STUDY ON TORSION COUPLED RESPONSE OF NON-PROPORTIONALLY DAMPED ECCENTRICALLY ISOLATED STRUCTURE UNDER EARTHQUAKE

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Abstract: This paper presents a numerical investigation into non-proportionally damped eccentric isolated structures. The non-proportional damping characteristics of the isolated system is described by Rayleigh's sub-structural damping model, a simplified equation of motion (EOM) is derived. The result of the numerical simulation shows that the eccentricity ratio of the isolation layer has apparent effect on torsional angle of the isolation layer, while has little effect on the horizontal displacement of the isolation layer and the inter story drift of superstructure and the torsional angle of the isolated layer. The increase of the eccentricity ratio of the superstructure has little effect on the horizontal displacement and torsional angle of the isolation layer, but will result in an increase in the horizontal displacement and torsional angle of the superstructure. The extent of increase of the inter story drift and the torsional angle rises with the extent of increase of the eccentricity ratio. The response of structure can be decreased markedly by enhancing damping ratio of isolation layer properly, but excessive damping in isolation layer may cause the response of superstructure to increase.

Key words: base isolation; eccentrically isolated structure; translation-torsion coupled vibration; non-proportional damping; time-history analysis

1. INTRODUCTION

In the design of an earthquake resistant building, the structural layout are often required to be regular and symmetric, aiming at reducing eccentricity and avoiding torsional effect. However, eccentricity is usually ineluctable in practical engineering. In a eccentric structure, the mass and stiffness centers are not coincident, and the inertial force exerted at the mass center would bring forth torsional moment with respect to the stiffness centre, which resulting in translation-torsion coupled vibration under earthquake action. Therefore, the torsional effect should be considered for the dynamic analysis of the eccentric structure under earthquake action. Base-isolation is one of the most popular techniques for mitigating seismic response. Wang and Yao (2004) presented an analysis of torsional response of a single story eccentric isolated structure, and investigated the effect of eccentric distance of the isolation layer and the super-structure on the torsional response. Deng et al. analyzed translation-torsion coupled response of multistory isolated structures, and found out that introducing base isolation technique can reduce the torsional response and inter-story drift of the superstructures (2002).Wu and Li (2003) studied the seismic response of eccentric base isolated structures on different ground conditions, and analyzed the effect of different parameters through large number of numeric simulations. The author (2000a) proposed a practical damping model for non-proportionally damped isolated structures using a sub-structural Rayleigh damping model, and developed several engineering methods for time domain analysis (2002, 2000b). This paper will investigate the effect of non-proportional damping on the eccentric isolated MDOF structures based on the equation of motion (EOM) with translation-torsion coupled vibration.

2. EOM OF ISOLATED STRUCTURES CONSIDERING TRANSLATION-TORSION COUPLING

2.1 EOM of eccentric isolated structure

Taking an isolated building with single eccentricity as a research object, assuming the earthquake action is along *x* direction and the eccentricity occurs along *y* direction, the EOM of the eccentric isolated structure is given by

$$\begin{bmatrix} [M] & 0 \\ 0 & [J] \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + [C] \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} [K_{xx}] & [K_{x\theta}] \\ [K_{\theta x}] & [K_{\theta\theta}] \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = - \begin{bmatrix} [M] & 0 \\ 0 & [J] \end{bmatrix} \begin{Bmatrix} \delta \\ 0 \end{Bmatrix} \ddot{u}_g \tag{1}$$

Where, $[M] = diag[m_1 \quad m_2 \quad \cdots \quad m_n]^T$ is the mass matrix, and $[J] = diag[J_1 \quad J_2 \quad \cdots \quad J_n]^T$ is the moment of inertia matrix. $[K_{xx}]$, $[K_{x\theta}]$, $[K_{\theta x}]$, and $[K_{\theta\theta}]$ are the translational movement stiffness, translation-torsion coupled stiffness, torsion stiffness matrix respectively, which are tri-diagonal matrices. The elements of the translational movement stiffness matrix are:

$$(k_{i,i})_{xx} = k_i + k_{i+1}\,,\quad (k_{i+1,i})_{xx} = -k_{i+1}\,,\quad (k_{i,i+1})_{xx} = -k_{i+1}$$

The elements of the translation-torsion coupled stiffness matrix are:

$$(k_{i,i})_{x\theta} = k_i e_i + k_{i+1}(e_{i+1} - \bar{e}_{i+1})\,,\quad (k_{i+1,i})_{x\theta} = -k_{i+1}(e_{i+1} - \bar{e}_{i+1})\,,\quad (k_{i,i+1})_{x\theta} = -k_{i+1} e_{i+1}$$

The elements of the torsional movement stiffness matrix are:

$$(k_{i,i})_{\theta\theta} = k_{\theta,i} + k_{\theta,i+1} + k_{i+1}\bar{e}_{i+1}(\bar{e}_{i+1} - 2e_{i+1})\,,\quad (k_{i+1,i})_{\theta\theta} = k_{\theta,i+1} + k_{i+1} e_{i+1}\bar{e}_{i+1}\,,\quad (k_{i,i+1})_{\theta\theta} = -k_{\theta,i+1} + k_{i+1} e_{i+1}\bar{e}_{i+1}$$

$$k_{\theta,i} = R_i + k_i e_i^2$$

k_{i} is the shear stiffness coefficient of the i^{th} story; $k_{\theta,i}$ is the torsional stiffness coefficient of the i^{th} story with respect to the mass center; R_i is the torsional stiffness coefficient of the i^{th} story with respect to the stiffness center; e_i is the eccentric distance of i^{th} story; $\bar{e}_i$ is the coordinate of the mass center of the i^{th} story with respect to the mass center of the i-1th story. $r_i = \sqrt{R_i / k_i}$ is the radius of gyration; $e_i' = e_i / r_i$ is the eccentricity ratio; $r_i\theta_i / x_i$ is torsion radius ratio, which is an important index for judging the overall torsion of the structure.

2.2 Expression of non-proportional damping matrix

Using sub-structural Rayleigh damping model, the non-proportional damping matrix can derived as:

$$[C] = [C_0] + [C_r], \qquad [C_0] = \alpha_s [M] + \beta_s [K] \tag{2}$$

$$(C_{1,1})_r = C_{brx} = \alpha_{bx} m_1 + \beta_{bx}(k_{1,1})_{xx}\,, \qquad (C_{n+1,n+1})_r = C_{br\theta} = \alpha_{b\theta} J_1 + \beta_{b\theta}(k_{1,1})_{\theta\theta} \tag{3}$$

Where, $[C_0]$ stands for classical Rayleigh damping matrix, and $[C_r]$ is the remainder term damping matrix of non-proportionally damping. The other elements of $[C_r]$ being 0, *n* is number of stories of the structure. α_s, β_s are the Rayleigh damping coefficients of the superstructure. α_{bx}, β_{bx} are the Rayleigh damping coefficients of the isolation layer corresponding to translational movement, and $\alpha_{b\theta}$, $\beta_{b\theta}$ are the Rayleigh damping coefficients of the isolation layer corresponding to torsional movement.

$$\begin{Bmatrix} \alpha_s \\ \beta_s \end{Bmatrix} = \frac{2\xi_s}{\omega_1 + \omega_2} \begin{Bmatrix} \omega_1\omega_2 \\ 1 \end{Bmatrix}, \qquad \begin{Bmatrix} \alpha_{bx} \\ \beta_{bx} \end{Bmatrix} = \frac{2\xi_{bx}}{\omega_1 + \omega_2} \begin{Bmatrix} \omega_1\omega_2 \\ 1 \end{Bmatrix}, \qquad \begin{Bmatrix} \alpha_{b\theta} \\ \beta_{b\theta} \end{Bmatrix} = \frac{2\xi_{b\theta}}{\omega_1 + \omega_2} \begin{Bmatrix} \omega_1\omega_2 \\ 1 \end{Bmatrix} \tag{4}$$

ξ_s is the damping ratio of the superstructure, and ξ_{bx}, $\xi_{b\theta}$ are the damping ratio of the isolation layer corresponding to translational and torsional movement:

$$\xi_{bx} = \frac{\sum k_{bj}\xi_j}{\sum k_{bj}}, \qquad \xi_{b\theta} = \frac{\sum k_{bj}(x_j^2 + y_j^2)\xi_j}{\sum k_{bj} y_j^2} \tag{5}$$

3. NUMERICAL EXAMPLE AND RESULT DISCUSSION

A 3 DOF eccentric isolated structure, which is a real isolated building, is chosen as numerical example, with the structural parameters given in table 1, in which the story number "b" stands for the isolated layer, and the other number *j*=1, 2 stands for the superstructure, counting from base block towards the top of the structure.

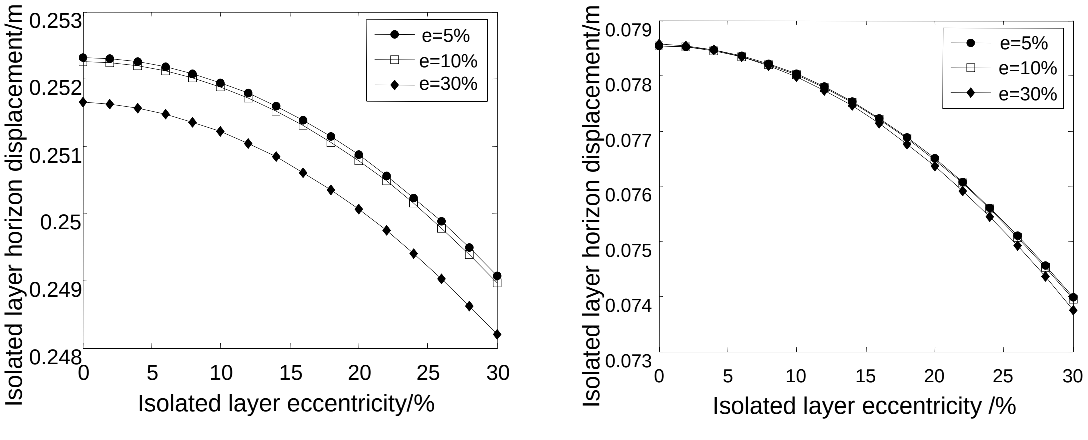
Table 1 Parameters of non–proportionally damped isolated structure

story No. *j*	m_j /kg	J_j / kg.m^2	k_{xj} /kN.m^{-1}	$k_{\theta j}$ /kN.m
b	742700	7.3137e07	2.646e04	3.5834e6
1	788500	7.7648e07	4.1197e05	5.712e7
2	788500	7.7648e07	4.1197e05	5.712e7

The effect of eccentricity on the seismic response of this example will be discussed using the abovementioned example based on time domain analysis method. The El Centro and Taft wave are chosen as excitation signal, and the amplitude of the acceleration is adjusted to 4.0ms^{-2} representing the level of major earthquake in a region with a fortification intensity of 8 degree and a PGA of 0.2g. The damping ratio of the superstructure is set to be $\xi_s = 0.05$, and the damping ratios of the isolation layer corresponding to both the translational and torsional movement are set to be $\xi_{bx}=\xi_{b\theta}=0.15$.

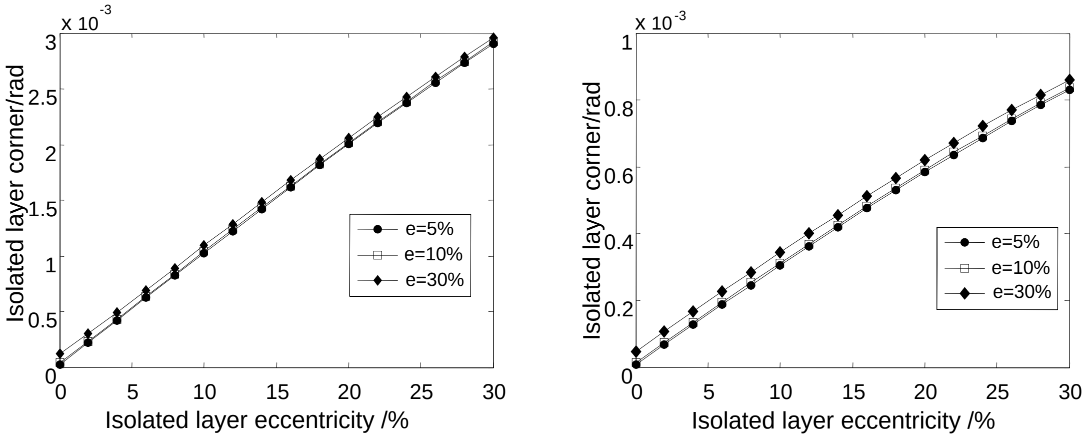
3.1 Effect of eccentricity of isolation layer and superstructure on seismic response

The relative eccentricity ratio *e* of the superstructure is chosen to be 5%, 10% and 30%, and set the eccentricity of the isolation layer e_b to change in the range 5%~ 30%, the variation of horizontal displacement, torsional angle of isolation layer, inter story drift and of isolation layer, and torsional angle of superstructure is shown in Figures 1~4, respectively.



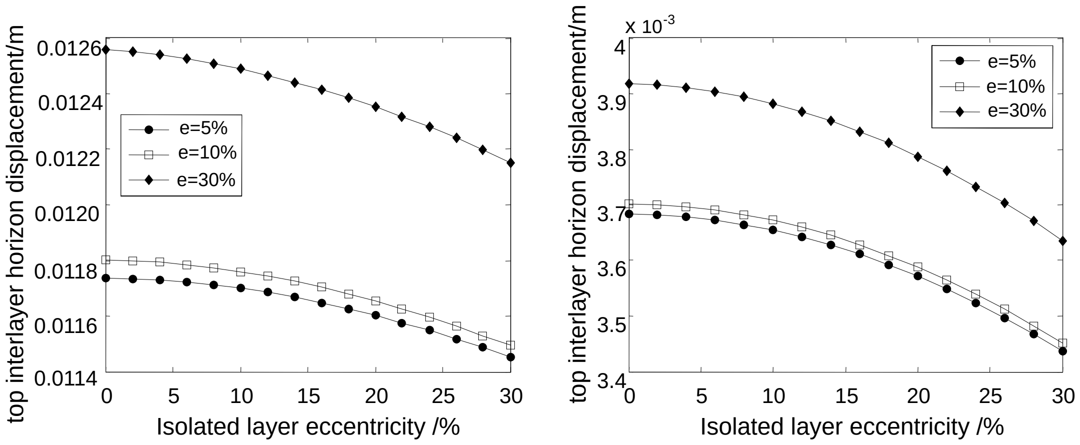
(a) El Centro wave (b) Taft wave

Fig. 1 Variation of displacement of isolation layer with the eccentricity ratio



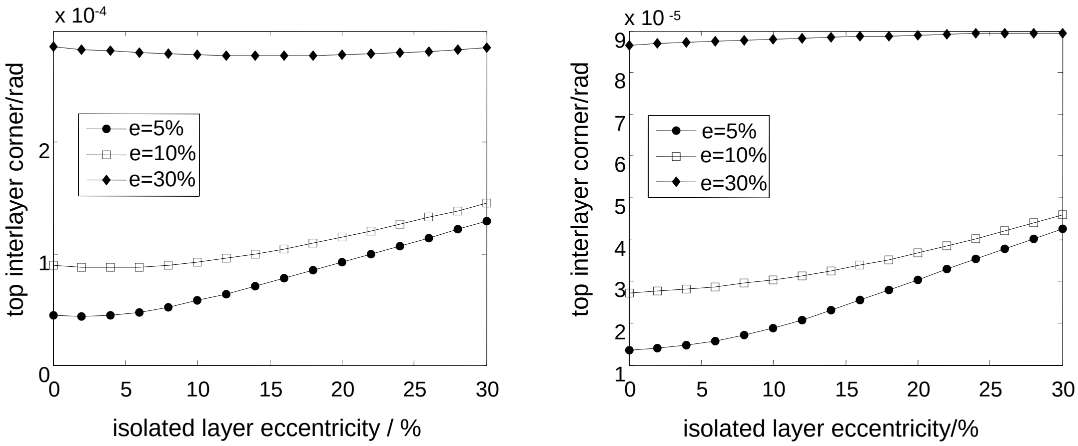
(a) El Centro wave (b) Taft wave

Fig. 2 Variation of torsional angle of isolation layer with eccentricity ratio



(a) El Centro wave (b) Taft wave

Fig. 3 Variation of inter story drift with eccentricity ratio



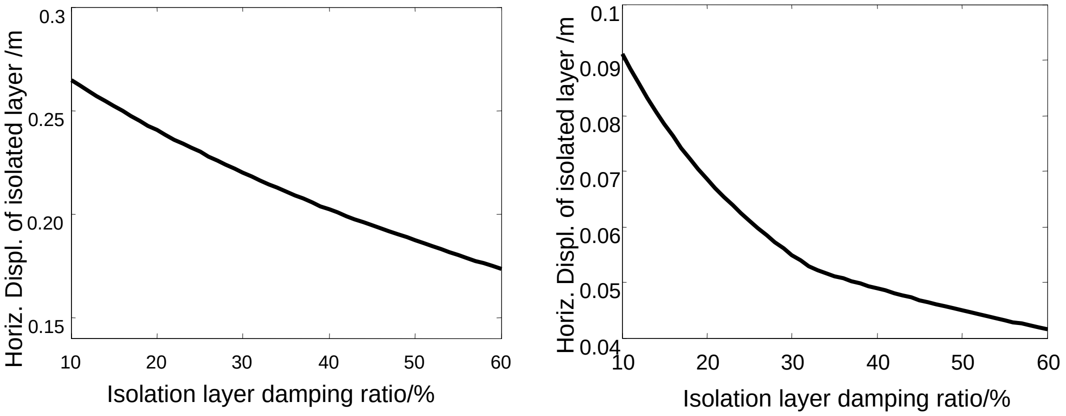
(a) El Centro wave (b) Taft wave

Fig. 4 Variation of torsional angle of superstructure with eccentricity ratio

One may infer from Figures 1-4 that the torsional angle response increases obviously with the increase of eccentricity ratio of the isolation layer, with very little decrease in the horizontal displacement of the isolation layer and the inter story drift of superstructure, which shows that eccentricity ratio of the isolation layer has apparent effect on torsional angle and has little effect on the horizontal displacement of the isolation layer and the inter story drift of superstructure. The increase of the eccentricity ratio of the superstructure has little effect on the horizontal displacement and the torsional angle of the isolation layer, but does increase the inter story drift and the torsional angle of the superstructure. More over, the extent of increase of the inter story drift and the torsional angle rises with the extent of increase of the eccentricity ratio. When the eccentricity ratio of the superstructure is small, the increase of eccentricity ratio of the isolation layer will cause an apparent increase of the torsional angle of the superstructure. While the eccentricity ratio of the superstructure is large, on the other hand, the effect of the increase of eccentricity ratio of the isolation layer will become smaller.

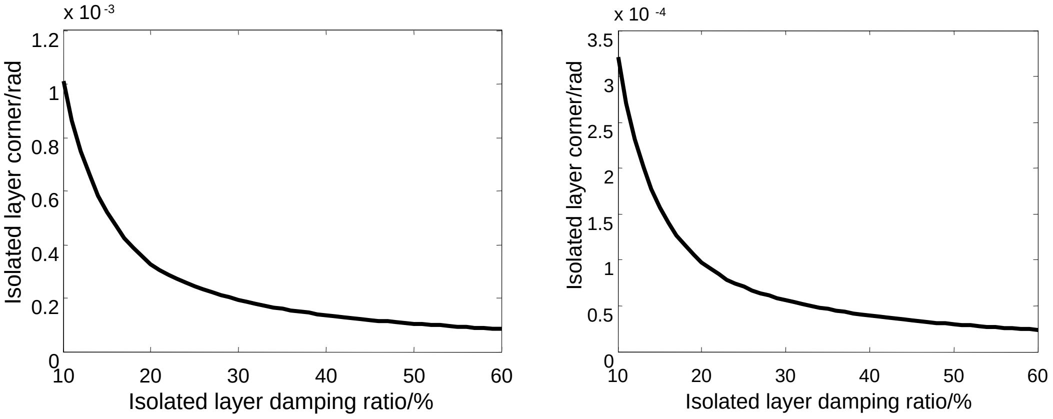
3.2 Effect of isolation damping on seismic response of eccentric isolated structure

Set the eccentricity ratios to be 5% for both the superstructure and the isolation layer, which corresponds to an eccentric distance of $e_z = 1$m, the effect of isolation damping on seismic response of eccentric isolated structure is shown in Figure 5~9.



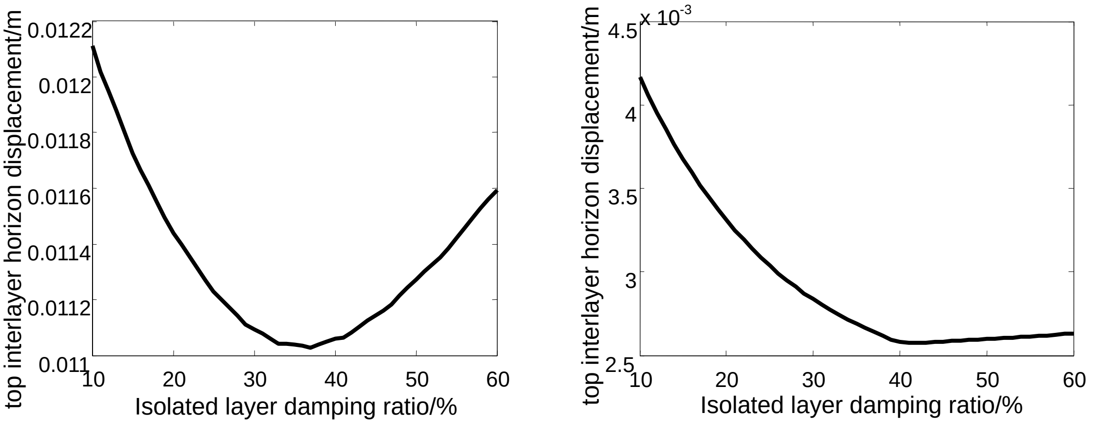
(a) El Centro wave (b) Taft wave

Fig. 5 Variation of displacement of isolation layer with isolation damping



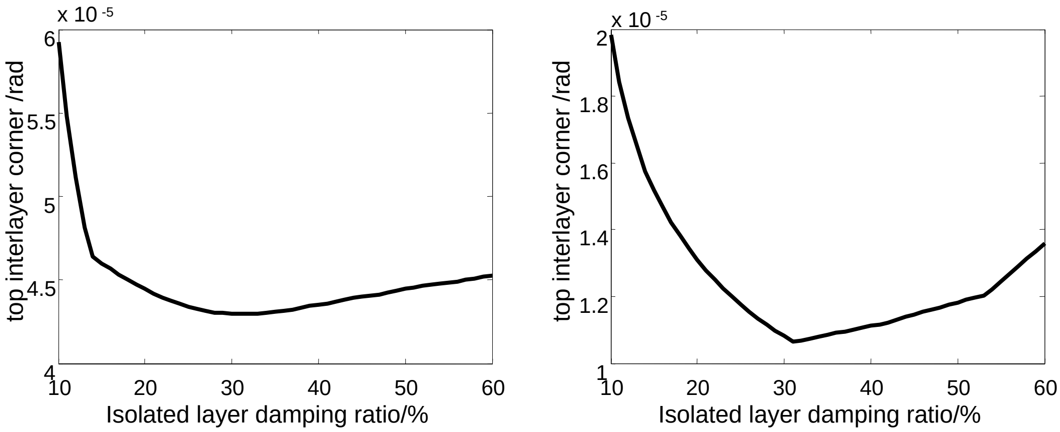
(a) El Centro wave (b) Taft wave

Fig. 6 Variation of torsional angle of isolation layer with isolation damping



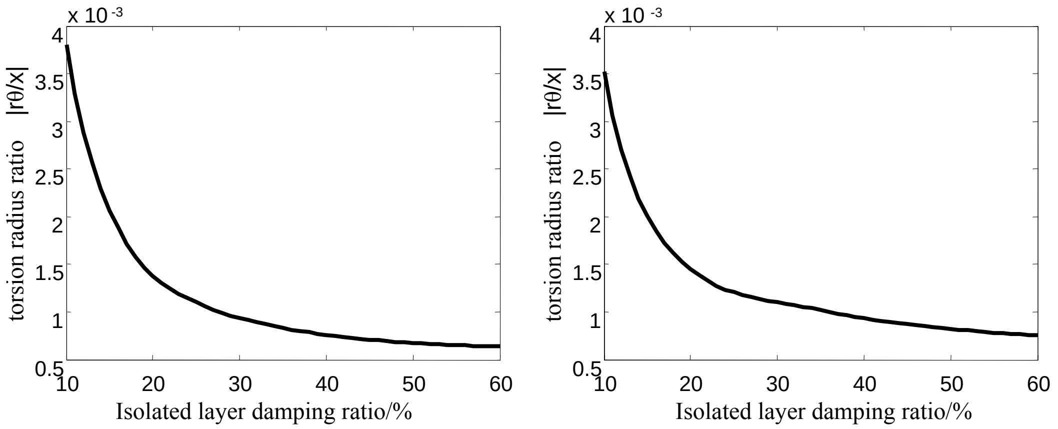
(a) El Centro wave (b) Taft wave

Fig. 7 Variation of horizontal displacement of top story with isolation damping



(a) El Centro wave (b) Taft wave

Fig. 8 Variation of torsional angle of the top story with isolation damping



(a) El Centro wave (b) Tafe wave

Fig. 9 Variation of torsion radius ratio of isolation layer with isolation damping

From Figures 5~9 one may obtain that the increasing of isolation damping apparently decreases horizontal displacement of the isolation layer; when the damping ratio in the isolation layer is $\xi_b < 0.20$, the increasing of isolation damping decreases the torsional angle; While the decrease in the torsional angle with the increase of isolation damping can also be easily observed for the damping ratio of the isolation layer lying between $0.2 \le \xi_b < 0.35$, the effect of increasing the isolation damping on the decreasing the torsional angle will not be apparent. For $\xi_b < 0.35$, the increase of isolation damping ratio will apparently decrease the horizontal displacement of the top story, while for $\xi_b > 0.35$, increasing the isolation damping ratio will result in an increase in the horizontal displacement of the top story. For $\xi_b < 0.3$, increasing the isolation damping ratio will apparently decrease the torsional angle of the top story; If $\xi_b > 0.3$, on the contrary, the increase of the isolation damping ratio will increase the torsional angle of the top story. If the isolation damping ratio $\xi_b < 0.20$, the increase of the isolation damping ratio will apparently decrease the torsion radius ratio, while for $\xi_b > 0.20$ the increase of the isolation damping ratio will have little effect on decreasing the torsion radius ratio.

4. CONCLUSIONS

According to the dynamic behavior of isolation structure, a simplified model for eccentric isolated structure is established, and EOM is derived. The effect of eccentricity ratio of the superstructure and the isolation layer and the damping ratio of the isolation layer on the dynamic response is investigated through numerical simulation.

(1) The eccentricity ratio of the isolation layer has apparent effect on torsional angle of the isolation layer, while has little effect on the horizontal displacement of the isolation layer and the inter story drift of superstructure and the torsional angle of the isolated layer. The increase of the eccentricity ratio of the superstructure has little effect on the horizontal displacement and torsional angle of the isolation layer, but will result in an increase in the horizontal displacement and torsional angle of the superstructure. The extent of increase of the inter story drift and the torsional angle rises with the extent of increase of the eccentricity ratio.

(2) An appropriate increase of isolation damping ratio may decrease the seismic response of eccentric isolated structure apparently. However, excessive damping will not only decrease the response of the structure, on the contrary, it may increase the response of the structure.

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