

PERFORMANCE AND RELIABILITY OF ELECTRICAL POWER GRIDS UNDER CASCADING FAILURES

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ABSTRACT:

The stability and reliability of electrical power grids are essential for the continuous operation of modern cities and also critical for preparedness, response, recovery and mitigation in emergence management. Due to the fact that present power grids in China are usually running near their critical operation points, they are vulnerable and sensitive to disturbances from natural and man-made hazards such as hurricanes, earthquakes and terrorist attacks, etc., which often trigger cascading failures. Using the provincial power grid data from Chinese power utilities including 118 buses (substations) and more than 180 branches (transformers and transmission lines), we first present the properties of the power network with modern complex network concepts such as state transition graph and characteristic length. Then the dynamic power flow over the power grid is calculated with the BPA load flow analysis software. System performance and reliability of the power grids are evaluated under an earthquake scenario and a hypothetical terrorist attack. With the identified most vulnerable (critical) edges and nodes, the robustness of the power network is evaluated under the triggered cascading failures. The findings could be useful for power industries and policy makers to evaluate the risk of blackout induced by cascading failures, and identify the vulnerability of power systems and improve the resilience of power systems to external disturbances.

KEYWORDS: Cascading failure, system reliability, performance, power grids, state transition graph

1. INTRODUCTION

Urban infrastructure systems such as power networks are critical backbones of modern societies. The stability and reliability of electrical power grids are indispensable for the continuous operation of modern cities and also critical for preparedness, response, recovery and mitigation in emergence management. Under extreme events such as earthquake impact or intentional disruption, failure of power system infrastructure not only disrupts the residential and commercial activities, but also impairs the post-disaster response and recovery, resulting substantial socio-economic (U.S.-Canada Power System Outage Task Force, 2004; Chang et al., 1996; Chang et al., 2001). Cascading failure, also known as avalanche of power systems, has drawn intensive research interests in recent years, especially after the large-scale August 14 cascading failure in North America in 2003 (U.S.-Canada Power System Outage Task Force, 2004). Due to the fact that present power grids in China are usually running near their critical points, they could be vulnerable and sensitive to disturbances from natural and man-made hazards which could potentially trigger cascading failures.

Evaluation of the performance and reliability of infrastructure systems is complex in nature due to the large number of network components, complex network topology, and component/system interdependency. Currently, the most successful study of power network cascading failures is based on OPA and CASCADE models (Carreras et al., 2002; Dobson et al. 2002; Nedic, et al., 2006). Although some results can be drawn based on these models, we are still unable to explain whether and how a certain disturbance can cause a cascading failure

(Wu et al. 2007). Complex system theories have been recently employed to understand the cascading failures. Watts (1999) showed that the topology of modern power grid is always a small-world graph and the propagation of failures within the network was studied with complex system theories. Focusing on the topologies of power systems, these approaches, however, ignored the weights of nodes (generators and substations) and links (transformers and transmission lines), which might be more important in practice (Wu et al. 2007).

Stakeholders and emergency managers are also concerned the availability of electricity services, or the connectivity between power generator and the service areas. The total number of households without electricity services is one of the measures of the severity of impact (FEMA, 2003). Network reliability, or the probability of connectivity between node-pairs can be evaluated based on the network configuration with graph theory.

System reliability of lifeline networks can be measured in three perspectives (Li, 2005): (1) structural component reliability; (2) connectivity reliability between node-pair; and (3) performance reliability. In this paper, the scope of network reliability analysis presented is limited to connectivity reliability. The basic idea of analytic network reliability method is to convert complex network to combination of simply networks such as parallel or series systems, and then system reliability could be computed by finding the union and intersection of these simply networks, e.g. generic substation can be modeled with a series system of macro-components (Vanzi, 1996). Kroft (1967) first describe shortest path algorithm to compute system reliability. Panoussis (1974) and Taleb-Agha (1977) proposed general network reliability can be computed by converting complex lifeline networks to SSP (series systems in parallel) networks. But finding the shortest paths is not always an easy task, especially for large-scale networks. Aggarwal and Misra (1975) proposed disjoint shortest path algorithm. Later, researchers (Dotson and Gobien, 1979; Yoo and Deo, 1988; Torrieri, 1994; Li and He, 2002) improved this algorithm and found it can give exact reliability for large-scale complex networks. Full probability analytic algorithm (Wu and Sha, 1998) and Ordered Binary Decision Diagram (OBDD) algorithm (Kuo et. al, 1999) are also able to find exact network reliability but neither of them is able to handle large-scale networks. RDA is efficient and applicable to all networks (Li and He, 2002) but not able to handle dependence issue.

On the other hand, since complete information is not always available, especially for complex systems, researchers chose to describe the system reliability approximately with reliability bounds. Ditlevsen (1979) gave theoretical bounds for general system reliability problems. But the theoretical bounds are often too wide for practical uses. Song et al. (2003 and 2004) obtained much narrower reliability bounds of the reliability of power substation systems by linear programming. Recently, a matrix-based system reliability (MSR) method is proposed to account for dependence and incomplete information (Song and Kang, 2007) and utilized to bridge networks and natural gas networks (Chang and Song, 2007). MSR is easy to implement and flexible to handle dependence and incomplete information, which make it advantageous to give more insight results.

This paper studies the cascading failure of the power system with the specific emphasis on its satiability and network reliability. First, the stability performance is evaluated with the state transition graph. Next, we analyzed the system reliability of the power system under cascading failures. We then demonstrate the analysis with a provincial power network in China. The results and observations from the case study are summarized, and the identified future research topics are also presented.

2. PERFORMANCE AND THE STATE TRANSITION GRAPH OF POWER GRIDS

The capability of power grids to deliver electric power depends on boundary conditions (e.g., power and voltage from supply nodes and to demand nodes), the admittance of transmission lines, and the working states (operating or failure) of substation components. Boundary conditions are specified at the nodes in terms of active and reactive components at load nodes, real power and voltage amplitude at the generation nodes, and voltage amplitude and phase angle at the swing node. In general, the steady state of a power system is described by power flow, which is the solution of a typical nonlinear equation sets:

$$\dot{V}^* Y V = \dot{S} = P + jQ \quad (3.1)$$

where \dot{V} is the voltage phasor vector as a complex value, Y is the admittance matrix corresponding to the power network, \dot{S} is the vector of complex power injected into each nodes, “*” is the conjugate operator. Electrical power transferred through a given edge (i.e., transmission line or transformer) can be calculated once the power flow equation is solved. The power flow model is theoretically acceptable since the power system runs normally for most of time, and most blackouts always happen with a long-period dynamics, which may also be described by power flow (Taylor, 1994).

The power grid can be treated as a weighted digraph from a macroscopic viewpoint: generators and load motors as the nodes to inject electric power and loads into power grids, and transmission lines and transformers as the edges to transfer electrical power. To quantify the security of power system, we use capacity factor of each edge other than mere electrical power or circuit as the weight. For example, the weight of a transformer is the ratio of apparent power transferred over its rated capacity, while the weight of a transmission line is usually the ratio of actual transmitted current over its rated current.

Though most electrical equipments are designed with over-loading protection capacities, it is assumed that the overloaded equipments shall quit the network and the working state be “failure”, and vice versa. Electrical power grid can be viewed as a discrete dynamical network when all the buses, transmission lines and transformers have binary working state, that is, operating state denoted by “1” and failure state by “0”. By describing the states of nodes and edges with binary vectors, the state transition graphs (Wuensche, 2003; Wu et al., 2007) can capture dynamic behaviors of both nodes (i.e., power plants and substations) and edges (e.g., transmission lines) and can illustrate the cascading failure of power networks. In nature, the state transition graph is an enumeration of all possible initial states of the system. Since the state transition graphs are difficult for direct uses for large-scale networks, characteristic lengths of the state transition graphs provide us a feasible measure to understand cascading failures.

3. SYSTEM RELIABILITY UNDER CASCADING FAILURES

The MSR method subdivides the sample space of component events with s_i distinct states, $i = 1, \dots, n$, into $m = \prod_{i=1}^n s_i$ mutually exclusive and collectively exhaustive (MECE) events. The probability of any general system event is then described by the inner product of two vectors:

$$P(E_{sys}) = \mathbf{c}^T \mathbf{p} \quad (3.2)$$

where \mathbf{c} is the “event vector” whose element is 1 if its corresponding MECE event is included in the target system E_{sys} event, and 0 otherwise; and \mathbf{p} is the “probability vector” that contains the probabilities of all the MECE events. Efficient matrix-based procedures were proposed to efficiently obtain these vectors by use of matrix computing languages such as Matlab® (Kang et al., 2007). The MSR method has the following merits over existing system reliability methods. First, the probability of a system event is always calculated by a simple matrix multiplication as in Equation (3.2) regardless of the complexity of the system event definition. Second, the MSR method separates the tasks of identification of system event (\mathbf{c}) and computation of probability calculations (\mathbf{p}), which allows for an easy integration with other computation modules, e.g. geographic information system (GIS) or network analysis algorithms. Moreover, the matrix-based procedures proposed along with the method help obtain \mathbf{c} and \mathbf{p} vectors efficiently. Third, even if one has incomplete information on the component failure probabilities and/or their statistical dependence, the matrix-based framework still enables us to obtain the narrowest possible bounds on any general system event (Song and Der Kiureghian, 2003) based on the available information. Fourth, once $P(E_{sys})$ is obtained, one can calculate the probabilities of other system events of interest, conditional probabilities and component importance measures (Song and Der Kiureghian, 2005) without additional probability calculations.

A drawback of the MSR method is the size of the vectors increases exponentially with the number of component events. However, this can be overcome by subdividing the system event into multiple disjoint link sets or cut sets constituted by smaller number of components. When disjoint cut sets or link sets, S_i , $i = 1, \dots, N_{set}$ are identified, the system failure probability or reliability is computed by summing up the results of the MSR analyses of individual sets, i.e.

$$P(E_{sys}) = \sum_{i=1}^{N_{set}} P(S_i) = \sum_{i=1}^{N_{set}} \mathbf{c}_i^T \mathbf{p}_i \quad (3.3)$$

where \mathbf{c}_i and \mathbf{p}_i are the event and probability vectors of the i -th disjoint cut set or link set. These disjoint sets can be efficiently identified by using the RDA (Li and He, 2002). After the Boolean descriptions of the disjoint cut sets and link sets are identified, the corresponding event vectors \mathbf{c}_i 's are obtained by use of the following matrix-based procedures (Kang et al., 2007):

$$\begin{aligned} \mathbf{c}^{\bar{E}} &= \mathbf{1} - \mathbf{c}^E \\ \mathbf{c}^{E_1 \cdots E_n} &= \mathbf{c}^{E_1} * \mathbf{c}^{E_2} * \dots * \mathbf{c}^{E_n} \\ \mathbf{c}^{E_1 \cup \dots \cup E_n} &= \mathbf{1} - (\mathbf{1} - \mathbf{c}^{E_1}) * (\mathbf{1} - \mathbf{c}^{E_2}) * \dots * (\mathbf{1} - \mathbf{c}^{E_n}) \end{aligned} \quad (3.4)$$

where $\mathbf{1}$ denotes a vector of 1's; and “*” is the Matlab[®] operator for element-by-element multiplication. When component events are statistically independent of each other, the probability vectors \mathbf{p}_i 's are constructed based on the failure probabilities of the network elements by the following recursive matrix-based procedure:

$$\begin{aligned} \mathbf{p}_i^{[1]} &= [P_1 \quad 1 - P_1]^T \\ \mathbf{p}_i^{[j]} &= \begin{bmatrix} \mathbf{p}_i^{[j-1]} \cdot P_j \\ \mathbf{p}_i^{[j-1]} \cdot (1 - P_j) \end{bmatrix} \quad \text{for } j=2, 3, \dots, n_i \end{aligned} \quad (3.5)$$

where n_i is the number of the components in the i -th cut set or link set, P_j is the failure probability or reliability of the j -th component in the set; and $\mathbf{p}_i = \mathbf{p}_i^{[n_i]}$. The probability vectors can be obtained even in case the components have statistical dependence (Kang et al., 2007).

4. NUMERICAL EXAMPLE

Earthquake impact on power system may cause not only regional blackout, but also impair post-disaster response and recovery. The target area, with more than 1.36 million customers, is serviced by the state-owned Hainan Power Grid Company (HPGC), part of the China Southern Power Grid (CSPG). The electrical facilities operated by HPGC cover more than 33,920 square km service territory through more than 1,339 km of 220 kV transmission lines and more than 5,015 km of distribution lines (HPGC, 2007).

The HPGC power grid consists of 118 buses (substations) and more than 180 branches (transformers and transmission lines). For the purpose of demonstrating the proposed methodology without undue complexity, we use a simplified power network model as shown in Figure 1(a). The simplified network consists of 48 nodes (thermal power plants and substations) and 59 links (transmission lines). The arrows indicate the directivity of the electric current flow, from source to customers, and from high voltage substations to low voltage ones. The 19 thermal power plants are considered the only supply facilities or source nodes for the given power network. Power transmission lines are considered distributing elements while substations as demand nodes. Although the network only contains 107 elements, a multi-scale approach (Der Kiureghian and Song, 2008) can be used to assess the system reliability by aggregating or disaggregating service areas into equivalent service nodes.

The system reliability of the power grids is evaluated under two hypothetical scenarios: an intentional disruption and an earthquake scenario. The system performance of the target power grids is evaluated by analyzing the

characteristic length of the state transition graphs. Since the state transition graph is the enumeration of all possible states of the system and not feasible for practical uses for large scale network, the characteristic lengths of the state transition graph are employed to analyze the power grid's performance.

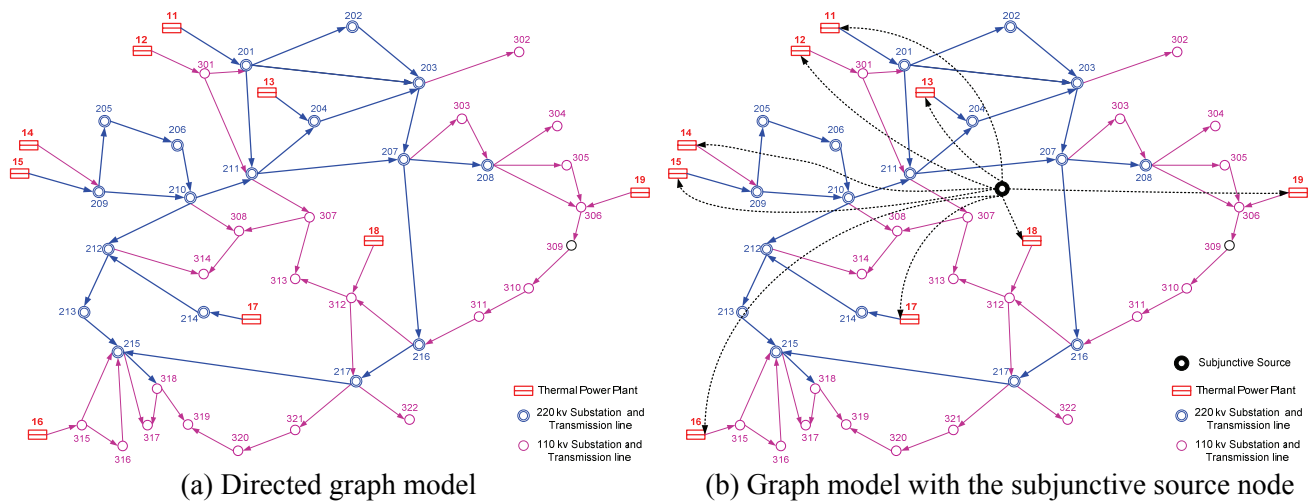


Figure 1 Graph model of HPGC power grid

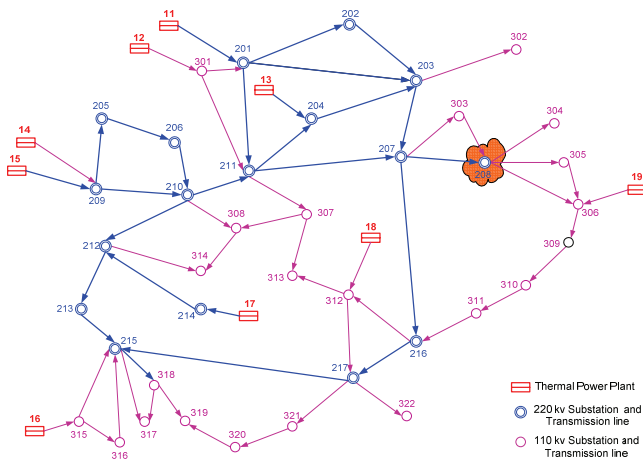
4.1 Scenario 1: Intentional Disruption on the Power System

In this scenario, we assume that consequences of the disruption on the specific node are substantial so that the node and its adjacent edges will quit from the power system immediately after the attack, which is equivalent to deleting the node and its adjacent edges from the network by changing the network's topology.

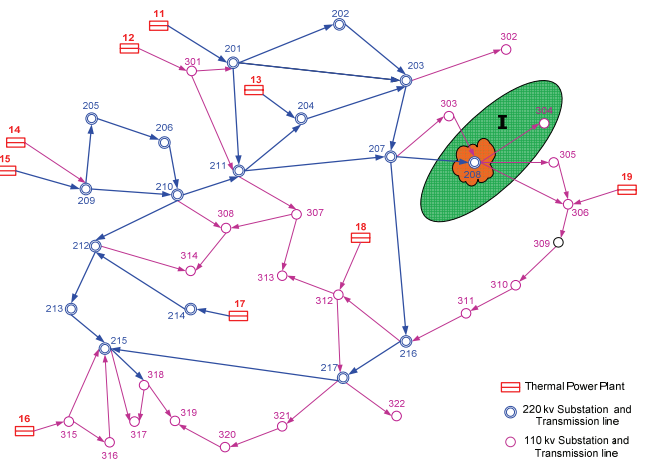
Under the intentional disruption, a 220 kV substation (208) is attacked in a simulated intentional disruption. The attack results in the immediate failure of the Substation 208, causing a series of power system cascading failures in east part of the study area. The three-step cascading failure propagated on the power network and changed the network structure as shown in Figure 2. The first step is that Substation 304 quitted working from the network due to the failure of Substation 208. Subsequently, the Power Plant 19 and three 110 kV substations (305, 306, and 309) failed and service regions covered by these facilities experienced a power outage. In the last step of the cascading failure, substations 310 and 311 stopped operating since the only power source (substation 306) feeding these two substations failed in the second step of cascading failure. Redistribution of power flow with the changed network topology is calculated in each step of the cascading failures with BPA flow analysis software. Power flow over the power grid reached a stable state and the system equilibrium equations reached convergent solutions after a three-step cascading failure.

To apply RDA to the HPGC power network, a subjunctive source node feeding all power plants is added to the graph to facilitate finding the node-pair connectivity in the multi-source network. The network in Figure 1(a) is then represented by the 49-node and 68-arc network as shown in Figure 1(b). Different electrical equipments in the power grids have various operating reliabilities. And even for the same type of equipment, the reliability may not be identical. For demonstration purpose the reliability of electrical equipments is assumed identical (0.95) in this numerical example. When accurate information of equipment reliabilities is available later, it can be used to get refined results.

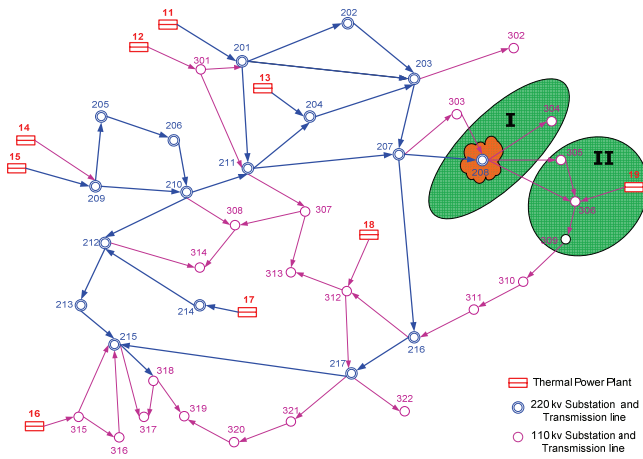
The event (**c**) and probability (**p**) vectors are obtained for each link set or cut set by use of matrix-based procedures in Equations (3.4) and (3.5). The probability of outage at each distribution node, i.e. its disconnection from all the sources is computed by Equation (3.3). Figure 3 shows the reliability and failure probabilities of the network nodes with system reliability analysis while Figure 4 compares the change of probabilities of outage at several substations during the cascading failures.



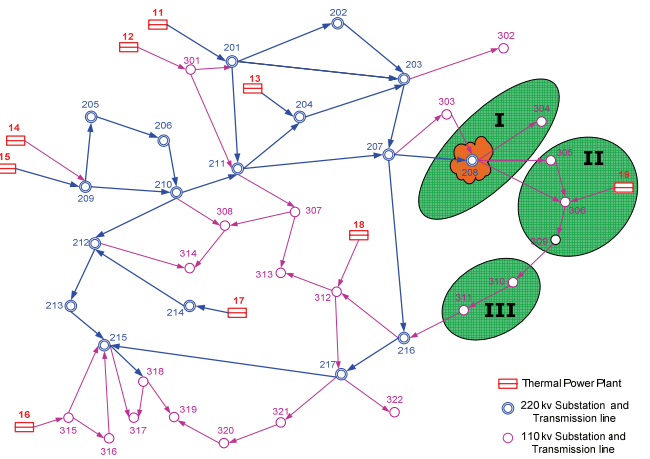
(a) Intentional disruption to Substation 208



(b) Step I: Substation 304 quit the network



(c) Step II: substations 305, 306 and 309 and power plant 19 quit the network



(d) Step III: substations 310 and 311 quit the network

Figure 2 Cascading failure of the HPGC power network under intentional disruption

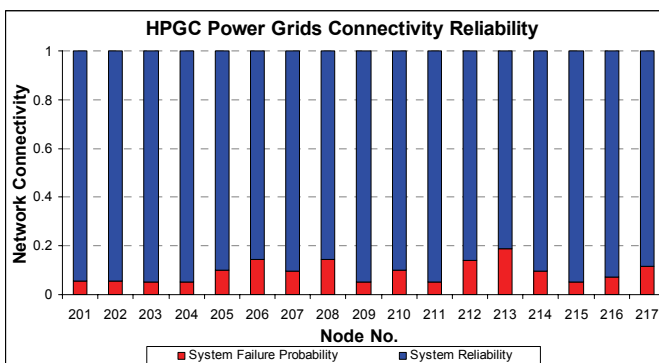


Figure 3 System connectivity reliabilities and failure probabilities of network nodes

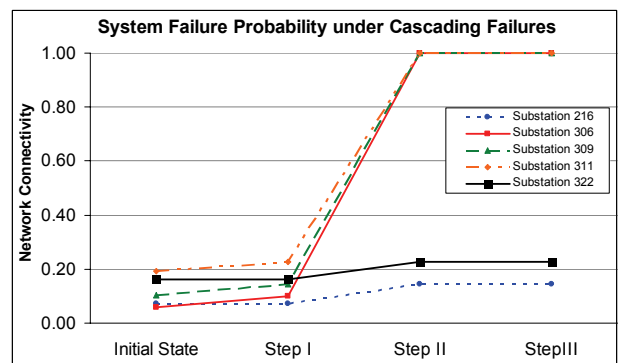


Figure 4 System failure probability under cascading failures

4.2 Scenario 2: Seismic Impact on the Power System

In a hypothetical magnitude 7.0 earthquake scenario, the epicenter is set near the 220 kV Substation 208. The circular attenuation relationship (Wu et al., 2006) was used to calculate the ground motion intensity. As show in Figure 5, we found six sub-stations (208, 303, 304, 305, 306 and 309) locate in high intensity regions (Intensity > 7) and three of them (208, 304, and 305) fail and quit from the network under the hypothetical seismic impact. Converging solution was reached after 5 iterations by changing the parameter of the network

and recalculating the power flow, system is convergent and can operate stably with partially damaged power grid. In this scenario, no cascading failure is observed. Figure 6 compares the probabilities of outage at selected substations before and after the earthquake impact.

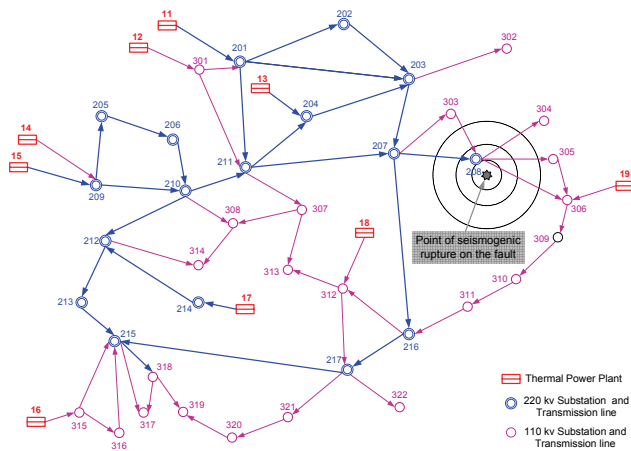


Figure 5 HPGC power network under earthquake impact

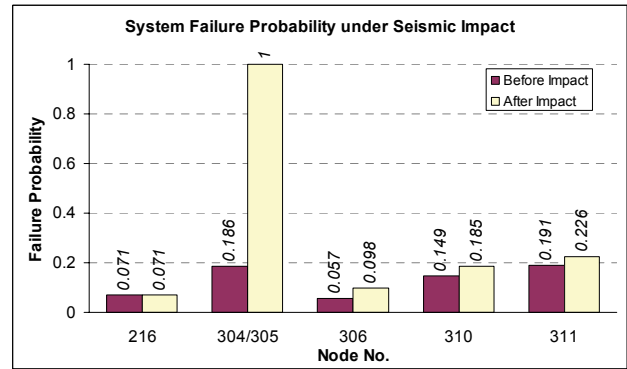


Figure 6 System failure probability under cascading failures

4.3 Performance of the Power Systems

To simulate the cascading failure of power grids, we utilized the 6-step cascading failure simulation algorithm which can capture the state changes from disturbances and illustrates the cascading failure as the state transition graph (Wu et al., 2007). Since there are more than $2^{118+180} \approx 5 \times 10^{89}$ states in the state space, it is infeasible to show all the states in a single state transition graph. However, we can illustrate the global behavior of the state transition graph by characteristic length of the graph, as shown in the following formula:

$$l = \frac{1}{n(n+1)/2} \sum_{i \geq j} d_{ij} \quad (4.1)$$

in which, d_{ij} is the length of the shortest path between node i and node j ; l is the average distance between arbitrary two nodes in the graph, which is used to measure state transition possibility of power grids.

Figure 7 illustrates the characteristic length of state transition graph at various average usage ratios of the HPGC network. Since there are huge amount of disconnected parts in a certain state transition graph, the entire l is somewhat small in the Y axis. However, it is noteworthy that few dots (or states) are connected by the state transfer lines when the power grid has a low utilization ratio (e.g., 0.1). This indicates that the power grid is stable and robust to intentional disturbances.

The characteristic length increases when the power grid operates at higher load levels (or larger utilization ratios), and the state transit graph shows higher degrees of connectivity. This suggests the power grid is prone to cascading failures at higher load levels, which is parallel to what one might deduce from intuitive reasoning. It can be seen from

Figure 7 that the value of characteristic length declines when the load level exceeds a certain critical point. This is the case when the real power system operates near the critical point, most part of the power system crashes directly except for these involved in the cascading failure. From the system point of view, parameters other than the load level (e.g., number of initially failed equipments) may also cause phase transition. In essence, external disturbances, no matter they are man-made or natural, change the states of network. Although it is still not clear whether and how a disturbance can cause cascading failure, the characteristic length of state transition graph could help us better understanding the nature of cascading failures.

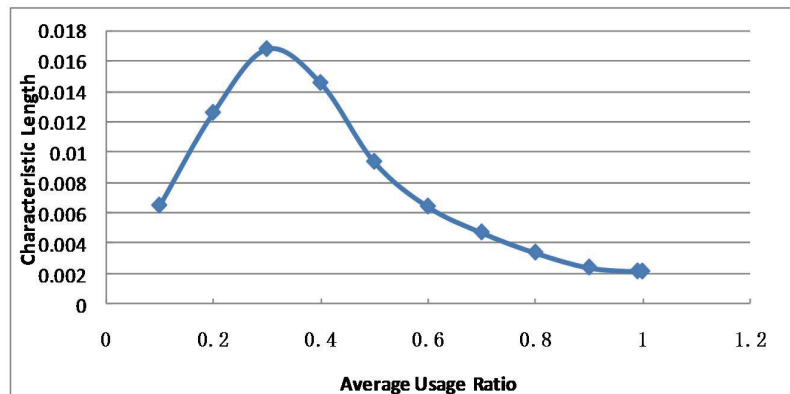


Figure 7 Characteristic length at various average usage ratios of the HPGC power network

5. CONCLUSIONS

This paper analyzed the performance and system reliability of power network under cascading failures. Characteristic length of the state transition graphs is employed to measure the stability performance of power network at various load levels. Outage of distribution nodes are identified as disjoint sets by use of the RDA. The probability of each disjoint set is computed by use of the MSR method. The proposed methodology is demonstrated through a case study of the HPGC power network. The findings are useful for power industries to evaluate risk of blackout by cascading failures, and to identify the vulnerability of power systems and improve the resilience of power systems to external disturbances.

We limit our scope to high voltage transmission network only. Nevertheless, this methodology can be extended to include distribution networks with multi-scale analysis approach. The reliability of overloaded equipments and uncertainties of external disturbances are not considered either. Further research is in progress to incorporate internal (i.e., reliability electrical equipments) and external (i.e., disturbances) uncertainties.

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