

DAMAGE IDENTIFICATION OF FATIGUE CRACK IN STEEL TRUSS STRUCTURE

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ABSTRACT :

Structural Health Monitoring (SHM) has emerged as a new research area in the field of civil engineering. Localized characteristics of damage in steel truss require local measurements. The main difficulties of obtaining the local parameters include: (1) exciting higher modes in an element of the truss; (2) predominance of the lower frequency global modes for the truss vibrations; (3) choice of health baseline for the truss. In this project, using theoretical and experimental methods, free vibration characteristics of an elastic simple beam with a fatigue notch crack damage located any where of the beam is investigated. And then, new health baseline based on symmetry for steel truss structure is defined. Based on this baseline, experimental data are obtained to show that the local and relative small damage can be detected even when the global dynamic characteristics of the steel truss do not change significantly.

KEYWORDS:

Structural Health Monitoring (SHM); health baseline; crack damage; modal frequency

1. INTRODUCTION

Recent years, many projects in infrastructural construction are developing rapidly. Many infrastructures such as highways, bridges, habitations and considerable construction items have been set up. And with the development of economy and the improvement of living standard, security of civil structures in their life-span is regarded as more and more important. So Structural Health Monitoring and damage identification of civil structures are popular areas of research in recent years. Housner et al. introduced some non-destructive identification methods in 1997, such as acoustics method, electrical stream method, ultrasonic method, as well as eyeballing method. The Non-destructive Identification method requests that the damage region must be prior to know (Doebeling et al. 1996, 1998). However, the instruments used by these methods are expensive, and the on-line monitoring can not be carried out. It is difficult to identify the point that can not be approach easily and the damage which hide inside the structure can not be detected either, and it would take a long time to have an overall identify of a structure. Ahn pointed out in 2003 that to identify a connective point of a building by using the non-destructive technology cost 800 to 1200 dollars. Therefore, this method has its disadvantage in either financial power or man power or overall identification. Certainly, its advantage is more direct by which the visible damage can be identified intuitively.

As regards to the issue of structural health baseline, most of the previous scholars choose the original information of structure as the baseline of damage identification, for example, Ahmed Elgamal et al. thought in 2001 that the on-line monitor to the structure which using the traditional or modern advanced health monitoring technology should be done immediately after the completion of structure, which was aimed to establish baseline data of structural health state, then using the data in hand to make a contrast with the data in use to identify the damage in structure. Shi et al.(2002) calculated and compared modal strain energy change of the structure before and after damage to identify and locate damage. As Fig. 1 showed, actually it is a method to choose complete structure as health baseline. Model update method to identify damage is also using the original information of structure as the baseline to identify damage by contrast with damaged structure. Such choice tends to be limited as followed: (1) primitive information of the structure is not complete and unobtainable sometimes; (2) primitive information and the current state information of the structure are not obtained on the same noise level. The two limitations may arouse our doubt with the conclusion that using such healthy baseline

to identify damage. How to choose the damage baseline and factor? If choose the whole structural parameters as damage factors, fine identification consequence may not be obtained by using the damage identification method vibration-based; Good identification consequence may be obtained by choosing local vibration parameter as damage factor. This is the very key point of the paper: structural damage detection baseline.

Fairly speaking, global and local diagnosis technology both have their own advantages and disadvantages, and they should be used together to complete damage detection task of the complicate and huge civil structure. The global diagnosis technology vibration-based can detect whether the damage happen or not and where the damage location possibly happen, then utilizing local diagnosis technology to locate the damage more accurately and to evaluate the degree of damage.

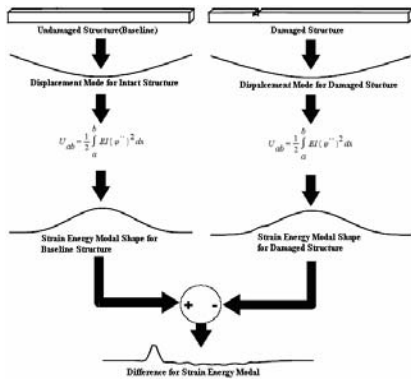


Figure 1 damage identification process of modal strain energy(Left)

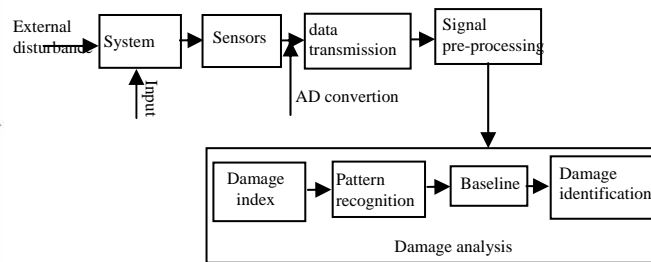


Figure 2 damage identification process of vibration-based method (Right)

2. KEY POINT OF STRUCTURAL DAMAGE DETECTION

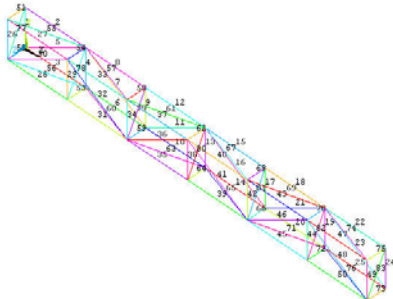


Figure 3 profile of truss bridge

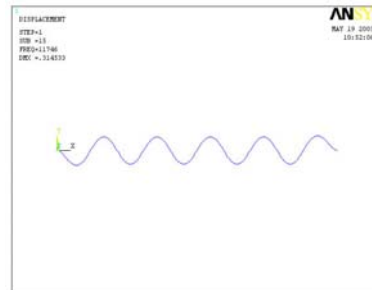


Figure 4 mode shape of a bar of truss model

The basic process of the damage identification that vibration-based can be generalized as Figure 2. The key point of structural damage identification lies on what kind of signal and establish one to one relationship between structural damage and these signals. The sensitive degree to the structural damage is different among frequency, mode shape (displacement, strain or curvature and so on), damping or other parameters which represent the structural dynamic characteristics, of which strain mode or curvature mode are more sensitive, because compared with displacement mode, strain or curvature is one-order differential of displacement, and tiny change caused by damage will be magnified by the differential process, which leads to a sudden change of strain mode. The modal frequency we usually mention is a parameter that represents the structural global particularity, and local damage of the structure has little effect on global stiffness of the structural, so the change of modal frequency is unobvious. But the vibration frequency of local member of the structure will be decreased as the increase of damage degree. Figure 3 shows us a model of truss, its vibration frequency is the global frequency; Figure 4 shows a member of the truss model, which is vibrating on the frequency of the global frequency of truss model and its own frequency. The frequency of its own is called the global vibration frequency.

3. THE DEFINITION OF STRUCTURAL HEALTHY BASELINE

3.1 The western medical baseline of diagnosis

The diagnostic process in western medical practice can be divided into the following four stages: (1) The doctor sees the patient, mainly gives the patient a preliminary examination (blood pressure, pulse and body state, etc.) and uncomfortable symptom described by the patient; (2) Have a relative physical examination according to the first stage (CT, chemical examination of blood, etc.); (3) Synthesize the examination of the first and second stages, and compare the physical standards (baselines) with those of a normal person, to give a relative diagnose; (4) Do necessary pharmacological experiment before using medicine.

The western medical method usually makes a comprehensive consideration of the above 4 stages to judge what kind of disease a man has and taking what kind of medicine to treat. Thus the baselines used in the western medical judgments are physical standards of a normal man, most of which are obtained through statistic method.

3.2 The Chinese diagnostic baseline

As an alternative to western medical diagnosis, traditional Chinese medical practitioners have relied on the so-called four diagnoses, which could be dated as far back as five thousand years, (1) Inspection: including the patient's complexion, eyes, tongue, nails, gait (overall physical appearance, openness, and emotional demeanor); (2) Listening and smelling: including the sound of the patient's voice and breathing, as well as odors associated with the body or breath; (3) Inquiring: asking for information on present and past complaints, including appetite, digestion, bowel movement, bladder, sweat, pain, patterns of sleep, family health history, work, living habits, physical environment, and emotional life; (4) Palpation: touching the body to determine temperature, moisture, pain or sensitivity, and the taking of the pulse.

Interestingly, the western and the eastern (Chinese) diagnostic procedures correlate with each other rather closely, though the processes are named differently. The most significant difference between the two diagnostic processes is in the last step, where the west resorts to the medical laboratory tests and the east relies on the non-invasive palpation. While hundreds of medical lab tests have been developed with each targeting the diagnosis of one specific disease or organ, the palpation provides non-invasive diagnosis of various vital internal organs based on the single method of taking pulses at selected locations on the human body. The fundamental difference between these two diagnostic methods is that while western medicine tends to advance towards increasing diagnostic technology, eastern medicine aims to sharpen the direct powers of observation, listening, feeling, and interrogation of the patient (Tierra, 2002).

3.3 Vibration-based Structural health baseline

Rytter (1993) classified damage detection techniques by their exactness and applicability into four levels, Level 1: Determination that damage is present in the structure; Level 2: Level 1 plus determination of the geometric location of the damage; Level 3: Level 2 plus quantification of the severity of the damage; Level 4: Level 3 plus prediction of the remaining service life of the structure.

We first set an example of healthy baseline of man to explain the problem. The blood pressure for a normal man should be 80 mm/hydrargyrum or so, diastolic pressure should be about 120mm/hydrargyrum, which is the standard for a doctor to judge whether a man has got a high-pressure disease. To judge whether a man has got other diseases also have many different quantification standards, which is a common problem in medicine. Certainly, this healthy standard of human being can be conveniently obtained through statistics. But according to the traditional Chinese medical theory, it is uneasy to obtain the baseline. An experienced old Chinese doctor can judge the healthy statement of a man by examining his pulse, but the baseline of a healthy man in his mind can not be easily obtained through statistics.

Structural health diagnosis is similar to the fact that using the Chinese medical theory to examine the patient by examining his pulse, which is to examine the structure. If we can get a health baseline that the structure is similar to the health baseline of human body, then to compare the particularity that expressed in use with this health baseline so that you can make a conclusion that whether the structure has been damaged. According to the process that the old Chinese doctor checks a patient, for a civil engineering structure, if without the coordination of modern sensor theory, signal analysis technology, and structure reflection analysis theory, it is difficult for a good structure examinant to judge and diagnose a structural damage of a huge civil engineering structure.

Usually, the civil structure has some characteristics, such as huge volume, complicate structure, etc. Therefore, it is incomparable among single structure, which means that it is uneasy to obtain the health baseline of structure by using the method of the statistics. However, how to definite the health baseline of an engineering structure? We definite the structural health baseline according to the health baseline of man as followed: such a signal, which represents a class of or a kind of or a single structure's health state, and can be used as a compare standard of some state's information in the using process of a structure, so that to examine the damage of the structure clearly, and whether the damage happened and where it happened.

3.4 The establishment of structural health baseline

Though the civil engineering structure have some particularities such as various kinds, huge volume and complicate structure, a kind of structure has a relatively simple particularity in the form of structure, which is a structure in accordance with the symmetry principle. For example, the railway steel truss bridge. As a result of the application of the symmetry principle, the mechanical character of a pair of bars in the symmetry location should be symmetrical. As for such structure, which is under the common condition, the possibility that a pair of bars in the symmetry location have the same damage is fairly small. Thus, if choose the symmetrical bars in the symmetry structure as the compare standard for each other, and to have a one by one contrast test for structural bars, and we can find that when the difference of the bars in pairs relate to the signal extends a range (damage threshold), and we can make a conclusion that someone of the bars has got a damage (usually is the poor one), then a health baseline used for local damage detection will be established.

Though the civil engineering structure have some particularities such as various kinds, huge volume and complicate structure, a kind of structure has a relatively simple particularity in the form of structure, which is a structure in accordance with the symmetry principle. For example, the railway truss bridge (Figure 6), from the geometry point of view, the bridge has vertical and horizontal axis of symmetry in whole, in local meaning, bottom and top bars, vertical or tilting bars are arranged in pairs. As a result of the application of the symmetry principle, the dynamic characteristics that are showed by a pair of members in the symmetry location also should show a symmetrical principle, which should have the same dynamic particularities. As for such structure, which is under the common condition, the possibility that a pair of bars in the symmetrical location have the same damage is fairly small. Thus, if choose the pair bars in the symmetry structure as the compare standard for each other, and to have a one to one contrast test for structural bars, and we can find that when the difference of the bars in pairs relate to the signal extends a range (damage threshold), and we can make a conclusion that someone of the pairs has got a damage (usually is the poor one), then a health baseline used for local damage diagnosis of structure will be established.

4. A CONTINUOUS CRACKED SIMPLY-SUPPORTED BEAM

4.1 The Crack Model

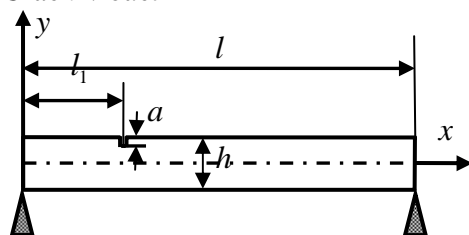


Figure 6: Schematic of a simply-supported beam with a notch crack (left)

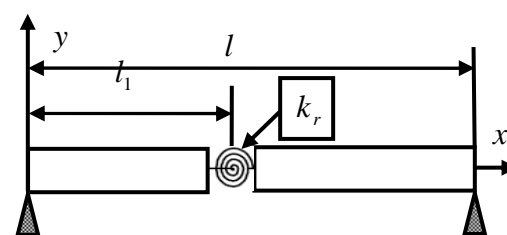


Figure 7: A simply-supported beam with a transverse crack located at position l_1 (right)

Cracks have been modeled in several different ways. In most of these methods, the crack is treated as a reduction in the stiffness of the structure at its location. The most common methods used are a reduction in the modulus of elasticity at the intended crack location, an abrupt change of the cross-sectional area and the use of a rotational crack compliance matrix coefficient. The inverse of this coefficient, in the latter method, can be used to define an equivalent massless torsional spring. The method of using a torsional spring to define a crack will be used in this analysis. This approach has been used in studying prismatic bars and has given good results in the modeling of the crack. Reference to use of this technique may be found published by Chondros , Dimarogonas and Rizos.

The crack, in this analysis, is considered to be open. In other words, the crack is wide enough so that it does not completely close during a vibration. A torsional spring, having a constant k_r , will be used to model a transverse crack of depth a . This crack is located at the distance l_1 on the simply-supported beam, with orthogonal cross-section of width, b , and height, h , as shown in Figure 6.

The spring constant k_r , which describes a transverse crack of depth a , is determined by the following formula obtained from previous work performed by Rizos et al., Robert Y. Liang et al., T. G. Chondros et al., Bovsunovskii A. P. et al.,

$$k_r = \alpha EI$$

$$\alpha = \frac{1}{5.346 \cdot h \cdot F(s)} \quad (4.1)$$

In Eqn. 4.1, E is the modulus of elasticity of the beam material, I is the second moment of area of the undamaged cross-section, and the dimensionless local compliance function $F(s)$ is computed from the following equation,

$$F(s) = 1.86s^2 - 3.95s^3 + 16.38s^4 - 37.23s^5 + 76.81s^6 - 126.90s^7 + 172.0s^8 - 143.97s^9 + 66.56s^{10} \quad (4.2)$$

where $s = a/h$ is the relative depth of the crack.

The simply-supported beam, with a crack modeled as a torsional spring, is shown in Figure 7.

4.2 The Frequency Equation of the Cracked Simply-supported Beam

The mathematical model of the physical system is an Euler-Bernoulli beam with a crack. The crack will be modeled as a torsional spring and is located at an arbitrary position l_1 . The beam properties are assumed to be homogeneous and isotropic. The beam vibration is measured from the static equilibrium position, constrained to the plane of motion, and assumed to be transverse. The mathematical model used to describe this motion will be broken into two separate functions. These two functions, $y_1(x)$ and $y_2(x)$, are used to describe the motion of the beam on the left and right sides of the crack respectively and are spliced together with the restriction that the displacement at the location l_1 is the same for both functions.

According to the shape function of deflection of intact simply-supported beam, the deflections of the left and right side of the beam and their first to third partial derivative with respect to x can be expressed,

$$y_1(x) = C_1 \sin kx + C_2 \cos kx + C_3 \operatorname{sh}kx + C_4 \operatorname{ch}kx$$

$$\frac{dy_1(x)}{dx} = C_1 k \cos kx - C_2 k \sin kx + C_3 k \operatorname{ch}kx + C_4 k \operatorname{sh}kx \quad (4.3)$$

$$\frac{d^2 y_1(x)}{dx^2} = -C_1 k^2 \sin kx - C_2 k^2 \cos kx + C_3 k^2 \operatorname{sh}kx + C_4 k^2 \operatorname{ch}kx$$

$$\frac{d^3 y_1(x)}{dx^3} = -C_1 k^3 \cos kx + C_2 k^3 \sin kx + C_3 k^3 \operatorname{ch}kx + C_4 k^3 \operatorname{sh}kx$$

$$y_2(x) = D_1 \sin kx + D_2 \cos kx + D_3 \operatorname{sh}kx + D_4 \operatorname{ch}kx$$

$$\frac{dy_2(x)}{dx} = D_1 k \cos kx - D_2 k \sin kx + D_3 k \operatorname{ch}kx + D_4 k \operatorname{sh}kx \quad (4.4)$$

$$\frac{d^2 y_2(x)}{dx^2} = -D_1 k^2 \sin kx - D_2 k^2 \cos kx + D_3 k^2 \operatorname{sh}kx + D_4 k^2 \operatorname{ch}kx$$

$$\frac{d^3 y_2(x)}{dx^3} = -D_1 k^3 \cos kx + D_2 k^3 \sin kx + D_3 k^3 \operatorname{ch}kx + D_4 k^3 \operatorname{sh}kx$$

in which $k^4 = \omega^2/a^2$, and that $a = \sqrt{EI/\rho A}$, ω is angular frequency, C_1 to C_4 and D_1 to D_4 are the real constants must be evaluated so as to satisfy the known boundary conditions (displacement, slope, moment, or shear) at the ends of the cracked beam and the known continuum and compatibility conditions at the cracked location.

Considering the uniform cracked simply-supported beam shown in Fig. 1. The four known boundary conditions

and the four continuum and compatibility conditions of the cracked simply-supported beam are,

$$\begin{aligned}
 y_1(x)|_{x=0} &= 0 & y_1(x) &= y_2(x)|_{x=l_1} \\
 \frac{d^2 y_1(x)}{dx^2}|_{x=0} &= 0 & \frac{dy_1(x)}{dx} + \frac{EI}{k_r} \frac{d^2 y_1(x)}{dx^2} &= \frac{dy_2(x)}{dx}|_{x=l_1} \\
 y_2(x)|_{x=l} &= 0 & \frac{d^2 y_1(x)}{dx^2} &= \frac{d^2 y_2(x)}{dx^2}|_{x=l_1} \\
 \frac{d^2 y_2(x)}{dx^2}|_{x=l} &= 0 & \frac{d^3 y_1(x)}{dx^3} &= \frac{d^3 y_2(x)}{dx^3}|_{x=l_1}
 \end{aligned} \tag{4.5}$$

Substituting Eqn. 4.5 into Eqn. 4.3 and Eqn. 4.4, the following equation can be derived,

$$\frac{EI}{2k_r} k(\sin kl_1 \cos kl_1 - shkl_1 chkl_1 + cthklsh^2 kl_1 - ctgkl \sin^2 kl_1) = 1 \tag{4.6}$$

Substituting Eqn. 4.1 into Eqn. 4.6, and let $h = \beta l$ (β is a constant), $l_1 = \xi l$ ($0 \leq \xi \leq 1$), one obtains,

$$2.673 \beta kl F(s) (\sin \xi kl \cos \xi kl - sh \xi kl ch \xi kl + cth kl sh^2 \xi kl - ctg kl \sin^2 \xi kl) = 1 \tag{4.7}$$

In Eqn. 4.7, there are three variables which respectively are s which represents damage extent, ξ which represents damage position and kl which is natural frequency coefficient of cracked simply-supported beam.

The solution of this transcendental equation provides the values of kl which represents the frequency coefficient of vibration of the cracked simply-supported beam. It can be obtained that the natural frequency coefficients of the beam vary with crack position along the beam under different damage extents.

4.3 Numerical Example

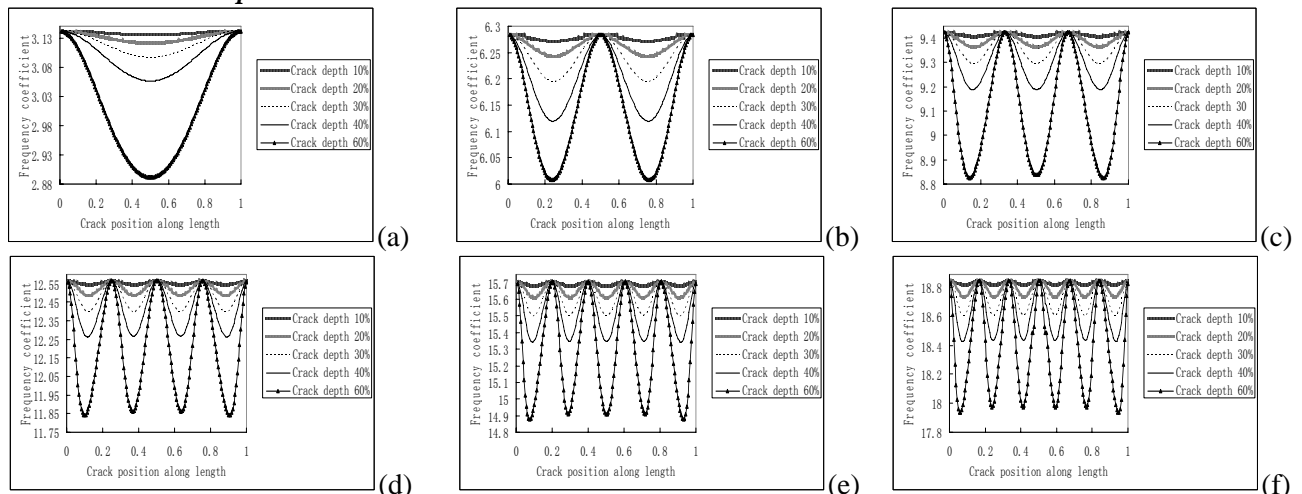


Figure 8: the figures from a to f are the 1st to 6th order frequency coefficients vary with the crack position under five damage scenarios

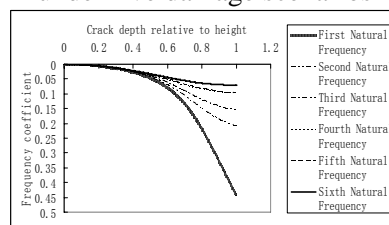


Figure 9 the curve of natural frequency coefficients versus damage extent from 1st order to 6th order

Eqn. 4.7 is a transcendental equation whose solution is a function of crack position, crack depth, and other beam properties. Because we are interested in determining the crack position and crack depth of the simply-supported beam from the natural frequencies of beam vibration, let us examine how the natural frequencies vary with the crack depth and position on the beam. A hypothetical case will be considered. The properties of this beam are obtained from the beam which will be used in the numerical calculation. The beam will have the following

properties: a length of 100 cm, a modulus of elasticity of 2.05×10^5 Mpa, a height of 4 cm, a width of 1 cm, and a density of 7850 kg/m^3 . By choosing a crack depth and crack position, the natural frequency of the cracked beam is obtained by finding the value of w which will make the. The first six order natural frequencies coefficients of the cracked simply-supported beam are determined for crack depths of 10%, 20%, 30%, 40%, and 60%, of the height of the beam, versus the crack position. These results have been plotted and are shown in Figure 8 numbered from No. a to No. f.

Several interesting observations can be made by examining Fig. 3 numbered from No. a to No. f. The first natural frequency coefficient of the cracked beam, which is observed in No. a of Figure 8, tends to drop as the crack depth increases when the crack position moves from the left end to the middle point of the beam and tends to rise when the crack position moves from the middle point to the right end of the beam. However, this is different from other five natural frequency coefficients. Here the other five natural frequency coefficients tend to rise and drop depending on the crack location. In some cases the crack depth is indistinguishable from the frequency coefficients given (for example it occurs around the middle point in the second natural frequency coefficient graph). This behavior occurs when the crack locates in the inflexion of the corresponding modal shape namely the second derivative of the modal shape is zero. The slope at the right side of the crack is the same as the left side, and the effects made by the torsional spring become zero. For all the six curves in Figure 8, it is the same phenomena that the natural frequency coefficients tend to decrease with the crack depth increasing.

From Figure 9, it can be found that the relative decrease of the 1st natural frequency coefficient is ultimate. On the other hand, the 1st natural frequency coefficient is the most sensitive to the crack damage.

5. TEST VERIFICATION OF A TRUSS MODEL HEALTH BASELINE



Figure 10 truss model arranged on shake-table (Left) and sensors placed on the test model (Right)

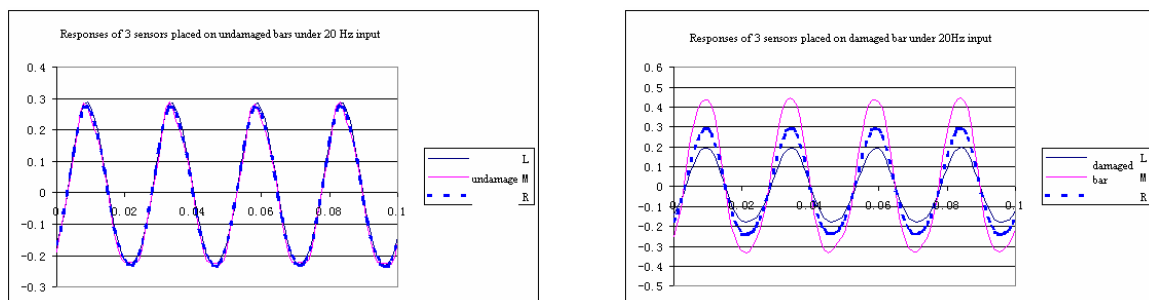


Figure 11 acceleration responses of undamaged (Left) and damaged (Right) bar under sine wave with 20Hz input, solid and dashed lines with blue color represent the responses of the two ends of the bar, solid line with pink color represent the responses of the middle of the bar.

The tests of steel truss bridge model are undertaken on the shaking table at Institute of Engineering Mechanics (IEM), CEA, China (Figure 10). Preliminary results have shown local frequency measurement of some promise for structural damage detection. One example is shown below. The bridge model is designed by 1:8 of scaling down a 48 m long steel truss bridge to fit the 5 m by 5 m shake-table of IEM. The model has 6-sections with 6 m in length, 0.5 m in height and 0.25 m in width, and is made of 83 30 mm by 3 mm (25 mm by 3 mm) steel angel-bars. Figure 10 shows the acceleration response recorded on a pair of bars located on the bottom middle of the model with one cut and the other intact. The obvious differences between the two graphs of figure 11 were observed. The acceleration response recorded on the middle of damaged bar is larger than that recorded on

the middle of undamaged one. The results of preliminary tests coincide with that from the baseline established in 4th section

6. SUMMARY

In this passage, we generally introduced the methods to diagnose the global and local damage of the structure, and mentioned that how to choose the health baseline in diagnosing the structural health, and gave a definition of structure health baseline through the analogy of the health baseline of human, that is a signal which represent a structural health state, and through which engineers can decide conveniently whether and where the structure is damaged; and under the background of the railway stain bridge to establish a structural health diagnose baseline which is based on symmetry principle. The results from the preliminary tests coincide with that from the baseline established in this paper.

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