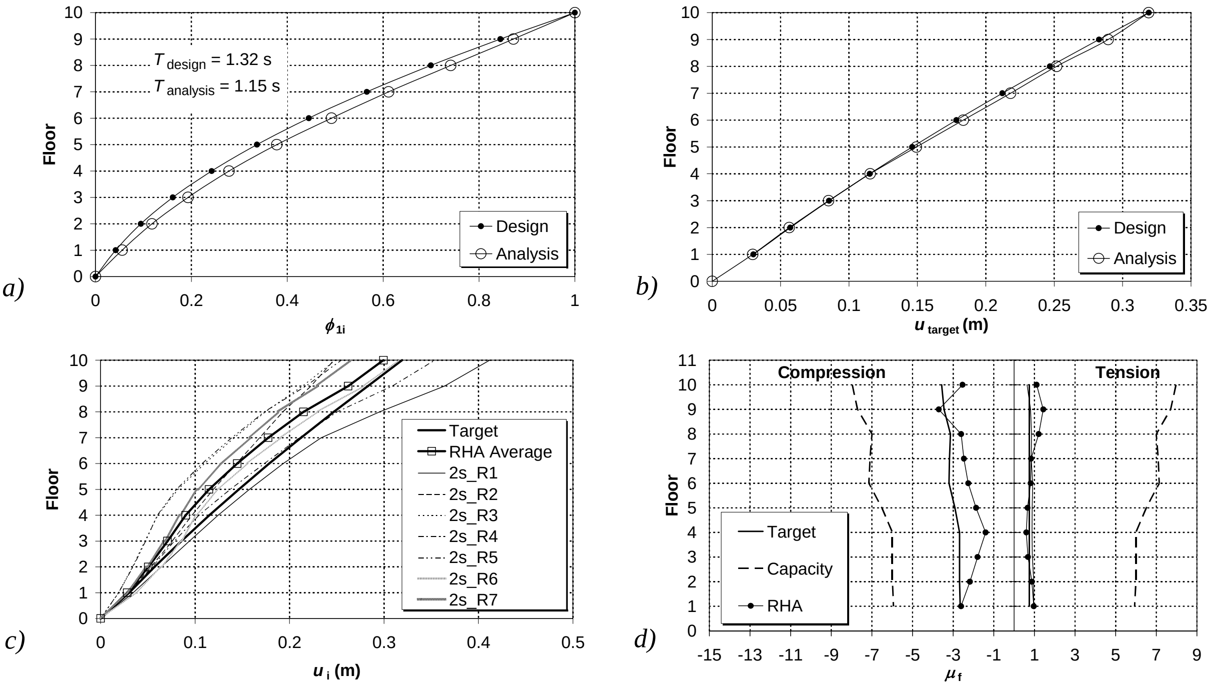

Figure 6. Results from numerical analysis of an inverted V-bracing.

5. CONCLUSIONS

A novel methodology for displacement based design of steel braced structures has been presented. The main distinctive feature of the procedure is the direct calculation of the structure displacements corresponding to selected limit states. This is made on the basis of simple analytical relationships between storey drift and member deformations, based on the selection of the yielding zones. The proposed approach permit to control both the elastic and inelastic displacements. Results from one application to a case study have also been shown in the paper. The comparison of the analytical design-stage predictions and the numerical finite element modeling results shows that the method may work properly. Further research is needed in order to assess the effect of higher modes of vibration.

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