Seismic Reliability Analysis and Topology Optimization of Lifeline Networks

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ABSTRACT:

In this paper, in order to analyze seismic reliability of large-scale lifeline networks, the concept of structural function and complementary structural function are introduced. Then, a path-based recursive decomposition algorithm and a cut-based recursive decomposition algorithm are derived to calculate the seismic reliability of lifeline networks. These two algorithms can enumerate the disjoint minimal paths and the disjoint minimal cuts simultaneously. As the result, a probabilistic inequality can be used to give the approximate reliability. Based on seismic reliability analysis of lifeline networks, a topology optimization model is established. The goal of the model is to find the least-cost network which satisfies specified seismic reliability. Combining with the path-based recursive decomposition algorithm, three optimization algorithms, genetic algorithm, simulated annealing algorithm and simulated annealing genetic algorithm, are presented to obtain the optimal solutions. Above approaches have been used to a practical gas network.

KEYWORDS: Lifeline Network, Recursive Decomposition Algorithm, Topology Optimization

1. Introduction

The lifeline systems, including water distribution, gas supply and power networks etc, are the arteriae of modern cities. With the development of modern society, lifeline systems play more and more important roles in urban life (Li, 2005). The investigations of many previous earthquakes indicated that the performance of lifeline systems may have important effects on the property losses and casualties of cities during the disasters. In fact, almost all the lifeline systems suffered serious damages during many previous strong earthquakes.

For lifeline networks, just like building structures, seismic analysis and seismic optimization or design are two important research fields. In this paper, the concept of network structural function and complementary structural function are introduced. By taking of the recursive decomposition of structural function and complementary structural function, a path-based recursive decomposition algorithm and a cut-based recursive decomposition algorithm are deduced to evaluate network seismic reliability. A number of case studies show that these two algorithms form high efficient and accurate methods to calculate the large-scale networks seismic reliability. Based on these technologies, the topology optimization model is further explored. The goal of the model is to find the least-cost network which satisfies specified seismic reliability. Three optimization algorithms, genetic algorithm(GA), simulated annealing algorithm(SAA) and simulated annealing genetic algorithm(SAGA), are developed to get the optimal solutions. An actual gas network is investigated to indicate the validity of the proposed approaches.

2. Seismic Reliability Evaluation of Lifeline Networks

2.1 Structural Function and Complementary Structural Function

A network is a graph with a weight assigned to each edge or node. In a network, the weight denotes the success probability (reliability) of corresponding edge or node. In this paper, only edge-weighted networks, i.e. edges are assigned weights, are considered. However, for the node weighted networks and general weighted networks, the proposed algorithm is also available after making small changes.

For an edge weighted network, each edge can be in either of two states, operative state or failed state. Therefore

the network also owns two states, operative state or failed state. By introducing Boolean algebra, the operative state and failed state are represented by 1 and 0 respectively. The terminal nodes are denoted as source and sink. Then network structural function is represented by $\Phi(G)$ defined as follows

$$\Phi(G)=\begin{cases}1 & \text{if network operates}\\ 0 & \text{if network fails}\end{cases} \tag{2.1}$$

When all edges of any path of the network are in operative state, the network operates. View network structural function and all edges in the network as boolean variants, the network structural function can be written as

$$\Phi(G)=\bigcup_{k=1}^{m}A_k \tag{2.2}$$

where m is the number of minimal paths (MPs) of G and A_k is a MP of G.

Correspondingly, a complementary structural function of network can be defined as follows

$$\Phi'(G)=1-\Phi(G)=\begin{cases}1 & \text{if network fails}\\ 0 & \text{if network operates}\end{cases} \tag{2.3}$$

Similarly, the complementary structural function can be expressed as the union of all minimal cuts (MCs)

$$\Phi'(G)=\bigcup_{k=1}^{m'}D_k \tag{2.4}$$

where m' are the number of MCs of G and D_k represent a MC of G.

2.2 Path-based Recursive Decomposition Algorithm

In order to simplify the calculation, define a shortest MP from the source to the sink as $A_1=a_{11}a_{12}\cdots a_{1m_1}$, where a_{1i} is an edge of the network and m_1 is the number of edges in A_1. From A_1, according to Boolean laws and De Morgan's Law, the network structural function $\Phi(G)$ becomes

$$\begin{aligned}\Phi(G)=A_1+\bar{a}_{11}\Phi(G_{11})+a_{11}\bar{a}_{12}\Phi(G_{12})+\cdots\\ +a_{11}a_{12}\cdots a_{1i-1}\bar{a}_{1i}\Phi(G_{1i})+\cdots+a_{11}a_{12}\cdots\bar{a}_{1m_1}\Phi(G_{1m_1})\end{aligned} \tag{2.5}$$

where G_{1i} represents a subgraph after removing edge a_{1i} from G and merging the nodes connecting with the edge $a_{1j}(j=1,2,...i-1)$ into the source in sequence.

Above subgraphs can be classified into connected subgraphs and disconnected subgraphs. If taking the number of connected subgraphs as m_{1p}, then the number of disconnected subgraphs is $m_{1c}=m_1-m_{1p}$. Meantime, if G_{1i} is a disconnected subgraph, the coefficients in front of $\Phi(G_{1i})$ forms a disjoint minimal cut (DMC) of G. Let $C_{1j}=a_{11}a_{12}\cdots a_{1i-1}\bar{a}_{1i}$, then complementary structural function can be written as

$$\Phi'(G)=\sum_{j=1}^{m_{1c}}C_{1j}+Q_1 \tag{2.6}$$

where Q_1 is the remainder term, denoting that C_{1j} do not form whole DMC set.

As disconnected subgraphs exist no path, their structural functions equal to 0 and $\Phi(G)$ can be rewritten as

$$\Phi(G) = A_1 + \sum_{i=1}^{m_{1p}} B_{1i}\Phi(G_{1i}) \tag{2.7}$$

where $B_{1i} = a_{11}a_{12}\cdots a_{1i-1}\bar{a}_{1i}$ and G_{1i} denotes the connected subgraph.

Decompose above connected subgraphs step by step until no connected subgraph exists, then all disjoint minimal paths (DMPs) will be enumerated. Finally $\Phi(G)$ can be expressed as

$$\Phi(G) = \sum_{i=1}^{M} L_i \tag{2.8}$$

where L_i is a DMP and M is the number of all DMPs.

At the same time, all DMCs are also enumerated, that is

$$\Phi'(G) = \sum_{j=1}^{K} C_j \tag{2.9}$$

where C_j is a DMC and K is the number of all DMCs.

After enumerating all DMPs and DMCs, the reliability and the failure probability of the network can be deduced as

$$R = P[\Phi(G) = 1] = \sum_{i=1}^{M} P(L_i = 1) \tag{2.10}$$

$$F = P[\Phi'(G) = 1] = \sum_{j=1}^{K} P(C_j = 1) \tag{2.11}$$

For a large network, it might be impossible to enumerate all disjoint products. In these cases, the upper bound and the lower bound of the network reliability can be calculated using the probabilistic inequality (Li, 2005)

$$\sum_{i=1}^{M_s} P(L_i = 1) \le R \le 1 - \sum_{j=1}^{K_f} P(C_j = 1) \tag{2.12}$$

where M_s and K_f are the number of calculated DMPs and DMCs, $M_s \le M$ and $K_f \le K$.

When the difference between the upper bound and the lower bound is smaller than a specified error bound, the approximate network reliability can be given as

$$R = 0.5\left(1 - \sum_{j=1}^{K_f} P(C_j = 1) + \sum_{i=1}^{M_s} P(L_i = 1)\right) \tag{2.13}$$

2.3 Cut-based Recursive Decomposition Algorithm

Similar to the path-based recursive decomposition algorithm, define a MC of G as $D_1 = \bar{a}_{11}\bar{a}_{12}\cdots\bar{a}_{1m_1}$. Using De Morgan's Law and Boolean laws, network complementary structural function becomes

$$\Phi'(G) = D_1 + a_{11}\Phi'(G_{11}) + \cdots + \bar{a}_{11}\bar{a}_{12}\cdots a_{1m_1}\Phi'(G_{1m_1}) \tag{2.14}$$

where G_{1i} represents a subgraph after removing edge $a_{1j}(j=1,2,...i-1)$ from G and merging the nodes connecting with the edge a_{1i} into the source.

Above subgraphs can be classified into absorbing subgraphs (the subgraph that sink is merged into source) and non-absorbing subgraphs. For each absorbing subgraph, the coefficients in front of its complementary structural function forms a DMP of network. For non-absorbing subgraphs, decomposing them repeatedly until no non-absorbing subgraph exists, then all DMCs will be enumerated. Meantime, all DMPs are also enumerated. Correspondingly, the probabilistic inequality can be used to calculate the approximate value.

3. Seismic Reliability Optimization of Lifeline Network

3.1 Optimization Model

Apparently, the seismic reliability of the lifeline network systems is determined by its edge seismic reliability and its topology. Usually, the strategies to improve edge reliability include using ductile pipeline materials and adopting larger diameter pipelines and so on. But these methods may be not suitable for existing lifeline networks because some pipelines have to be discarded before they are out of service. So modifying the network topology by adding several edges to or removing several edges from the network is a feasible way to improve the network seismic reliability.

The network topology can be set as an optimization model. As its optimization object is actually to find the least-cost network topology structures which satisfy prescribed seismic reliability constraint, the optimization model can be mathematically formulated in the following general form

$$\begin{aligned} &\text{minimize} \quad C(G^*) = \sum \gamma_j c_j \\ &\text{subject to} \quad P_k \geq P_0 \quad k = 1,2,...,n \\ &G^* \text{ is a subgraph of } G \end{aligned} \tag{3.1}$$

where G represents a network and is usually generated empirically, G^* is a solution of the model, γ_j takes value of 1 if edge *j* exists in G^* and 0 inversely, P_k represents the seismic reliability between sources and terminal *k* and can be calculated using above path-based recursive decomposition algorithm, P_0 represents the reliability constraint and c_j represents the cost of edge *j* and can be evaluated in an actual lifeline network.

Obviously, above problem is a typical combinatorial optimization problem in which γ_j is the optimization variable. Considering a network with 60 edges and 30 nodes, the number of all potential networks is $\sum_{i=29}^{60} C_{60}^{i} \approx 7.5\times10^{17}$. Assuming to use a computer that can deal with 100 networks a second, it will take about 2.38×10^{8} years to calculate all feasible networks. It is impossible in practice. Therefore, the modern optimal algorithms, such as GA, SAA and SAGA, should be introduced to solve this optimization model.

3.2 Genetic algorithm

GA was pioneered by J. H. Holland during the mid-1970s in the field of machine learning (Holland, 1975). It is a method of searching solutions in the solution space by imitating the natural selection process (Holland, 1991).

GA has been widely used in various optimization tasks, including numerical optimization, combinatorial optimization problems such as knapsack problem and airline crew scheduling problem (Xuan et al, 2004). In order to use GA in the topology optimal problem of lifeline networks, a generation including many genes is created initially where each gene represents a network. Then by using selection, crossover and mutation operators, a new generation is evolved. The fitness of each gene determines whether it will survive or not. After a number of iterations or when some criteria are met, a near-global optimal solution could usually be found.

3.2.1 Representing graphs as genes

Manipulating a network with GA requires that the network is properly represented. Note that any gene in GA is a subgraph of original *G*. The simplest 0-1 binary coding can be adopted here. An *n* bits array is used to represent a graph and each bit represents an edge of the graph *G*, where *n* is the number of edges in *G*. A '1' in the array means that the gene consists of a corresponding edge of *G* while a '0' means not. For example, Fig.1 is a bridge network. In this figure, the original *G* includes all edges while the subgraph doesn't include the edge 5(dash line). Then the corresponding gene of the subgraph can be written as 11110.

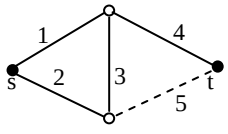

Figure 1 A bridge network

3.2.2 Generating initial graphs

Because all the other generations in the genetic algorithm evolve from the first one, so it is reasonable to hope the first generation contains as many diverse genes as possible. Therefore generating a series of random graphs is the main job in this step. To generate a random graph, a gene is initialized to contain no edges. That means the bits in the array of the gene are all '0's. Then each bit in the array is chosen in turn and changed to '1' at a prescribed probability. In other words, each edge in *G* is added to the graph at random. It must be noted that not all graphs generated from above method are feasible in practice. Disconnected graphs have no practical meanings. Therefore one must first judge the connectivity of each graph in the initial generation and modify the disconnected graphs by adding several edges at random. In order to keep the schema of the genes, only a small number of edges can be added to a disconnected graph. If the graph is still a disconnected graph after modifying, it is discarded and a new gene is generated to replace it.

3.2.3 Evaluating the graphs

At each generation, the genes need to be evaluated to determine their fitness. Since the model is to calculate the least-cost system, evaluation requires calculating the cost of each edge. However, it must be noted that not all genes in the generation are feasible solutions because some of them may not satisfy the reliability constraints. For these genes, a penalty factor, a function of the system seismic reliability, is applied to those unfeasible solutions. The penalty factor can be defined as

$$Q(X)=\begin{cases} 0 & P_{\min i}\geq P_0 \\ \left[\max(P_{\min j})-P_{\min i}\right]\dfrac{C_{\max}-C_{\min}}{\max(P_{\min j})-\min(P_{\min i})} & P_{\min i}<P_0 \end{cases} \tag{3.2}$$

where $P_{\min j}$ represents the minimum nodal seismic reliability of gene *j*, $C_{\max}$ and $C_{\min}$ represent the maximum and minimum cost of the gene in one generation respectively.

The fitness of gene *k* can be written as

$$T(k)=S-C(k)-Q(k) \tag{3.3}$$

where *S* is a large value and *C(k)* represents the cost of gene *k*.

3.2.4 GA operators
There are three operators, selection operator, crossover operator and mutation operator, to be used on the genes in current generation to produce the genes in the next generation.

The selection operator is a core operator of GA. It selects the superior genes, individuals with high fitness, at a high probability and inferior genes, individuals with low fitness, at a low probability and passes them to next generation. Although many selection operators are available, roulette wheel selection operator and elitist selection operator (Chen et al, 1996) are adopted here.

The crossover operator is an operator which guarantees the global searching capability of GA. This operator takes two genes at a prescribed probability and produces two offsprings. In this paper, one-point crossover operator (Chen et al, 1996) is adopted for producing new genes.

The mutation operator is used to guarantee the local searching capability of GA by perturbing the genes generated from crossover operator. The process of this operator is very simple, selecting a gene and changing each bit of gene into 0 if it was 1 and 1 if it was 0 at a prescribed mutation probability.

3.2.5 Stopping criteria
In this paper, the algorithm stops when the number of iterations reaches to maximum iteration number.

3.3 Simulated annealing algorithm
SAA was first introduced by Kirkpatrick et al (1983) and independently by Cerny (1985) as a problem-independent combinatorial optimization technique. SAA has been applied to a wide range of difficult combinatorial optimization problems, such as traveling salesman problems (Aarts et al, 1998), large-scale integration computer-aided design (Wong et al, 1988), computer communication networks design (Samuel et al, 1995) and so on.

SAA is a search procedure in which the current solution is continually compared to solutions which are obtained by carrying out a perturbation. The perturbation result is accepted at a probability described as followings:

$$P(i \Rightarrow j) = \begin{cases} 1 & f(j) \le f(i) \\ \exp([f(i) - f(j)]/t) & f(j) > f(i) \end{cases} \qquad (3.4)$$

where $f(i)$ is energy function of solution *i* determined by its cost and seismic reliability and *t* is current temperature, a control parameter which decreases as the process of SAA goes on and approaches 0 at last.

Apparently if the perturbation result is an improved solution, it is accepted and the current solution is updated accordingly. Otherwise, it can also be accepted at a probability described in Eq.(3.4). By accepting a worsening solution, SAA avoids being trapped too early in a local optimal solution. On the other hand, the probability of accepting a worsening perturbation solution decreases because *t* decreases as the process of SAA goes on, which guarantees the algorithm will eventually converge and be less likely to move away from a global optimal solution after having approached it.

For the network topology optimization problem, a solution is represented as the same as the gene in GA and the process of SAA can be described as followings.
①Produce an initial solution using the same method as producing the genes in initial generation of GA;
②Determine current temperature *t* based on the initial temperature *T* and cooling schedule. If the current

temperature is lower than the terminal temperature, stop.
③Perturb current solution and generate a new solution. Calculate its energy function and determine the accepting probability of the new solution.
④Generate a number varied from 0 to 1 at random and compare it with above accepting probability. If the random number is smaller than the accepting probability, the new solution is accepted and the current solution is updated. Otherwise, the new solution is discarded and current solution is preserved.
⑤Judge whether the number of perturbations has reached prescribed value or not at current temperature, if yes, go to step ②, or go to step ③.

3.4 Simulated annealing genetic algorithm

Although GA is a very useful algorithm for combinatorial optimization problem, it has a major limitation that premature convergence will occur when the genetic algorithm cannot find the optimal solution due to loss of some important characters. The reason is that GA depends heavily on crossover operator, and the mutation probability is generally too small to move the search into another space. To overcome this problem, many researchers (Ilkwon et al, 1996; Yu et al, 2000) notice that SAA is good at widening the searching space and its operator is very similar with the mutation operator in GA. So a hybrid algorithm which replaces the mutation operator in GA with the perturbation in SAA at the same temperature is developed. In other words, each gene generated by crossover operator will be perturbed to produce several new solutions and be updated according to the accepting probability in Eq.(3.4). Herein the initial temperature is used for the first generation and the temperature doesn't change in one generation but decreases in next generation.

4. Case Study

A gas network locatedin a City of China, which consists of 463 nodes and 977 pipelines, is investigated in this paper. Using path-based recursive decomposition algorithm, the seismic reliability of network is calculated and the result is shown in Fig.2. Just for such a complexity network, path-based recursive decomposition algorithm can give the results accurately and quickly. Also, let P_0 equal to 0.8, GA and SAGA have been used to solve the seismic topology optimization problem of the gas network and the result is shown in table 4.1. Also the result calculated by GA is shown in Fig.3. Apparently, after optimization, the network seismic reliability of the network increase much more, all nodes are in the state of reliable (reliability is between 0.9 and 1) or lightly damaged (reliability is between 0.7 and 0.9).

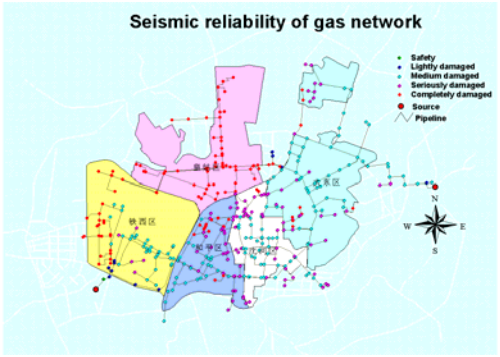

Figure 2 Seismic reliability of the network

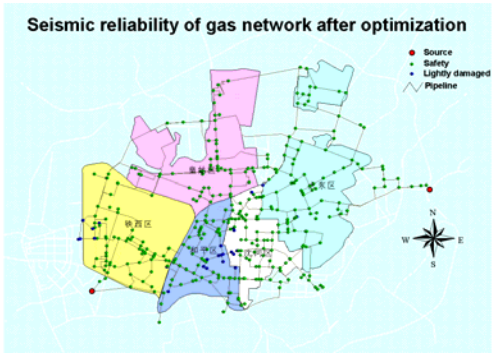

Figure 3 Seismic reliability of the network after optimization

Table 4.1 Comparison of GA and SAGA for the actual network

Algorithm	Cost
GA	￥573,013,000 ($74,417,272)
SAGA	￥570,820,000 ($74,132,467)

5. Conclusion

Seismic reliability analysis and optimal design of lifeline systems is important for modern cities that located on earthquake areas. Based on structural function and complementary structural function, several systematical recursive decomposition algorithms were developed to give the seismic reliability of lifeline networks. Combining with these algorithms, different optimal algorithms, genetic algorithm, simulated annealing algorithm and simulated annealing genetic algorithm, are explored to solve the topology optimization problems of lifeline systems. Case study indicates that above approaches provide an efficient route on seismic reliability analysis and optimization of lifeline networks.

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