A STUDY ON DAMAGE CONCENTRATION IN LOW-RISE STEEL MOMENT FRAMES UNDER SEVERE SEISMIC MOTIONS

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ABSTRACT:

A steel moment frame is categorized as the most ductile structure, therefore, basically has an excellent earthquake (EQ) resistant capacity. However, in 1995 Kobe EQ, premature collapses such as the first-story local sway collapse mechanism, or brittle fractures in the welding joints were widely observed in low-to-middle rise steel moment frames due to damage concentration.
The damage (plastic strain energy) concentration to a specific story or the inter-story damage distribution phenomena are studied through dynamic elasto-plastic response analyses using trial modeling of low-rise (3 stories) steel moment frames.
The results are summarized and discussed in terms of the story-drift responses, the cumulative rotation capacities of plastic hinges, or the plastic-energy absorbed quantities and its concentration patterns.

KEYWORD: Column and beam strength ratio, Type of column-base, Cumulative plastic deformation ratio η

1. INTRODUCTION

A steel moment frame is known as the most ductile structure, hence has an excellent earthquake (EQ) resistant capacity. However, in the 1995 Kobe EQ, premature collapse modes such as the local sway collapse mechanism (1st story collapse mode) or brittle fractures in the welding joints were widely observed in low-to-middle rise steel moment frames.
In the other hand, the complete failure caused from local sway collapse mechanism was not observed in the 1994 Northridge EQ, although brittle fracture occurred in the welding joints of steel moment frames.
This was considered as the sake of damage distribution effect of the gravity column.
It is an interior column with pin-jointed beams that only sustained gravity loads in the perimeter-moment frames (commonly used in the United State (US)), and had contributed to prevent damage concentration to a specific story by making moment frames with highly strong-column and weak-beam condition.
The most utilized steel building in Japan is a low-rise (about 3 stories) spatial-moment frame (almost all beam-column joints are moment joints) that uses cold-formed rectangular hollow section (RHS) column and is characterized as follows;
①The story height of the 1st story is mostly higher than another upper stories.
②The column section is mostly the same section from bottom to the top story, because of the convenience of construction without field column joint.
③The column-base is mostly exposed type (semi-rigid joint) with poor ductility.
Therefore, the 1st story has more deteriorated horizontal stiffness and strength than the upper stories, which would cause the damage concentration to the 1st story.
The response in these low-rise structures, with its fundamental period less than 1.0 second, is highly changeable according to the soil conditions or the spectrum characteristics of EQ waves, therefore the seismic resistant capacity against severe EQ depends on the plastic strain energy absorbing ability of the structural members.
This paper aims to clarify the damage concentration phenomena and the required plastic deformation capacity (ductility μ or cumulative deformation η) of each plastic hinge of columns and beams in low-rise steel moment frames under severe seismic motions through dynamic elasto-plastic response analyses by trial modeling of 3-story moment frames (a total of 16 models were designed).

The strong-column and weak-beam condition is able to prevent the damage (the plastic strain energy) concentration, therefore, the column and beam strength ratio is an important index to evaluate these phenomena. However, there are 2 definitions in the column and beam strength ratio.
One is the member-level moment strength ratio defined at each column-beam joint, and the other is from the floor moment strength ratio α as shown in equation (1).
The parameters to be considered here are the column and beam strength ratio (α) defined in Eq.(1) and determined from the framing type, the column-base type (rigid or pin), and the ultimate strength level of frames (C_B=Qu/W, where, C_B :the ultimate retaining story-shear strength expressed in the base-shear coefficient, Qu: the retaining story-shear strength, and W: the total building weight).
Results are summarized and discussed in terms of the story-drift angle R responses, the cumulative rotation capacities of plastic hinges η , the absorbed plastic strain energy Ep, or, its concentration patterns.

$$\alpha i = \left(\Sigma \left({}_{B\text{-}C}Mp(i) + {}_{T\text{-}C}Mp(i-1)\right) / \Sigma \left({}_{L\text{-}B}Mp(i) + {}_{R\text{-}B}Mp(i)\right)\right) \quad (1)$$

where, αi : column and beam strength ratio of i-th floor
${}_{B\text{-}C}Mp(i)$,${}_{T\text{-}C}Mp(i)$: full plastic moment of column in the i-th story, bottom and top, respectively(not the reduced plastic moment by the axial force, because of low-rise buildings).
${}_{L\text{-}B}Mp(i)$,${}_{R\text{-}B}Mp(i)$: full plastic moment of beam in the i-th floor, left and right end, respectively

2. SEISMIC DESIGN OF ANALYTICAL MODELS

The analytical models are 3-story symmetrical moment frames (a rectangular shape plan with 3 spans of 7.2m), and the typical floor height is 4.0m (4.5m in the ground story), as shown in Fig. 1 and 2., where, “US” indicates the perimeter moment frame with gravity column commonly used in the United States, and “JP” the spatial moment frame without gravity column commonly used in Japan.

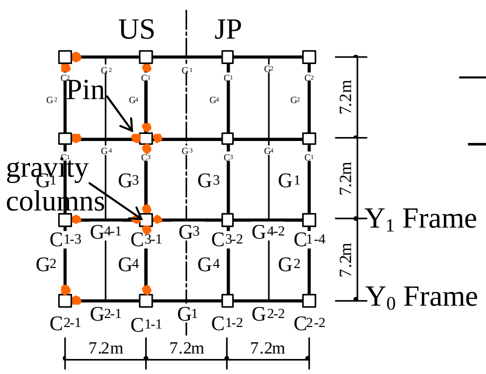


Fig.1 Structural Plan

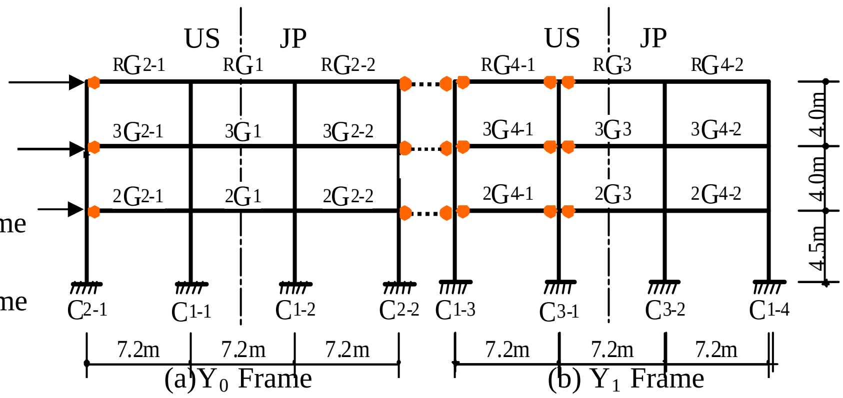


Fig.2 Structural Framing

The designed story-shear force Qi determined from Ai distribution (defined in the current Building Standard law of Japan) and 0.2 of the base-shear coefficient, or the story mass Wi are shown in Table 1.
The materials used are the cold-formed RHS column (BCP235) and H-sectional beam (SN400).
The columns of 1-3 story and beams of 2-3 floors have basically the same section, however, the 2nd floor beams have slightly larger dimensions to be compatible with the stress distribution in the frame with the pin column-base type.
Table 2 shows the 16 analytical models whose names indicate the parameters in order of “**the ultimate strength level**”, “**the type of column-base**”, and “**the column and beam strength ratio** α ”, which are as follows;.
① The ultimate strength level (0.3, or 0.5)
C_B=Qu/W≒0.3 (models 1-8), and ≒0.5 (models 9-16), where, Qu is defined when the maximum story drift angle reaches to 2% by push-over analyses. The fundamental vibration periods are 0.96-1.04sec.(models 1-8) and 0.63-0.84sec.(models 9-16), which are longer as C_B level decreases.
② The type of column-base (r, or p)
r: rigid (fixed joint) column-base (models 1-3,7,9-11,15), and p: pin column-base (models 4-6,8,12-14,16).

③ The column and beam strength ratio α (=average value of the 2nd and 3rd floors)
α<1.0 (models 1,4,9,12), α≑1.3 (models 2,5,10,13), α≑1.7-2.0 (models 3,6,11,14), and α>3.0 (models 7,8,15,16), where, α is larger in the perimeter moment frame (models with gravity column, 7,8,15,16,) comparing to the spatial moment frame (models without gravity column, 1-6, 9-14).

Table 1 Wi, Ai, and Qi

Story	Wi(kN)	Ai	Qi(kN)
3	3,342	1.51	1,008
2	3,477	1.2	1,641
1	3,554	1.0	2,075

Qi : story-shear-force determined from C_0 (base shear coefficient) =0.2
W : Total building weight=10,372kN (≑7-7.5 kN/m^2 in unit floor area)

Table 2. Analytical Models

Analytical models	Column : C1 (BCP235)	Beam : G1 (SN400)	Analytical models	Column : C1 (BCP235)	Beam : G1 (SN400)
1: 0.3-r-0.9	□-350x12 (350x12)	H-450x250x9x22 (9x16)	9 : 0.5-r-0.8	□-450x12 (450x12)	H-650x250x12x28 (12x25)
2: 0.3-r-1.3	□-350x12 (350x12)	H-400x200x 9x22 (9x12)	10: 0.5-r-1.3	□-400x16 (400x16)	H-500x250x12x22 [9x19]
3: 0.3-r-1.7	□-350x16 (350x16)	H-400x200x9x22 (9x19)	11: 0.5-r-1.8	□-400x19 (400x19)	H-500x200x 9x19 [9x16]
4: 0.3-p-0.9	□-500x12 (500x 9)	H-600x250x 9x22 (9x19)	12: 0.5-p-0.8	□-500x19 (500x12)	H-700x300x16x28 (14x22)
5: 0.3-p-1.3	□-500x12 (500x 9)	H-600x250x12x22 (12x16)	13: 0.5-p-1.3	□-550x16 (550x12)	H-650x250x16x28 (12x25)
6: 0.3-p-2.0	□-450x22 (450x16)	H-600x200x12x22 (12x16)	14: 0.5-p-1.8	□-500x22 (500x19)	H-700x200x12x25 (12x22)
7*:0.3-r-4.1	□-500x22 (500x19)	H-600x200x12x22 (*1)	15* :0.5-r-3.5	□-500x36 (500x28)	H-700x250x14x28 (*3)
8* :0.3-p-3.0	□-700x28 (600x19)	H-750x300x16x32 (*2)	16* :0.5-p-3.5	□-700x36 (700x22)	H-850x300x16x32 (*4)

① Columns in the table are the ground story columns, and ( ) indicate the 2nd and 3rd stories
② Beams in the table correspond to the 2nd floor, ( ) indicates the 3rd and R-th floors, [ ] indicates the R-th and 3rd floors are the same sections of the 2nd floor
③* Models(7,8,15,16) are perimeter moment frames with gravity columns, where all gravity columns are □-300x9
④ (*1) : 3rd floor beam H-600x200x12x22(19), where,( ) indicate R-th floor.
(*2), (*3) : 3rd floor beam H-700x300x12x22 (19), where, ( ) indicate R-th floor
(*4) : 3rd floor beam H-850x300x16x25 (22), where,( ) indicate R-th floor

3 STATICAL ANALYSES

3.1 PUSH-OVER ANALYSES

Two frames (Y_0 and Y_1) connected with pin-jointed rigid beams, according to the rigid-slab assumption and symmetrical framing system, are analyzed using "clap. f[1]", which is an elasto-plastic computer program that considers both material and geometrical non-linear behaviors utilizing the generalized plastic-hinge concept (strain-hardening effect of steel material Et/E=0.01 is included).
Horizontal loads of Ai distribution are proportionally applied to each story after the vertical loads are applied.

3.2 STORY-SHEAR FORCE to STORY DRIFT RELATIONSHIP and COLLAPSE MECHANISM

The story shear force to story drift relationship and the collapse mechanism of models 2,3,5-8 (models with C_B≑0.3) are shown in Fig.3, and the numbers in the figures indicate the incremental steps of push-over analyses. Push-over analyses were performed until the maximum story drift reached 15cm or 100 steps under the control of the building top horizontal deformation increment (⊿=3mm).

3.2.1 Models without gravity column (Models 2,3,5,6)

Premature plastic hinges are formed at the column-bases in rigid column-base type models (2,3), or at the column-top of the 1st story in pin column-base models (5,6) when α is small (≑1.3), where local sway collapse mechanism occurred at the 1st story.
The local sway collapse mechanism in a specific story is defined here as the plastic hinges formed at both the top and bottom of columns in a specific story and the story drift angle of this specific story is extremely larger than the other stories.
Although the premature plastic hinges are formed at the column-bases in the rigid column-base models, plastic hinges are also observed in the beams of upper stories, which is considered to be the shifting to a global collapse mechanism as α becomes larger (≑1.7).

In the pin column-base models, the plastic hinges were formed at the column-top of the 1st story, and a local sway collapse mechanism by damage concentration was observed.

3.2.2 Models with gravity column (Models 7,8)

Plastic hinges are observed in the beams of all the perimeter moment joints.

A global collapse mechanism (not a local sway collapse mechanism) is observed, because of the high strong-column and weak-beam condition (α >3) realized by the contribution of the gravity columns.

These behaviors are the same in the larger C_B (≒0.5) models (9-16), therefore the plastic hinge locations are distributed and the damage concentration into specific stories is prevented as the column and beam strength ratio α becomes larger.

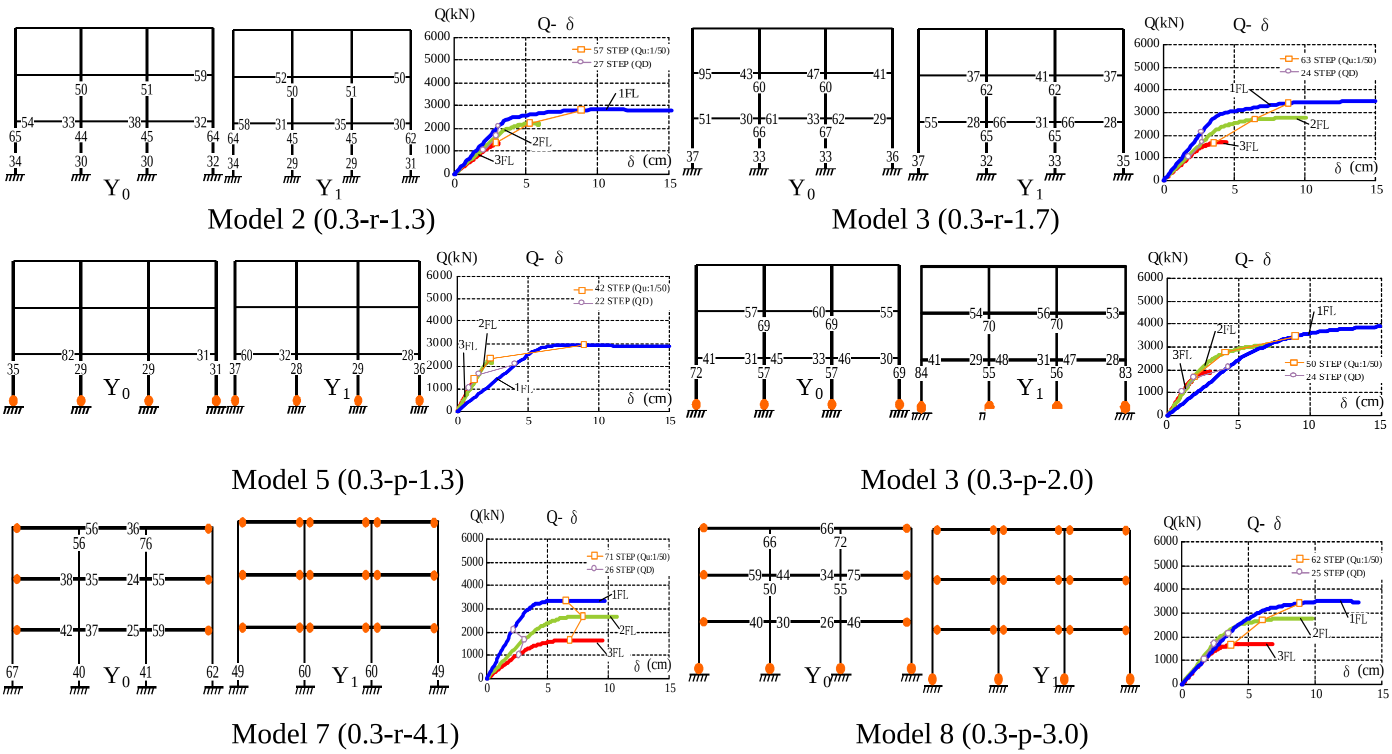


Fig 3. Story-Shear-Force to Story-Drift relationships and its Collapse Mechanisms

4. DYNAMIC ANALYSES

4.1. TIME-HISTORY DYNAMIC RESPONSE ANALYSES

The computer program "clap f[1]" is also used with the assumption of stiffness proportional viscous damping of 2% of the critical damping for the fundamental period, and the normal-bi-linear restoring force characteristic of structural members.

The input EQ waves are, as shown in Table 3, the El Centro and the Hachinohe by scaling to levels 2, and 3 (E2, E3, H2, H3, respectively), and the JMA Kobe as recorded (K3), with 30 seconds duration time.

Table 3 Maximum Acceleration of Input EQ wave (gal)

EQ waves	**Level 2**	**Accel.**	**Level 3**	**Accel.**
El Centro 1940 NS	**E2**	**511**	**E3**	**766**
Hachinohe 1968 NS	**H2**	**333**	**H3**	**500**
JMB Kobe 1995 NS			**K3**	**817**

* Level 2 indicates the maximum velocity (V_{max})=25cm/s, and V_{max}=50cm/s in Level 3

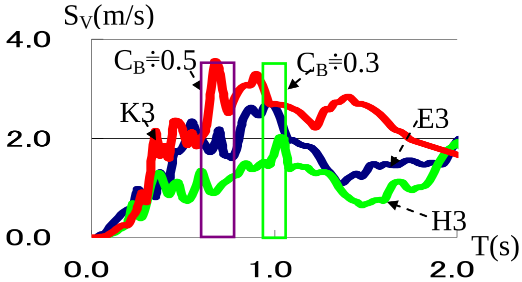


Fig 4. Velocity Response Spectrum of Input EQ (h=0.02)

4.2. MAXIMUM STORY-DRIFT RESPONSE

Figure 5 shows the maximum story-drift angle (R: x10^{-2} rad.) of 1^{st}-3^{rd} story.
Except for models with gravity columns (7,8,15,16), the maximum story-drift is tending to concentrate in the 1^{st} story, especially in the pin column-base models.
The responses are smaller as the structural strength level (C_B=Qu/W) become larger as shown in E2, E3, H2, and H3. However, in K3, the responses of larger C_B(≑0.5) models (9-16) are not smaller than those of smaller C_B (≑0.3) models (1-8). In other words, the increasing of the structural strength level in this order (C_B≑0.3 to 0.5) did not always decrease the responses. The dynamic correlations between the buildings and EQ waves (S_V spectra shown in Fig.4) have more significant effect.

4.3. EQUIVALENT VELOCITY OF INPUT EARTHQUAKE ENERGY V_E AND DAMAGE ENERGY V_D

Table 4 shows the equivalent velocity of the input EQ energy V_E and that of the damage energy V_D (sum of the plastic strain energy: Σ Ep).
As shown in "av.1" and "av.2", V_E and V_D are almost of the same order as in the case of the ultimate strength level (C_B) and the EQ wave, in spite of the differences of the column and beam strength ratio α or the type of column-base. It seems that the energy balance-based seismic design concept is effective, and that V_E and V_D can be estimated from the natural vibration period of the structure and the energy spectrum of EQ.
V_D is the dominant part of V_E (V_D/V_E=58-86%), and become larger as V_E increases and C_B decreases, therefore, the EQ energy is mainly absorbed by the plastic strain energy of the structural members, in other words, the hysteresis damping is more important than viscous damping in low-rise steel buildings.

Table 4 Equivalent Velocity of Input EQ Energy V_E and Damage Energy V_D

Models	T(S)	E2		E3		H2		H3		K3	
		V_E	V_D	V_E	V_D	V_E	V_D	V_E	V_D	V_E	V_D
1	0.96	113	88	163	131	86	67	120	100	170	147
2	1.06	111	80	158	128	85	64	122	107	167	144
3	0.98	113	84	163	130	81	56	122	100	173	145
4	0.95	114	88	165	129	86	65	122	100	170	145
5	1.01	114	86	160	126	86	64	126	108	171	145
6	1.04	115	79	159	124	89	59	128	101	173	150
7	0.99	110	82	160	127	78	56	122	101	169	142
8	1.02	113	85	159	127	85	61	127	104	172	146
"av.1"	1	113	84	161	128	85	62	124	102	171	146
9	0.63	128	98	188	156	65	37	95	72	225	191
10	0.75	132	95	197	163	54	29	98	69	206	177
11	0.75	130	89	199	163	52	25	95	64	206	181
12	0.75	136	99	200	165	55	29	104	70	210	180
13	0.78	137	108	194	161	67	33	108	77	208	174
14	0.79	139	109	200	164	68	40	106	80	212	185
15	0.8	132	101	193	156	66	42	102	78	198	173
16	0.84	154	120	206	172	83	62	117	96	205	179
"av.2"	0.76	136	102	197	163	64	37	103	76	209	180

(*) "av.1" are the average values of models (1-8), and "av.2" are of models (9-16).

4.4. Plastic Strain Energy Distributed to Each Story

Figure 6 shows the plastic strain energy (Ep(%)) distributed to each story by the severe EQ waves (E3, H3, and K3), where, the Ep of the 1^{st} story consists of the Ep for the 1^{st} story-columns and 2^{nd} floor-beams, and the Ep of the 2^{nd} story consists of the Ep for the 2^{nd} story-columns and 3^{rd} floor-beams, etc.
More than 90% of the damage (Σ Ep) concentrates to the 1^{st} story, except for the models with gravity columns (7,8,15,16).

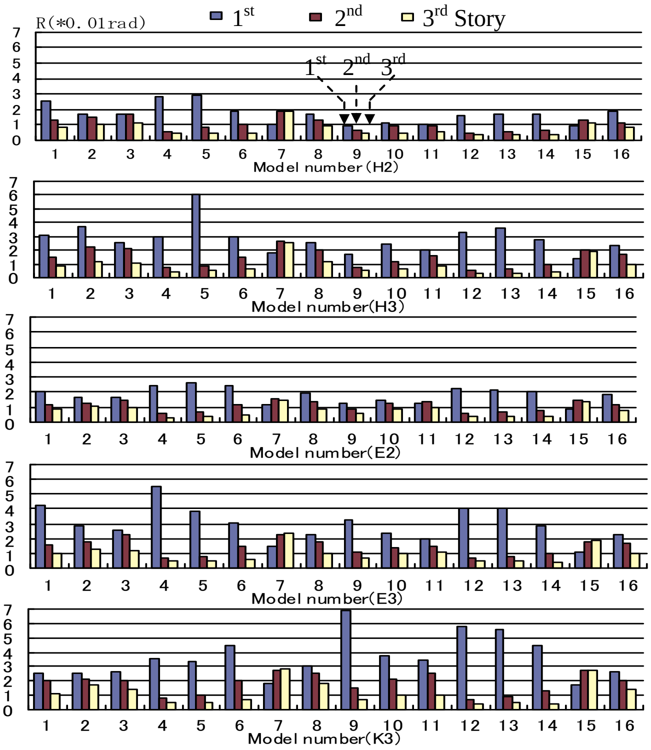


Fig.5 Maximum Story-Drift Angle (R)

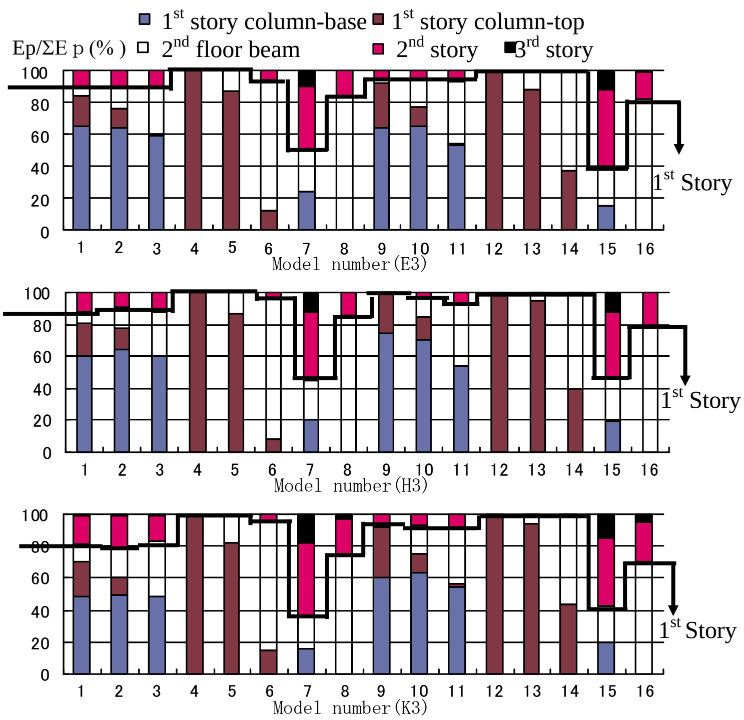


Fig.6 Plastic Strain Energy Distributed to each Story

4.5. Ductility Factor μ and Cumulative Plastic Deformation Ratio η of Each Member

Table 5 shows the maximum response in the plastic hinge of columns and beams in terms of the ductility factor μ (Eq.(2)) and the cumulative plastic deformation ratio η (Eq. (3)) by E3 and K3 waves, where, V_E is of the same order in models 1-8 (V_E=158~173cm/s), and in models 9-16 (V_E=193-225cm/s).

There is some damage concentration, such as μ >4 or η >25 (emphasized in Table 5).

The structural members with the maximum response are as follows;

① in the models without gravity columns:

at the column-base of the interior column in the rigid column-base type models, and at the top of the 1st story-column or 2nd floor-beams in the pin column-base type models.

② in the models with gravity columns:

at the beams jointed to the perimeter moment frame.

$$\mu = \theta \max / \theta y \quad (2)$$

$$\eta = (|\Sigma \Delta\theta p^+| + |\Sigma \Delta\theta p^-|) / \theta y \quad (3)$$

where, $\theta \max = \max(|\theta \max^+|, |\theta \max^-|)$: the maximum rotation angle of plastic hinge

$\Sigma \Delta\theta p = |\Sigma \Delta\theta p^+| + |\Sigma \Delta\theta p^-|$: the incremental plastic component of rotation angle of plastic hinge occurred in structural member ($\Delta\theta p^+$: plus side (+), $\Delta\theta p^-$: minus (-) side)

The rotation angle θ is the sum of elastic and plastic rotation angle of plastic hinges (= $\theta e + \theta p$), where, the elastic rotation angle θe corresponds to the member elastic deflection, and the yield rotation angle θy is determined from Mp/K (where, Mp: the full plastic moment, K : the initial stiffness as shown in Fig.7).

4.5 .1 Effects of each parameter

The effects of each parameter are as follow;

① The effects of ultimate strength level (C_B=Qu/W≒0.3, and ≒0.5)

The responses to E3 and K3 waves are almost of the same order in spite of the differences of V_E.

② The effects of column-base type (rigid, and pin joint type column-base)

The damages concentrated to the column-base, especially in the weak-column frames (α<1) of the rigid column-base model.

In the pin column-base model, the damages concentrated to the 2nd floor-beams, and as α increased, μ or η of the 2nd floor-beams also increased.

③ The effects of column and beam strength ratio α

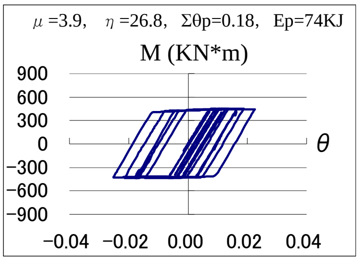
As α increased, μ and η of beams increased and for the case of columns decreased, in spite of the differences of the ultimate strength level C_B or the column-base type.

An extremely large (η >30) amount of μ and η are concentrated to the perimeter beams of the models with gravity columns (7,8,15,16,where, α >3)

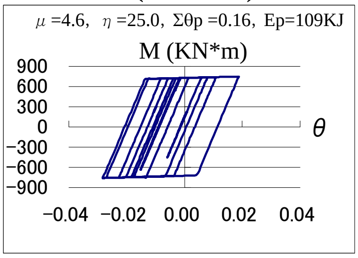
Figure 7 shows some of the hysteresis loops with large η (>25) in the responses to the K3 EQ wave, such as at the 1st story interior column-base of model 1 (*1), the 2nd floor beam of model 6 (*2), the 3rd floor-beam in the perimeter moment frame (model 7) (*3).

Table 5 Maximum Responses of μ and η

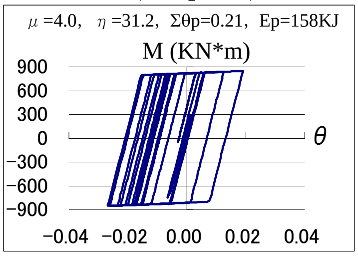
Analytical models		E3				K3			
		Beam		Column		Beam		Column	
		μ	η	μ	η	μ	η	μ	η
1	0.3-r-0.9	1.9	3.1	**5.9**	**29.8**	2.4	6.7	3.9	**26.8**(*1)
2	0.3-r-1.3	2.4	6.4	3.8	23.4	2.4	10.4	3.4	22.9
3	0.3-r-1.7	2.9	10.3	3.1	17.1	2.8	12.9	3.1	16.3
4	0.3-p-0.9	0.8	0	**6.4**	**25.2**	1.3	1.0	**4.1**	22.1
5	0.3-p-1.3	2.3	8.8	**4.2**	19.7	2.1	9.4	3.5	20.5
6	0.3-p-2.0	3.5	21.6	1.5	2.2	**4.6**	**25.0**(*2)	2.3	4.1
7	0.3-r-4.1	**4.0**	**31.7**	1.8	5.2	**4.0**	**31.2**(*3)	2.3	5.6
8	0.3-p-3.0	3.6	**26.4**	1.8	3.4	**5.1**	**29.3**	2.1	6.1
9	0.5-r-0.8	1.3	0.8	**5.8**	**30.6**	2.1	2.2	**12.4**	**42.8**
10	0.5-r-1.3	2.6	11.9	3.5	**26.7**	3.6	12.1	**5.5**	**29.0**
11	0.5-r-1.8	2.3	14.6	2.8	17.3	3.9	17.2	**4.6**	21.2
12	0.5-p-0.8	0.9	0	**4.6**	20.9	1.3	0.8	**6.5**	**25.2**
13	0.5-p-1.3	2.1	9.8	**4.6**	20.2	2.7	10.3	**6.6**	**26.0**
14	0.5-p-1.8	2.8	24.7	2.2	9.2	3.1	21.2	3.6	11.9
15	0.5-r-3.5	2.9	**32.8**	1.4	4.5	**4.6**	**31.7**	2.3	5.5
16	0.5-p-3.5	3.9	**42.8**	1.3	1.2	**4.6**	**35.9**	1.6	2.1



Model 1 (0.3-r-0.9) (*1)



Model 6 (0.3-p-2.0) (*2)



Model 7 (0.3-r-4.1) (*3)

Fig.7 Hysteresis Loops (corresponds to (* 1)-(*3))

4.5 .2 Cumulative plastic deformation ratio η of each member

Figure 8 shows the distribution of η (K3 responses of models with C_B=Qu/W≒0.3), which are as follows;

① In the models 1-3 (with rigid column-base), the maximum η occurred in the 1st story interior column-base. As α increased, the maximum η decreased and damages are distributed to the upper stories.

② In the models 4-6 (with pin column-base), the maximum η is observed in the 1st story column-top (models 4,5) or 2nd floor beams (model 6). The plastic hinge patterns changed as α increased, however, the 1st story local sway collapse mode took place .

③ The models with rigid column-base have the advantage to prevent the local sway collapse mode in comparison to the models with pin column-base. However, a large amount of ductility is necessary ($\Sigma\theta p$ >0.18rad. is required in this analysis) in the column-base.

④ In the models 7,8 (the perimeter moment frame with gravity columns, α >3), the damage concentration occurred in the perimeter beams, however, the damages are not concentrated in the 1st story but distributed to the upper stories. The maximum η is over 30 ($\Sigma\theta p$ is over 0.2rad.) as shown in the 3rd floor-beam (model 7, with rigid column-base), or in the 2nd floor-beam (model 8, with pin column-base).

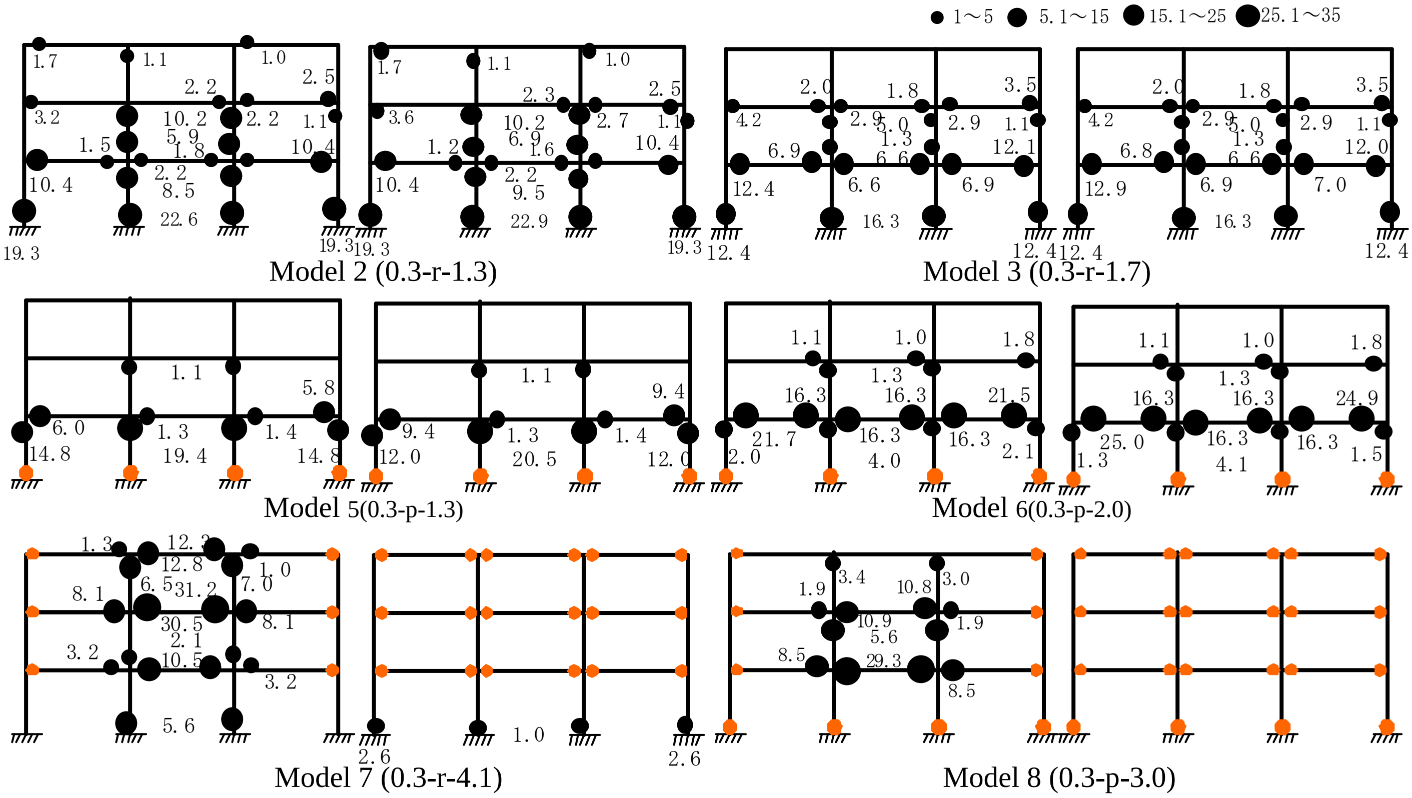


Fig.8 Cumulative Plastic Deformation Ratio η

5. Conclusions

This paper aims to clarify the damage concentration phenomena in low-rise (3 stories) steel moment frames through dynamic elasto-plastic response analyses using trial modeling with applicable ultimate strength levels (C_B=Qu/W÷0.3-0.5) under severe seismic motions (V_E=1.5-2.0m/s level). Following are the conclusions.

(1) V_D/V_E exceeds 80% when V_E=1.5-2.0m/s (severe seismic motions). Most of the input EQ energy is absorbed as plastic strain energy of structural members in low-rise steel buildings.

(2) The damages tend to concentrate in the 1st story when the column and beam strength ratio is $\alpha<2$ in the spatial moment frames (commonly used in Japan).

Large ductility (almost η =30) is required in the column-bases with rigid column-base models, and in the 1st story column-tops or the 2nd floor beams with the pin column-base models.

(3) The damage concentration in the 1st story is prevented and the story drift angle response is normalized in the perimeter moment frames (commonly applied in US) where $\alpha>3$ is easily realized from the contribution of gravity columns. However, a large cumulative plastic deformation ratio (η =50) is required in the perimeter beams, therefore, a careful detail design is needed in the width-thickness ratio of structural members or welding procedures.

(4) A column to beam strength ratio α÷1.7 is required to prevent the 1st story local sway collapse mode for low-rise spatial moment frames commonly used in Japan with rigid column-base. And a larger α(>1.7) is required when using pin column-base.

REFERENCES

[1] Ogawa K and Tada M., Combined non-Linear Analysis for Plane Frame (“clap”), Proc. of 17th Symposium on Computer Technology on Information Systems and Applications, pp.79-84, Architectural Institute of Japan, 1994.12